**Math 1426** **Fall 2003**

**Final Exam**

1. A particle moves along the circle with equation $x^2 + y^2 = 25$ in such a way that, when it is at the point (3,4), its $x$-coordinate is increasing at the rate of 5 units per second. How fast is the $y$-coordinate changing at that instant?
(A) 5 (B) $-\frac{5}{3}$ (C) $\frac{5}{3}$ (D) $\frac{15}{4}$ (E) $-\frac{15}{4}$

2. In a healthy person of height $x$ inches, the average pulse rate (in beats per minute) is modeled by the formula $p(x) = \dfrac{596}{\sqrt{x}}$, $30 \le x \le 100$. Estimate the change in pulse rate that corresponds to a height change from 59 in to 60 in.
(A) $\dfrac{-596}{2\sqrt{(59)^3}}$ (B) $\dfrac{-596}{\sqrt{59}}$ (C) $\dfrac{-596}{2\sqrt{59}}$ (D) $\dfrac{-596}{\left(\sqrt{59}\right)^3}$ (E) $-596\sqrt{59}$

3. If $\dfrac{dy}{dx} = x^2\sqrt{x^3}$, $y = 1$ when $x = 0$, find $y$.
(A) $\frac{7}{2}x^{5/2} + C$ (B) $\frac{2}{9}x^{9/2} + C$ (C) $\frac{7}{2}x^{5/2} + 1$ (D) $\frac{2}{9}x^{9/2} + 1$
(E) $\frac{1}{6}x^6 + 1$

4. Find an equation of the curve that satisfies the following conditions: at each point $(x, y)$ on the curve, the slope of the tangent line is $2x + 1$; the curve passes through the point $(-3, 0)$.
(A) $y = x^2 + x + 6$ (B) $y = x^2 + x - 6$ (C) $y = 2x^2 + 3x - 9$ (D) $y = 2x + 6$
(E) $y = -x^2 - x + 6$

5. Find an equation for the line perpendicular to $f(x) = 2\cos x + 3\sin x$ at the point $\left(\frac{\pi}{2}, 3\right)$.
(A) $2x - 4y = \pi - 12$ (B) $2x + y = \pi + 3$ (C) $4x - 2y = \pi - 6$
(D) $4x + 2y = \pi - 12$ (E) $2x + 4y = 2\pi + 3$

6. An object is traveling along a straight line with position at time $t$ given by $s(t) = 2t^3 - 21t^2 + 60t$. Find the distance traveled over the time interval [0,7].
(A) 107 (B) 77 (C) 97 (D) 131 (E) 119

7. If $y = f(u)$ is differentiable for all values of $u$, $\frac{dy}{du} = 6$ when $x = 1$, and $u^3 = x^3 + x + 6$, find $\frac{dy}{dx}$ at $x = 1$.

(A) $\frac{7}{3}$ (B) $\frac{2}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 3

8. Find the slope of the tangent to the graph of $\cos(xy) = x^2 - 1$ at the point $\left(1, \frac{\pi}{2}\right)$.

(A) $-\pi - 1$ (B) $\pi - 1$ (C) $-\frac{\pi}{2} + 1$ (D) $-\frac{\pi}{2} - 1$ (E) $-\frac{\pi + 4}{2}$

9. If $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 4 - x & \text{if } 0 \le x < 4 \\ (x-4)^2 & \text{if } x > 4 \end{cases}$, then $\lim_{x \to 0} f(x)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) does not exist

(E) not enough information given

10. If $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 4 - x & \text{if } 0 \le x < 4 \\ (x-4)^2 & \text{if } x > 4 \end{cases}$ then $\lim_{x \to 4^-} f(x)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) does not exist

(E) not enough information given

11. The function $f(x) = \frac{x-2}{x-2}$ is discontinuous

(A) nowhere (B) at 0 only (C) at $\frac{1}{2}$ only (D) at 2 only (E) at 0 and 2 only

12. If $g(x) = \begin{cases} 2x + 1 & \text{if } x \le -1 \\ 3x & \text{if } -1 < x < 1 \\ 2x - 1 & \text{if } x \ge 1 \end{cases}$, then $g$ is discontinuous at

(A) 0 only (B) 1 only (C) $-1$ and 1 only (D) no point of the domain
(E) every point of the domain

13. Constants $a$ and $b$ are chosen such that the function $f(x)=\begin{cases}\frac{ax-6}{x-2} & \text{if } x\neq 2\\ b & \text{if } x=2\end{cases}$ is continuous for all $x\in\mathbf{R}$. The product $ab$ is

(A) 3 (B) 2 (C) 6 (D) 9 (E) undefined

14. If $f(t)=\begin{cases}2+(t-1)^2\cos\left(\frac{1}{t-1}\right) & \text{if } t\neq 1\\ 2 & \text{if } t=1\end{cases}$, then $f'(1)$ is

(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) 2 (E) does not exist

15. Interpret $\lim_{h\to 0}\left(\dfrac{\cos h-1}{h}\right)$ as a derivative.

(A) $\dfrac{d}{dx}(\cos 0)$ (B) $\cos' x$ (C) $\cos'(h)$ (D) $\cos'(1)$ (E) $\cos'(0)$

16. Suppose $f$ is a function with the property that $f'(x)=\cos\left(x^2\right)$. Find $g'(x)$, where $g(x)=f(x^3)$.

(A) $g'(x)=3x^2\cos\left(x^6\right)$ (B) $g'(x)=\cos\left(x^6\right)$ (C) $g'(x)=\sin\left(x^6\right)$

(D) undefined (E) none of these

17. Suppose we know that a function $g$ has derivative $g'(x)=\sqrt{x^2+16}$ for all $x$ and that $g(3)=-2$. Use a tangent-line approximation to estimate the value of $g(3.05)$.

(A) $-2.01$ (B) $-1.75$ (C) $-1.95$ (D) $-1.9$ (E) none of these

18. If $\int_2^3 f(x)\,dx=7$ and $\int_2^1 f(x)\,dx=3$, what is $\int_1^3 f(x)\,dx$?

(A) $-4$ (B) 10 (C) 4 (D) $-10$ (E) none of these

19. The value of $\dfrac{d}{dx}\int_x^1 e^{-t^2}\,dt$ at $x=3$ is

(A) $e^9$ (B) $-e^9$ (C) $\dfrac{1}{e^9}$ (D) $-\dfrac{1}{e^9}$ (E) does not exist

20. Give an expression in terms of a limit that gives the area between the graph of $y = x^2$, the *x*-axis, and the lines $x = 1$ and $x = 5$.

(A) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{4k}{n}\right)^2\frac{5}{n}$ (B) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^2\frac{4}{n}$ (C) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^2\frac{5}{n}$

(D) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{4k}{n}\right)^2\frac{5}{n}$ (E) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{4k}{n}\right)^2\frac{4}{n}$

21. Two cars start at the same point at noon. One moves east at the rate of 65 mi/hr. The other moves north at the rate of 75 mi/hr. At what rate is the distance between them changing after 2 hours?

22. Given $f'(x) = \frac{x}{x^2 + a^2}$, $a \neq 0$, and $f(0) = 0$, show that $f(x) = \frac{1}{2}\ln\left(\frac{x^2 + a^2}{a^2}\right)$.

**Use the techniques learned in this class. Finding the derivative of the given *f* will earn 0 credit.**

23. Given that $x^y = y^x$, compute $\frac{dy}{dx}$ at the point (2,2).

24. Sketch the graph of one function *f* with **all** the following properties:

(A) $f'(x) > 0$ for $x < -1$ and $-1 < x < 1$; $f'(x) < 0$ for $1 < x < 3$ and $x > 3$

(B) the graph has only one critical point $(1, -1)$ and no inflection points

(C) $\lim_{x\to-\infty} f(x) = -1$; $\lim_{x\to+\infty} f(x) = 2$; $\lim_{x\to-1^+} f(x) = -\infty$; $\lim_{x\to 3^-} f(x) = -\infty$

25. A Norman window has the shape of a rectangle with a semicircle on top (i.e., the diameter of the semicircle is equal to the width of the rectangle). If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.