PRINT YOUR NAME LEGIBLY AS IT APPEARS ON CLASS ROLL

LAST name:_____ FIRST name:_____

ID NUMBER: XXX–XX– __ __ __ __

CHECK THE APPROPRIATE SECTION

☐ Dr. Epperson Section 204

☐ Dr. Lin Section 101

☐ Dr. Shan Section 103

☐ Dr. Souza Section 506

☐ Dr. Vancliff Section 114

ON YOUR SCANTRON FORM, FILL IN THE TABLE:

NAME	last, first		
SUBJECT	MATH 1426-____	TEST NO.	VERS A

TURN OFF ALL CELL PHONES AND BEEPERS

& PUT THEM OUT OF SIGHT

DO NOT WRITE BELOW THIS LINE — DO NOT START UNTIL SO INSTRUCTED

	Points Earned
Part I (45 points)	$\cdot 3 =$
16 (11 points)	
17 (10 points)	
18 (11 points)	
19 (11 points)	
20 (12 points)	
PART II (55 points)	
TOTAL SCORE (100 points)	

SUMMATION FORMULAE

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

The square brackets following an exam-question number refer to a section/problem number in the text or a lab worksheet. Problem numbers preceded by the symbol $\sim$ are modeled on that problem from the text or lab, but are not identical to it; problem numbers without the symbol are identical to, or very close to, the problem from the text or lab.

INSTRUCTIONS FOR PART I Write your answers for these questions on a scantron form (882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 3 points each (no partial credit), for a total possible score of 45 points. You may use an approved calculator. You may write on this exam or request scratch paper, if needed.

1. [§5.3:$\sim$27] If $\int_0^2 f(x)\,dx = -1$ and $\int_0^7 f(x)\,dx = 7$, then what is $\int_2^7 f(x)\,dx$?

   (a) 8 (b) -8 (c) 6 (d) -6 (e) 0.

2. [§5.3] If $\int_{-2}^4 f(x)\,dx = 3$ and $\int_{-2}^4 g(x)\,dx = -2$, then what is $\int_{-2}^4 (3f(x) - 2g(x))\,dx$?

   (a) -5 (b) 5 (c) 13 (d) 7 (e) 1.

3. [§3.1: $\sim$13-15] If $f(x) = 3x^2 + 2$, then the difference quotient $\dfrac{f(5+\Delta x) - f(5)}{\Delta x}$ equals

   (a) $10 + \Delta x$ (b) $\dfrac{30\Delta x + 3(\Delta x)^2 + 4}{\Delta x}$ (c) $3\Delta x$ (d) $\dfrac{3(\Delta x)^2 + 4}{\Delta x}$ (e) $3(10 + \Delta x)$.

4. [§3.5: $\sim$20, $\sim$50, $\sim$51] Suppose that f is a differentiable function on $\mathbb{R}$, and that $f(1) = 0$, $f'(1) = 5$ and $f''(1) = 7$. Find the derivative of $e^{f(x)}$ at $x = 1$.

   (a) $5e$ (b) 5 (c) 0 (d) $7e^5$ (e) 7.

5. [§5.5: $\sim$20] When using integration by substitution to find $\int \sin^3 t \cos t\,dt$, which of the following substitutions WILL help directly?

   (a) $u = \sin^3 t \cos t$ (b) $u = \sin t \cos t$ (c) $u = \cos t$ (d) $u = \sin t$ (e) $u = \sin^2 t \cos t$.

6. [§4.6:$\sim$4] A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. If no fencing is needed along the river, find the dimensions that would require the least amount of fencing, and state the value of the dimension that is perpendicular to the river.

   (a) 290 meters (b) 300 meters (c) 310 meters (d) 400 meters (e) $300\sqrt{2}$ meters.

7. [§5.1:∼Example 6] Find an equation for the curve that satisfies the following conditions: (i) at each point (x, y) on the curve, the tangent line has slope $2x+1$, and (ii) the curve passes through the point $(-3, 0)$.

(a) $y = x^2 - x - 12$ (b) $y = 2x^2 + 6x$ (c) $y = x^2 + x - 6$

(d) $y = -2x^2 - 6x$ (e) $y = -x^2 - x + 6$.

8. [§5.2:∼9] Find $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{3k}{n^2}$. (Recall that certain formulae are given on Page 1).

(a) 3 (b) $\frac{1}{2}$ (c) 0 (d) $\frac{3}{2}$ (e) $+\infty$.

9. [§5.4:∼6] Evaluate $\int_{-1}^{1} (x^3 + bx^2)\, dx$, where b is a nonzero constant.

(a) $\frac{2b}{3}$ (b) $6b$ (c) $\frac{b^2}{3}$ (d) $12b^2$ (e) 0.

10. [§5.4:∼42] Given $F(x) = \int_{x}^{2} \frac{e^{3t}}{t}\, dt$, find $F'(x)$.

(a) $-\frac{3e^{3x}}{x}$ (b) $-\frac{e^{3x}}{x}$ (c) $\frac{e^{3x}}{x}$ (d) $\frac{3e^{3x}}{x}$ (e) $\frac{e^{3x}(3x-1)}{x^2}$.

11. [§4.5:∼27] Evaluate $\lim_{x\to 0^+} \left(\frac{a}{\sin 2x} - \frac{1}{2x}\right)$, where a is a constant such that $a > 1$.

(a) $\frac{a-1}{2}$ (b) 0 (c) ∞ (d) $\infty - \infty$ (e) $\frac{2}{a-1}$.

12. [§4.5] The following give possible answers to $\lim_{x\to+\infty} \frac{e^x}{x^{20}}$ and reasons for the given answer.

I. $\lim_{x\to+\infty} \frac{e^x}{x^{20}} = \infty$ since exponential functions e^{kx} dominate any power x^n for $k > 0$ and $n > 0$.

II. $\lim_{x\to+\infty} \frac{e^x}{x^{20}} = \infty$ since we observe that it has form $\frac{\infty}{\infty}$ and apply L'Hôpital's rule 20 times and then evaluate the resulting limit.

III. $\lim_{x\to+\infty} \frac{e^x}{x^{20}} = 0$ since any power x^n dominates exponential functions e^{kx} for $k > 0$ and $n > 0$.

IV. $\lim_{x\to+\infty} \frac{e^x}{x^{20}} = 0$ since we observe that it has form $\frac{\infty}{\infty}$ and apply L'Hôpital's rule 20 times and then evaluate the resulting limit.

Which of I-IV are correct?

(a) III only (b) I only (c) IV only (d) III & IV only (e) I & II only.

13. [§4.3] Given the function $f(x) = x \ln x$. Which of the following is true for f?

(a) f has one critical number, is increasing on $(0, \infty)$ and is concave up on $(0, \infty)$.

(b) f has one critical number, is increasing on $(0, \infty)$, has a relative minimum, and is concave up on $(0, \infty)$.

(c) f has one critical number, is increasing on $[1, \infty)$, has a relative minimum, and is concave up on $(0, \infty)$.

(d) f has no critical numbers, is increasing on $(0, \infty)$ and is concave up on $(0, \infty)$.

(e) f has no critical numbers, is increasing on $[1, \infty)$ and is concave up on $(0, \infty)$.

14. [§2.2] If $\lim_{x \to -a} \left(\dfrac{a^2 - x^2}{x + a} \right) = 12$ and $\lim_{x \to 0} \left(\dfrac{b \sin x}{x^2 + ax} \right) = \dfrac{2}{3}$, for nonzero constants a and b, determine the value of $a + b$.

(a) $a + b = 8$ (b) $a + b = 9$ (c) $a + b = 10$ (d) $a + b = \dfrac{20}{3}$ (e) $a + b = 6$.

15. Suppose $g(x) = \begin{cases} 2 - e^{x-1} & \text{if } x \leq 1 \\ e^{2x-2} & \text{if } x > 1 \end{cases}$. Compute $\lim_{\Delta x \to 0^+} \dfrac{g(1 + \Delta x) - g(1)}{\Delta x}$.

(a) e^{-2} (b) 2 (c) 1 (d) 0 (e) does not exist.

INSTRUCTIONS FOR PART II For these questions, you must write down **all** steps in your solutions **as if you do not have a calculator**. Write legibly, and **label any graphs or pictures**. Draw a box around your solution. Partial credit will be given for those parts of your solution that are correct. Total possible score for this part is 55 points.

16. [§5.1] [11 points] Find an equation for the curve $y = f(x)$ that satisfies all the following conditions: (i) at each point (x, y) on the curve, f satisfies the condition $f''(x) = 6x$, and (ii) the line $y = 5 - 3x$ is a tangent line to the curve at the point where $x = 1$.

17. [§5.5:∼27] [10 points] Show that $\int x(x^2-1)^{1/3}\,dx = \frac{3}{8}(x^2-1)^{4/3} + C$, where C is a constant, by using a formal integration technique taught in this course.

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18. [§5.2,5.3] [11 points] Show that the definite integral $\int_1^3 x^2\,dx = \dfrac{26}{3}$ by evaluating it as the limit of a Riemann sum using right-hand endpoints. (Recall that certain formulae are on Page 1.)

19. [§3.6] [11 points] Suppose that f is a function that is differentiable for all real x, and satisfies $f(e) = \ln 7$, $f'(e) = 1 - \dfrac{\ln 7}{e}$. Find $\dfrac{dy}{dx}$ at $x = e$, where $y = x^{f(x)}$.

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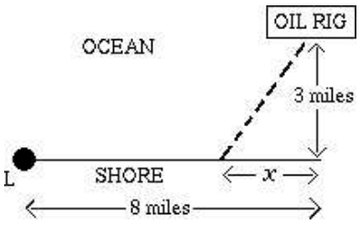
20. [§4.6:∼Example 4] [12 points] Oil from an offshore rig, located 3 miles from the shore, is to be pumped to a location L on the edge of the shore that is 8 miles West of the rig. The cost of constructing a pipe in the ocean from the rig to the shore is 1.5 times as expensive as the cost of construction on land. In order to minimize the cost of construction, what should be the value of the distance x in the picture below?



Have you shown all work in Part II? Write version A on scantron form.

Indicate instructor on front of exam.