

PRINT YOUR NAME LEGIBLY AS IT APPEARS ON CLASS ROLL

LAST name: \_\_\_\_\_ FIRST name: \_\_\_\_\_

ID NUMBER: XXX-XX- \_\_\_\_\_

CHECK THE APPROPRIATE SECTION

- Dr. Epperson Section 204
- Dr. Lin Section 101
- Dr. Shan Section 103
- Dr. Souza Section 506
- Dr. Vancliff Section 114

ON YOUR SCANTRON FORM, FILL IN THE TABLE:

NAME	last,	first	
SUBJECT	MATH 1426-_____	TEST NO.	VERS A

TURN OFF ALL CELL PHONES AND BEEPERS  
& PUT THEM OUT OF SIGHT

DO NOT WRITE BELOW THIS LINE — DO NOT START UNTIL SO INSTRUCTED

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	Points Earned
Part I (48 points)	$\cdot 4 =$
13 (10 points)	
14 (10 points)	
15 (10 points)	
16 (10 points)	
17 (12 points)	$\frac{\quad}{2} =$
PART II (52 points)	
TOTAL SCORE (100 points)	

The square brackets following an exam-question number refer to a section/problem number in the text or a lab worksheet. Problem numbers preceded by the symbol  $\sim$  are modeled on that problem from the text or lab, but are not identical to it; problem numbers without the symbol are identical to, or very close to, the problem from the text or lab.

**INSTRUCTIONS FOR PART I** Write your answers for these questions on a scantron form (882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 4 points each (no partial credit), for a total possible score of 48 points. You may use an approved calculator. You may write on this exam or request scratch paper, if needed.

1. [§3.5,  $\sim$ 30,  $\sim$ 50,  $\sim$ 51] Suppose that  $f$  is a differentiable function on  $(-\infty, \infty)$ , and that  $f(x) + x > 0$  for all  $x$ . Find the derivative of  $\ln(f(x) + x)$ .

(a)  $\ln\left(\frac{1}{f(x) + x}\right)$     (b)  $\frac{1}{f(x) + x}$     (c)  $\frac{f(x) + x}{f'(x) + 1}$     (d)  $\frac{f'(x) + 1}{f(x) + x}$     (e)  $\frac{f'(x) + 1}{f''(x)}$ .

2. [§3.2,3.3,3.5] The function  $f$  and its first derivative,  $f'$ , are defined on  $(-\infty, \infty)$ . If  $f(1) = e$  and  $f'(1) = e + 3$ , find  $h'(1)$ , where  $h(x) = \frac{\ln[f(x)]}{3}$ .

(a)  $\frac{1}{3e}$     (b)  $\frac{4}{3}$     (c)  $\frac{e + 3}{3e}$     (d)  $\frac{e + 1}{e}$     (e) does not exist.

3. [§3.2,3.3,3.5] The functions  $f$  and  $g$  and their first derivatives,  $f'$  and  $g'$ , are defined on  $(-\infty, \infty)$ . Their values at 0 and 1 are given in the table:

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	-1	3	2	6
1	$e$	$e$	$e+3$	$e-1$

If  $H(x) = f(x)g(e^x)$ , compute  $H'(0)$ .

(a)  $e + 1$     (b)  $e + 3$     (c)  $e - 1$     (d)  $3e - 1$     (e) does not exist.

4. [§3.5,  $\sim$ 13,  $\sim$ 50,  $\sim$ 51] Suppose that  $g$  is a differentiable function on  $(-\infty, \infty)$ , and that  $a$  is a number such that  $g(a) = 2 - 5\pi$  and  $g'(a) = -2$ . Let  $F(x) = (g(x) + 5\pi)^3$ . Find  $F'(a)$ .

(a) 36    (b) -24    (c) 12    (d) -12    (e) does not exist.

5. [§3.5,  $\sim$ 17,  $\sim$ 50,  $\sim$ 51] Suppose that  $g$  is a differentiable function on  $(-\infty, \infty)$ , and that  $g(1) = \frac{1}{2}$ ,  $g'(1) = \frac{2}{\pi} - 1$  and  $g'(0) = \pi$ . Let  $h(x) = \sin\left(\pi g(x) + \frac{\pi x^2}{2}\right)$ . Find  $h'(1)$ .

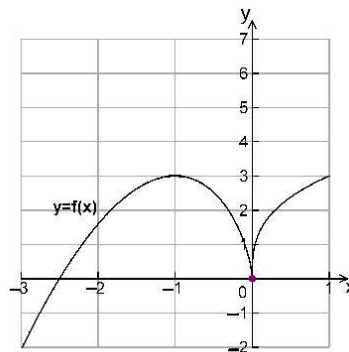
(a) 0    (b) 1    (c)  $\pi \cos 2$     (d)  $-(1 + \pi)$     (e) -2.

6. [§4.1] Find the absolute maximum value of  $f(x) = \begin{cases} 4x - 2 & \text{if } x \leq 1 \\ (x - 2)(x - 3) & \text{if } x > 1 \end{cases}$  on  $[\frac{1}{2}, \frac{7}{2}]$ .

(a) 3    (b) 2    (c) 1    (d) 4    (e) 5.

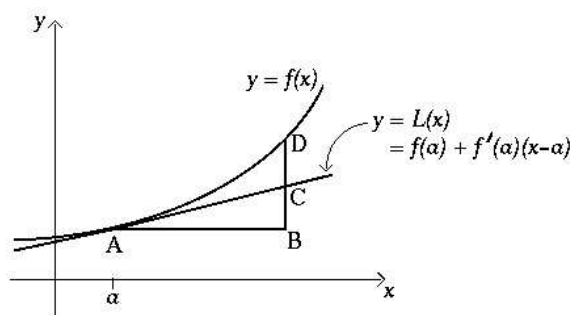
7. [§4.1] Let  $f(x) = x^2 + px + q$ . Find the values of  $p$  and  $q$  such that  $f(1) = 3$  is an extreme value of  $f$  on  $[0, 2]$ ; give the value of  $p + q$ .
- (a)  $-2$     (b)  $6$     (c)  $-6$     (d)  $2$     (e) no such values.
8. [§4.2] If you drive from here to Fort Worth, which theorem implies that your average speed for the whole trip will equal your instantaneous speed attained at some moment(s) during your trip?
- (a) the mean-value theorem    (b) the intermediate-value theorem    (c) the squeeze theorem  
 (d) the extremal-value theorem    (e) the constant-difference theorem.

9. [§4.3] In the following graph of  $y = f(x)$ ,



$f''(-1)$  is

- (a) undefined    (b) positive    (c) negative or zero    (d)  $\infty$     (e) not enough information given.
10. [§2.1,4.1,4.3] Referring to Question 9, determine which one of the following statements is FALSE.
- (a) the origin is an inflection point    (b) the origin is a critical point  
 (c) the origin yields a relative minimum    (d)  $f'(0)$  does not exist    (e)  $\lim_{x \rightarrow 0} f(x)$  exists.
11. [§4.3] Let  $f(x) = x^{\frac{2}{3}}(2x + 5)$ . Which of the following intervals contain an  $x$ -coordinate of an inflection point of  $f$ ?
- (a)  $f$  has no inflection points    (b)  $(-2, 0)$     (c)  $(-\frac{1}{2}, \frac{1}{2})$     (d)  $(0, 1)$     (e)  $(1, \infty)$ .
12. [§3.8] In the following graph of  $y = f(x)$ , which length represents the differential  $dy$  at  $x = a$ ?



- (a) the length between  $A$  &  $B$     (b) the length between  $B$  &  $D$     (c) the length between  $C$  &  $D$   
 (d) the length between  $A$  &  $C$     (e) The length between  $B$  &  $C$ .

**INSTRUCTIONS FOR PART II** For these questions, you must write down all steps in your solutions as if you do not have a calculator. Write legibly, and label any graphs or pictures. Draw a box around your solution. Partial credit will be given for those parts of your solution that are correct. Total possible score for this part is 52 points.

13. [§3.3,3.4,3.5] [10 points] The position of a particle moving along a straight line is given as  $s(t) = \sin\left(\frac{\pi t}{6}\right)$ , where  $s(t)$  is in meters and  $t$  is in seconds.

(a) Find the average velocity of the particle between  $t = 1$  second and  $t = 3$  seconds.

(b) Find the velocity of the particle at time  $t = 3$  seconds, and determine if the particle is accelerating or decelerating at this instant.

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14. [§3.7, ~38] [10 points] A weather balloon is rising vertically at the rate of 10 meters per second. An observer is standing on the ground 300 meters horizontally from the point where the balloon was released.

(a) Sketch/draw a picture of this situation; in the picture, mathematically include items of interest in solving part (b) of this question.

(b) At what rate is the distance between the observer and the balloon changing with respect to time when the balloon is 400 meters high?

15. [§4.1, Example 9] [10 points] A box with a square base is to be built so that the height of the box plus the length of one of the sides of the base is to be 8 meters. Find the EXACT dimensions for such a box that yield the maximal volume. Your solution should be fully detailed and complete.

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16. [§3.1,3.7] [10 points] Find the equation of the tangent line to the function  $y$ , which is a function of  $x$ , defined by the equation  $y + \sin^{-1} x = xy^2 + 3$  at the point  $(0, 3)$ .

17. [§4.3] [12 points] Let  $f(x) = x^3 - 3x^4$  on  $(-\infty, \infty)$ .

(a) Find the first derivative of  $f$ .

(b) Find any critical numbers and critical points.

(c) Find the interval(s), if any, where  $f$  is increasing and the interval(s), if any, where  $f$  is decreasing.

(d) Find the relative extrema (if any) and explain how you arrived at your answer(s).

(e) Find the second derivative of  $f$ .

(f) Find where the second derivative is zero or does not exist.

(g) Find the interval(s), if any, where  $f$  is concave up and the interval(s), if any, where  $f$  is concave down.

(h) List any inflection points of  $f$  and explain how you arrived at your answer(s).

(i) Sketch the graph of  $f$ .

Have you shown all work in Part II? Write version A on scantron form.

Indicate instructor on front of exam.