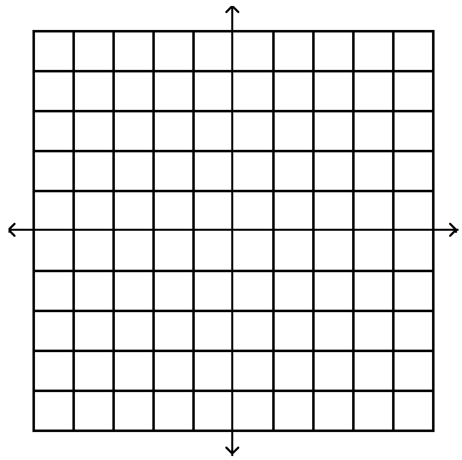
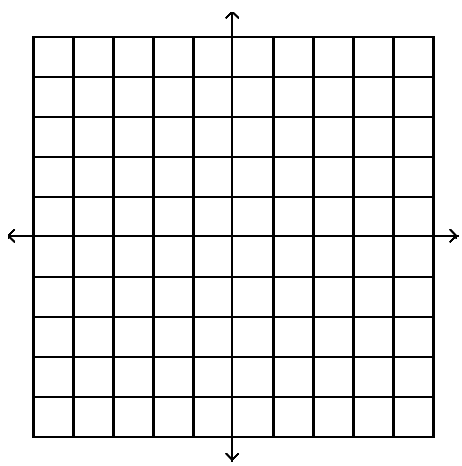
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Sketch the region bounded between the given curves and then find the area of the region.

1) $y = x^2$, $y = x^3$



2) $x = y - y^3$, $x = 0$



Use shells to find the volume of the solid formed by revolving the given region about the y–axis.

3) The region bounded by the curve $y = x^3$, y–axis, and $y = 1$

4) The region bounded above by the graph of $y = 2x - x^2$ and below by the x–axis

Find the volume of the solid formed by revolving the given region about the given line.

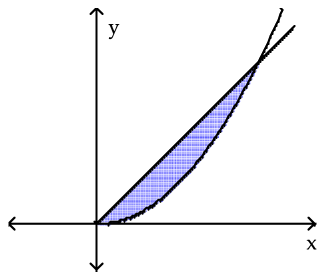
5) The region bounded by the curves $y = x^2$ and $y = 4$ about the line $y = 4$

Solve the problem.

6) Find the volume of the solid generated by revolving the region bounded by $x = y^2$ and $y = x^2$ about (a) the x–axis and (b) the y–axis.

Find the volume of the solid formed by revolving the given region about the given line.

7) The region bounded by the graphs of $y = x^2$ and $y = 2x$ about the line $x = 2$



Find all points of intersection of the curves given.

8) $r = 2$
$r = 2\cos\theta$.

Solve the problem.

9) Find the area enclosed by the graph of the polar equation $r = \sin\theta + \cos\theta$.

10) Find the area enclosed by the graph of the polar equation $r = 6\cos\theta$.

Find the length of the arc of the curve y = f(x) on the interval given.

11) $f(x) = \frac{x^3}{3} + \frac{1}{4x}$, on [1, 2]

Find the surface area generated when the graph of the function given on the prescribed interval is revolved about the x–axis.

12) $f(x) = 1 - x$ on $[-1, 2]$

13) $f(x) = x^3$ on [0, 1]

Find the surface area generated when the graph of the function given on the prescribed interval is revolved about the y–axis.

14) $f(x) = (3x)^{1/3}$ on $[0, \frac{1}{3}]$

Solve the problem.

15) Evaluate $\int \frac{dx}{\sqrt{4 - x^2}}$.

16) Evaluate $\int 2xe^x\, dx$.

17) Evaluate $\int t^3 \cos t\, dt$.

18) Use integration by parts to evaluate $\int xe^x \sin x\, dx$.

19) R is bounded on the left by the y–axis, on the right by the line $x = 1$, above by the curve $y = e^x$ and below by the curve $y = x^2$. Find the volume of the solid generated by revolving R around the y–axis by the method of cylindrical shells.

20) Use trigonometric substitution to evaluate $\int \frac{x}{\sqrt{1-x^2}}\,dx$.

21) Use trigonometric substitution to evaluate $\int \frac{\sqrt{x^2+1}}{x^4}\,dx$.

22) Use trigonometric substitution to evaluate $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$.

23) Use trigonometric substitution to evaluate $\int \frac{1}{x\sqrt{x^2-1}}\,dx$.

24) Evaluate $\int \frac{4}{x^2-1}\,dx$.

25) Evaluate $\int \frac{5x^3+x^2-3x-3}{x^3(x+1)}\,dx$.

26) Evaluate $\int \frac{x^5+1}{x^2}\,dx$.

27) Evaluate $\int x^3\sqrt{x^2+3}\,dx$.

28) Evaluate $\int \frac{x^2+x+1}{x^3-x^2+2x-2}\,dx$.

29) Evaluate $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$.

30) Evaluate $\int \frac{2x^4-1}{(x^2-1)^2}\,dx$.

31) Evaluate $\int_1^3 x \ln x \, dx$?

32) Evaluate $\int_0^2 x^3 e^{2x}\,dx$.

33)

Determine whether $\int_{-1}^{\infty} e^{-x}\,dx$ converges. If it does converge, evaluate the integral.

34)

Determine whether $\int_0^8 \frac{1}{2x\sqrt{x}}\,dx$ converges or diverges. If it does converge, evaluate the integral.

35)

Determine whether $\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx$ converges or diverges. If it converges, evaluate the integral.

36)

Determine whether $\int_3^5 \frac{1}{\sqrt{x^2-9}}\,dx$ converges or diverges. If it converges, evaluate the integral.

37) Find the area between the graph of $f(x) = xe^{-x^2}$ and the x–axis for $x \geq 0$.

38) Find the area between the x–axis and the graph of $y = (x + 1)^{-3}$ for $x \geq 2$.

39) Compute the limit of the convergent sequence $\left\{\left[1 + \frac{4}{n}\right]^n\right\}$.

40) Determine whether the following geometric series converges or diverges. If it converges, find its sum.

$$\sum_{j=1}^{\infty} -\left(\frac{-2}{e}\right)^j$$

41) Determine whether the following geometric series converges or diverges. If it converges, find its sum.

$$\sum_{k=1}^{\infty} 6 \cdot (0.7)^k$$

42) Determine whether the infinite series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{2^n} + \ldots$ converges or diverges. If it converges, find its sum.

43) Determine whether the infinite series $\sum_{n=1}^{\infty} \left(\frac{1}{2n} - \frac{1}{3^n}\right)$ converges or diverges. If it converges, find its sum.

44) Determine whether the infinite series $\sum_{n=0}^{\infty} \left(\frac{81}{80}\right)^n$ converges or diverges. If it converges, find its sum.

45) A ball is dropped from a height of 20 feet. After each bounce, it rises to 80% of its previous height. What is the total distance (up plus down) travelled?

46) Use the integral test to test the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)$ for convergence.

47) Use the integral test to test the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ for convergence.

48) Test the series for convergence: $\sum_{k=1}^{\infty} \frac{2k + 3}{(k^2 + 3k - 2)^4}$.

49) Test the series for convergence: $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$.

50) Test the series for convergence: $\sum_{k=1}^{\infty} k^{(-6/5)}$.

51) Test the series for convergence: $\sum_{k=1}^{\infty} \frac{2 \ln k}{k}$.

52) Test the series for convergence: $\sum_{k=1}^{\infty} \frac{1}{(0.45)^k}$.

53) Test the series for convergence: $\sum_{k=1}^{\infty} 5e^{-3k}$.

54) Test the series for convergence: $\sum_{k=1}^{\infty} \frac{1}{k^2 - 5k - 17}$.

55) Test the series for convergence: $\sum_{k=1}^{\infty} \frac{\left|\sin^2 k\right|}{k^2}$.

56) Test the series for convergence: $\sum_{k=1}^{\infty} \frac{3k}{e^k}$.

57) Test the series for convergence: $\sum_{k=1}^{\infty} \frac{\sqrt{k!}}{3^k}$.

58) Test the series for convergence: $\sum_{k=1}^{\infty} \frac{10^k}{k^k}$.

59) Test for convergence. State what convergence test you use.

(a) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ (b) $\sum_{n=1}^{\infty} 2^{-1/n}$ (c) $\sum_{n=1}^{\infty} \frac{10}{(n+1)^{3/2}}$

60) Determine the values of p for which the series $\sum_{n=1}^{\infty} \frac{1}{(2p)^n}$ converges.

61) Determine whether the series $\sum_{n=1}^{\infty} e^{-n^2}$ converges absolutely, converges conditionally, or diverges.

62) Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \sqrt[n]{n^2}$ converges absolutely, converges conditionally, or diverges.

63) Sum the indicated number of initial terms of the alternating series. Then apply the alternating series remainder estimate to estimate the error in approximating the sum of the series with this partial sum. Finally, approximate the sum of the series, writing precisely the number of decimal places that thereby are guaranteed to be correct after rounding.

$\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!}$, 3 terms

64) Find all x for which the series converges: $\sum_{k=1}^{\infty} \frac{x^{2k}}{k!}$.

65) Find the convergence set for the power series $\sum_{k=1}^{\infty} \sqrt{k+7}\,(2x)^k$.

66) Find the convergence set for the power series $\sum_{k=1}^{\infty} \frac{3^k(x+2)^k}{k!}$.

67) Find the convergence set for the power series $\sum_{k=1}^{\infty} \frac{(x-3)^k}{k}$.

68) Find $\int_0^x f(u)\,du$ by integrating the power series $f(x) = \sum_{k=0}^{\infty} (k+1)^2 x^k$.

69) Find the Maclaurin series for the function $\cos^2 x$.

70) Find the Maclaurin series for the function $\frac{1}{1-2x}$.

71) Write the Maclaurin series for $h(x) = \sin x \cos x$.

72) Find the Taylor series of the function at the indicated point a.

$f(x) = e^{-x}$, $a = 1$

73) Find the first four terms of the Taylor series of the function $f(x) = \sqrt[3]{x}$ at $c = 8$.

74) Find the Maclaurin series for the function $f(x) = \int_0^x e^{-t^3}\,dt$.

The initial point P and the terminal point Q of a vector are given. Write the vector in standard component form and find ||PQ||.

75) P(2, 2), Q(–3, 5)

Find a unit vector that points in the direction of the vector given.

76) $5\mathbf{i} - 12\mathbf{j}$

Solve the problem.

77) Let $\mathbf{u} = \langle 2, 3\rangle$ and $\mathbf{v} = \langle 1, -2\rangle$. Find scalars s and t so that the equation $s\mathbf{u} + t\mathbf{v} = \langle 0, 14\rangle$ is satisfied.

Find all real numbers x and y that satisfy the vector equation given.

78) $x^2\mathbf{i} + 4x\mathbf{j} = (y + 5)\mathbf{i} + 3y\mathbf{j}$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

79) Find the vector **v** given its magnitude and the angle it makes with the positive x–axis: $\|\mathbf{v}\| = 9$, $\alpha = 30°$

A) $\mathbf{v} = 9\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right)$
B) $\mathbf{v} = 9\left(\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}\right)$
C) $\mathbf{v} = 9\left(-\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$
D) $\mathbf{v} = 9\left(\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Determine whether the given points are collinear (that is, lie on a straight line)

80) (0, –5, 1), (1, 10, 4), and (6, 8, –1)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the expression.

81) $\mathbf{u} = -6\mathbf{i} - 7\mathbf{j}$, $\mathbf{v} = 8\mathbf{i} + 2\mathbf{j}$, $\mathbf{w} = -8\mathbf{i} + 2\mathbf{j}$; Find $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$.

A) 34
B) –62
C) –28
D) –24

82) $\mathbf{u} = 8\mathbf{i} + 3\mathbf{j}$, $\mathbf{v} = -3\mathbf{i} - 2\mathbf{j}$; Find $(5\mathbf{u}) \cdot \mathbf{v}$.

A) 80
B) –150
C) 30
D) –125

Find the dot product, u•v.

83) $\mathbf{u} = 10\mathbf{i} + 5\mathbf{j}$; $\mathbf{v} = 9\mathbf{i} - 9\mathbf{j}$

A) 90
B) 135
C) 45
D) –45

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the vector and scalar projections of v onto w.

84) $\mathbf{v} = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$; $\mathbf{w} = 5\mathbf{i} + 12\mathbf{j}$

Solve the problem.

85) Find all x such that the vectors $\mathbf{v} = 13\mathbf{i} - 2\mathbf{j} + x\mathbf{k}$ and $\mathbf{w} = x\mathbf{i} + 5\mathbf{j} + 3x\mathbf{k}$ are orthogonal.

Find v × w for the vectors given.

86) $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

87) $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

Find two different unit vectors, both of which are orthogonal to both v and w.

88) $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{w} = 3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$

Solve the problem.

89) Find a number, t, that guarantees the vectors $2\mathbf{i} + \mathbf{j}$, $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, and $\mathbf{i} + 2\mathbf{j} + t\mathbf{k}$ will all be coplanar.

Find the points of intersection of the line with each of the coordinate planes.

90) $x = 3 + t,\ y = 1 - 3t,\ z = 4t$

Tell whether the two lines intersect, are parallel, are skew, or coincide. If they intersect, give the point of intersection.

91) $\dfrac{x-5}{3} = \dfrac{y-2}{5} = \dfrac{z-3}{2};\ \dfrac{x-4}{-2} = \dfrac{y+6}{3} = \dfrac{z-4}{-3}$

92) $x = -3 + 3t,\ y = 2 + 5t,\ z = 3 - 2t;\ x = 4 - 3t,\ y = 3 - 5t,\ z = 2 + 2t$

Write the equation for the plane that contains the point P and has the normal vector N given.

93) $P(4, -5, 2)$; $\mathbf{N} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$

Find the distance between the point and the plane given.

94) $P(3, -2, 4)$; $2x + 5y - 3z = 7$

Find the distance between the point P to the line given.

95) $P(3, 0, 4)$; $\dfrac{x+1}{3} = \dfrac{y-1}{1} = \dfrac{z-2}{2}$

Find the distance between the lines given.

96) $\dfrac{x-2}{6} = \dfrac{y-5}{1} = \dfrac{z+1}{-2}$ and $\dfrac{x-4}{3} = \dfrac{y+1}{2} = \dfrac{z-2}{2}$

Solve the problem.

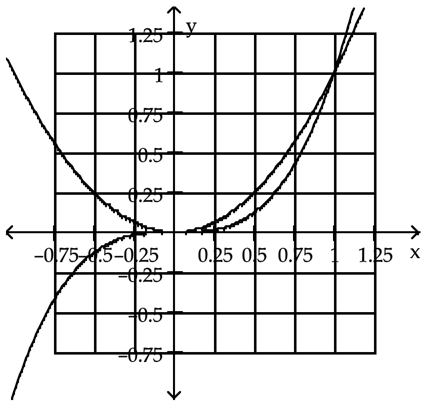
97) Find an equation for the line that passes through the point $P(3, 4, -2)$ and is parallel to the line of intersection between the planes $3x - 5y + 2z = 4$ and $2x + y - 2z = 7$.

98) Find an equation for the line of intersection between the planes $2x - y + 3z = 7$ and $5x + 2y - z = 5$.

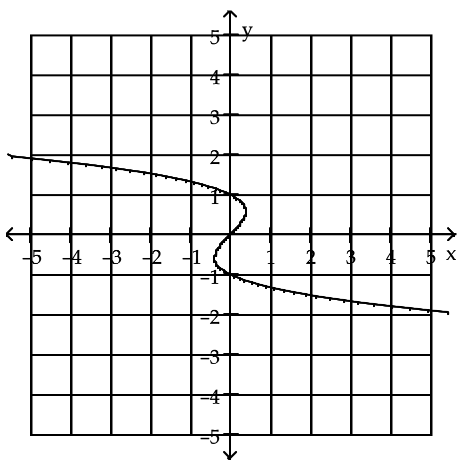
99) Determine whether the line $x = 5 - 4t,\ y = 16 + 6t,\ z = 2 + 5t$ and the plane $4x + y + 2z = 10$ intersect or are parallel.

Answer Key
Testname: 2425–S04–REVIEW.TST

1) Answer: $\frac{1}{12}$



2) Answer: $\frac{1}{4}$ each = $\frac{1}{2}$



3) Answer: $\frac{3\pi}{5}$

4) Answer: $\frac{8\pi}{3}$

5) Answer: $\frac{512\pi}{15}$

6) Answer: (a) $\frac{3\pi}{10}$; (b) $\frac{3\pi}{10}$

7) Answer: $\frac{8\pi}{3}$

8) Answer: (2, 0)

9) Answer: $\frac{\pi}{2}$

10) Answer: 9π

11) Answer: $\frac{59}{24}$

12) Answer: $3\pi\sqrt{2}$

13) Answer: $\frac{\pi}{27}(10\sqrt{10} - 1)$

14) Answer: $\frac{\pi}{9}(2^{3/2} - 1)$

15) Answer: $\sin^{-1}(\frac{x}{2}) + C$

16) Answer: $2e^{x}(x - 1) + C$

17) Answer: $(t^3 - 6t)\sin t + 3(t^2 - 2)\cos t + C$

18) Answer: $\frac{-e^{x}((x - 1)\cos x - x\sin x)}{2} + C$

19) Answer: $\frac{3\pi}{2}$

20) Answer: $-\sqrt{1 - x^2} + C$

21) Answer: $-\sqrt{x^2 + 1}(\frac{1}{3x^3} + \frac{1}{3x}) + C$

22) Answer: $\frac{(x^2 - 2)\sqrt{x^2 + 1}}{3} + C$

23) Answer: $\ln(\sqrt{x^2 - 1}) + x + C$ or $\arctan(\sqrt{x^2 - 1}) + C$

24) Answer: $-2\ln\left|\frac{x + 1}{x - 1}\right| + C$

25) Answer: $\frac{3}{2x^2} + \ln|x| + 4\ln|x + 1| + C$

26) Answer: $\frac{x^4}{4} - \frac{1}{x} + C$

27) Answer: $\frac{x^2}{5}(x^2 + 3)^{3/2} - \frac{2}{5}(x^2 + 3)^{3/2} + C$

28) Answer: $\ln|x - 1| + \frac{\sqrt{2}}{2}\tan^{-1}(\frac{x}{\sqrt{2}}) + C$

29) Answer: $\frac{x^2}{3}\sqrt{x^2 + 1} - \frac{2}{3}\sqrt{x^2 + 1} + C$

30) Answer: $2x - \frac{1}{4(x - 1)} - \frac{1}{4(x + 1)} + \frac{7\ln|x - 1|}{4} - \frac{7\ln|x + 1|}{4} + C$

31) Answer: $-2 + \frac{9}{2}\ln 3$

32) Answer: $\frac{17e^4 + 3}{8}$

33) Answer: converges to e

34) Answer: diverges to ∞

35) Answer: converges to 2

36) Answer: converges to ln 3

37) Answer: $\frac{1}{2}$

38) Answer: $\frac{1}{18}$

39) Answer: e^4

40) Answer: converges since $\left|\frac{-2}{e}\right| < 1$; sum is $\frac{2}{e + 2}$

41) Answer: converges to 14

42) Answer: converges; sum is 2

43) Answer: diverges

44) Answer: diverges

45) Answer: 180 feet

46) Answer: diverges

47) Answer: diverges

48) Answer: converges by the integral test

49) Answer: converges by the integral test

50) Answer: converges as a p–series; $p = \frac{6}{5}$

51) Answer: diverges by direct comparison to $\Sigma\frac{2}{k}$

52) Answer: diverges by the divergence test

53) Answer: converges by the integral test or as a geometric series

54) Answer: converges by limit–comparison with the p–series $\sum_{k=1}^{\infty} \frac{1}{k^2}$

55) Answer: converges by direct comparison to the p–series $\sum_{k=1}^{\infty} \frac{1}{k^2}$

56) Answer: converges by the ratio test ($L = \frac{1}{e}$)

57) Answer: diverges by the divergence test or ratio test

58) Answer: converges by the root test ($L = 0$)

59) Answer: (a) converges, ratio test (b) diverges, test for divergence
(c) converges, comparison test with a p–series

60) Answer: $p > \frac{1}{2}$, $p < -\frac{1}{2}$

61) Answer: converges absolutely

62) Answer: diverges

63) Answer: 0.54

64) Answer: converges for $|x| > 0$ by generalized ratio test;
converges for $x = 0$ because all terms are zero;
therefore, the series converges for all x

65) Answer: radius of convergence is $\frac{1}{2}$; interval of convergence is $(-\frac{1}{2}, \frac{1}{2})$.

66) Answer: radius of convergence is ∞; converges for all x

67) Answer: radius of convergence is 1; interval of convergence is [2, 4)

68) Answer: $\sum_{k=0}^{\infty} (k+1)x^{k+1} = \sum_{k=1}^{\infty} kx^k$

69) Answer: $1 + \sum_{k=1}^{\infty} \frac{(-1)^k 2^{2k-1} x^{2k}}{(2k)!}$

70) Answer: $\sum_{k=0}^{\infty} 2^k x^k$

71) Answer: $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n+1}}{(2n+1)!}$

72) Answer: $e^{-x} = \sum_{n=0}^{\infty} (-1)^n e^{-1} \frac{(x-1)^n}{n!}$

73) Answer: $f(x) = 2 + \frac{x-8}{12} - \frac{(x-8)^2}{288} + \frac{5(x-8)^3}{20736} - \ldots$

74) Answer: $f(x) = \int_0^x \sum_{k=0}^{\infty} \frac{(-1)^k t^{3k}}{k!} dt = \sum_{k=0}^{\infty} \frac{(-1)^k x^{3k+1}}{(3k+1)\, k!}$

75) Answer: $\langle -5, 3\rangle$; $\sqrt{34}$

76) Answer: $\langle \frac{5}{13}, -\frac{12}{13} \rangle$

77) Answer: s = 2; t = –4

78) Answer: x = 3 and y = 4, or $x = -\frac{5}{3}$ and $y = -\frac{20}{9}$

79) Answer: D

80) Answer: no

81) Answer: C

82) Answer: B

83) Answer: C

84) Answer: vector: $-\frac{155}{169}\mathbf{i} - \frac{372}{169}\mathbf{j}$; scalar: $-\frac{31}{13}$

85) Answer: $x = \frac{2}{3}$ or x = –5

86) Answer: $7\mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$

87) Answer: $-5\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$

88) Answer: $\frac{\sqrt{86}}{258}\mathbf{i} + \frac{11\sqrt{86}}{129}\mathbf{j} - \frac{17\sqrt{86}}{258}\mathbf{k}$; $-\frac{\sqrt{86}}{258}\mathbf{i} - \frac{11\sqrt{86}}{129}\mathbf{j} + \frac{17\sqrt{86}}{258}\mathbf{k}$

89) Answer: $t = -\frac{3}{7}$

90) Answer: $(0, 10, -12)$, $(\frac{10}{3}, 0, \frac{4}{3})$, $(3, 1, 0)$

91) Answer: intersect at $(2, -3, 1)$

92) Answer: parallel

93) Answer: $3x + 4y - 2z + 12 = 0$

94) Answer: $\frac{23\sqrt{38}}{38}$

95) Answer: $\frac{\sqrt{966}}{14}$

96) Answer: 7

97) Answer: $\frac{x-3}{8} = \frac{y-4}{10} = \frac{z+2}{13}$

98) Answer: $\frac{x-1}{-5} = \frac{y-1}{17} = \frac{z-2}{9}$

99) Answer: parallel