MATH 2425

SPRING 2004 **FINAL VERSION A**

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the 10 questions in this part counts 5 points each, for a total possible score of 50 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. [8.6/7] Determine if the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{k=3}^{\infty} \frac{(-1)^k k^2}{(k+1)(k-2)}$$

(a) absolutely convergent (b) conditionally convergent (c) divergent
(d) both (a) and (b) (e) none of these

2. [8.7/14] Find the interval of convergence for the series

$$\sum_{k=0}^{\infty} \frac{(2x+3)^k}{4^k}$$

(a) $\left(-\frac{7}{2}, \frac{1}{2}\right)$ (b) $\left[-\frac{7}{2}, \frac{1}{2}\right)$ (c) $\left(-\frac{7}{2}, \frac{1}{2}\right]$ (d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (e) $\left[-\frac{1}{2}, \frac{1}{2}\right)$

3. [9.6/~Example 6] Find the equation of the plane containing the lines

$$\frac{x-3}{4} = \frac{y-1}{2} = z+2 \text{ and } \frac{x-3}{5} = \frac{y-1}{-1} = \frac{z+2}{-2}$$

(a) $3x+13y+14z+6=0$ (b) $3x-13y-14z+6=0$
(c) $3x-13y-14z-32=0$ (d) $3x+13y+14z-6=0$
(e) $3x-13y+14z+32=0$

4. [6.2/~Ex. 2,7.7] Revolve the first quadrant region under the curve $y=\frac{1}{x}$ and to the right of $x=1$ about the *x*-axis and find the volume of the resulting solid.
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$ (e) the integral diverges

5. [6.4, 7.1/Ex.3]Find the arc length of the curve $y=\ln\cos x$ from $x=0$ to $x=\frac{\pi}{4}$.
(a) $\ln\left(\sqrt{2}+1\right)$ (b) $\ln\left(\sqrt{2}-1\right)$ (c) $\sqrt{2}+1$ (d) $\sqrt{2}-1$ (e) $\frac{\pi}{4}$

6. [6.1/~Ex.5] Find the area of the region to the right of the curve $x = y^2$ and to the left of the line $y = x - 2$.

(a) $\frac{2}{9}$ (b) $\frac{3}{4}$ (c) 2 (d) $\frac{9}{2}$ (e) 3

7. [6.2/~23] Revolve the first quadrant region enclosed by $y = x^2$ and $x = 1$ about the line $x = 1$. Find the volume of the resulting solid.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) π (e) 3π

8. (7.2/~28) Which of the following methods can be used to evaluate the integral $\int x(ax+b)^3\,dx$?

I. The substitution $u = ax + b$

II. Integration by parts with $u = x$ and $dv = (ax+b)^3\,dx$

III. Perform the indicated multiplication and integrate term by term.

(a) I only (b) II only (c) III only (d) II and III only (e) I, II, and III

9. [8.2] Find the sum of the infinite series $1 + 2r + r^2 + 2r^3 + r^4 + 2r^5 + r^6 + \cdots$ for those values of r for which it converges.

(a) $\frac{1-2r}{1+r^2}$ (b) $\frac{1-2r}{1-r^2}$ (c) $\frac{1+2r}{1+r^2}$ (d) $\frac{1+2r}{1-r^2}$ (e) $\frac{1+2r}{1-r}$

10. [9.3/30] Find x if the vectors $\mathbf{v} = 3\mathbf{i} - x\mathbf{j} + 2\mathbf{k}$ and $\mathbf{w} = x\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ are orthogonal.

(a) 0 (b) 2 (c) -2 (d) $\frac{1}{2}$ (e) $-\frac{1}{2}$

INSTRUCTIONS FOR PART II: For these questions, you must write down **all** steps in your solutions as if you did not have a calculator. Write legibly and carefully label any graphs or pictures. **Draw a box around your solution.** Partial credit will be given for those parts of your solution that are correct. Each of the questions in this part counts 10 points, for a total possible score of 50 points.

11. [7.4/16] $\int \frac{3x-1}{x^2-1}\,dx$

12. [8.4/58] Suppose $0 \le a_k \le 4$ and $b_k \ge k^2$. Explain why $\sum \frac{a_k}{b_k}$ converges.

13. [8.3/12] Determine the convergence or divergence of the following series:

$$\sum_{k=2}^{\infty} ke^{-k}$$

14a. [8.8/62] Find the Maclaurin series for $f(x) = x + \sin x$.

14b. Using (14a), show $\lim_{x \to 0} \dfrac{x + \sin x}{x} = 2$. (**DO NOT USE L'HÔPITAL'S RULE**.)

15. [9.5/35] Find the point of intersection of the lines :

$$\frac{x-3}{2} = \frac{y-1}{-1} = \frac{z-4}{1}; \quad \frac{x+2}{3} = \frac{y-3}{-1} = \frac{z-2}{1}$$