MATH 2425 **FALL 2003**

MIDTERM 1 – VERSION A

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question. On the scantron form, print your name legibly and indicate the exam version (either A or B).

Each of the questions in this part counts 5 points each, for a total possible score of 50 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

For problems 1-3, let R be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 1$.

1. The integral which represents the area of the region R is

A. $\int_0^1 (x^3 - 1)dx$ B. $\int_0^1 (1 - y^3)dy$ C. $\pi\int_0^1 (1 - x^6)dx$ D. $\int_0^1 y^{1/3}dy$

E. $\pi\int_0^1 y^{2/3}dy$

2. The integral which represents the volume of the solid of revolution of R about the x-axis is

A. $\pi\int_0^1 y^{2/3}dy$ B. $\pi\int_0^1 (x^3 - 1)dx$ C. $\pi\int_0^1 (1 - x^6)dx$ D. $\pi\int_0^1 (1 - x^3)dx$

E. $2\pi\int_0^1 x^4 dx$

3. The integral which represents the volume of the solid of revolution of R about the y-axis is

A. $\pi\int_0^1 (1 - x^6)dx$ B. $\pi\int_0^1 (x^3 - 1)dx$ C. $\pi\int_0^1 y^{2/3}dy$ D. $\pi\int_0^1 (1 - x^3)dx$

E. $2\pi\int_0^1 x^4 dx$

4. Find the arclength of the curve $y = \ln(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$.

A. $\ln\sqrt{2}$ B. $\ln(\sqrt{2} + 1)$ C. 0 D. 1 E. undefined

5. The integral which represents the area of the surface generated by rotating $x=\sqrt{y}, 1\le y\le 5$ about the *y*-axis is

A. $\pi\int_1^5 \sqrt{4y+1}\,dy$ B. $\pi\int_1^{\sqrt{5}} \sqrt{4x^2+1}\,dx$ C. $\pi\int_1^5 \sqrt{4x^2+1}\,dx$ D. $\pi\int_1^{\sqrt{5}} \sqrt{4y+1}\,dy$

E. $2\pi\int_1^5 \sqrt{4y+1}\,dy$

6. The length of the polar curve $r=4\sin\theta, 0\le\theta\le\frac{\pi}{2}$ is

A. π B. $\frac{\pi}{2}$ C. 1 D. 2π E. 3

7. Compute $\int_0^1 xf''(3x)\,dx$ given that $f'(0)=10,\ f(0)=1,\ f(3)=4$ and $f'(3)=-2$.

A. 1 B. 0 C. 10 D. undefined E. -1

8. A particle moves along the *x*-axis in such a way that its acceleration at time *t* is $a(t)=\sin^2 t$. What is the total distance traveled by the particle over the time interval $[0,\pi]$ if its initial velocity $v(0)=2$ units per second?

A. $\frac{\pi^2+4}{2}$ B. $\frac{\pi^2}{4}$ C. $2\pi-\frac{\pi^2}{4}$ D. $\frac{\pi^2}{4}-2\pi$ E. $\frac{\pi^2}{4}+2\pi$

9. Find a horizontal line $y=k$ that divides the area of the region in the first quadrant between $y=x^2$ and $y=9$ into two equal parts.

A. $k=\frac{9}{\sqrt[3]{4}}$ B. $k=\frac{4}{\sqrt[3]{9}}$ C. $k=\frac{9}{2}$ D. $k=\frac{4}{3}$ E. $k=3$

10. Which substitution is required to evaluate the integral $\int_0^1 \sqrt{1+x^2}\,dx$?

A. $x=\sin\theta$ B. $x=\tan\theta$ C. $x=\sec\theta$ D. $u=1+x^2$ E. $u=x^2$

INSTRUCTIONS FOR PART II: For these questions, you should write down **all** steps in your solutions. Write legibly and carefully label any graphs or pictures. Partial credit will be given for those parts of your solution that are correct. Each of the questions in this part counts 10 points, for a total possible score of 50 points.

11. Find the area of the region that lies inside both of the curves $r = 2\sin\theta$ and $r = 2\cos\theta$.

12. $\int \frac{dx}{x\sqrt{x^2 - 4}}$

13. $\int x^2 e^{-x} dx$

14. Revolve the region below the curve $y = \sin x$, $0 < x < \pi$, about the line $x = 2\pi$. Find the volume of the resulting solid.

15. Find the area of the region bounded by $y = x^3$ and its tangent line at the point $x = 1$.