MATH 2425

SPRING 2004 **MIDTERM 1A**

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the 8 questions in this part counts 6 points each, for a total possible score of 48 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. [7.2/~Example 3] How many applications of integration by parts are needed to evaluate $\int x^4 \sin x \, dx$?

   A.1 B. 2 C. 4 D. 6 E. none of these

2. [7.2/~Example 6] Which technique must be used to evaluate $\int x \tan^{-1} x \, dx$?

   A. one use of integration by parts with $u = x$
   B. one use of integration by parts with $u = \tan^{-1} x$
   C. no integration by parts, use the substitution $u = x$
   D. no integration by parts, use the substitution $u = \tan^{-1} x$
   E. no integration by parts, use the substitution $u = x \tan^{-1} x$

3. [6.3] Consider the two polar curves: $r = 3\sqrt{3}\cos\theta$ and $r = 3\sin\theta$. The points $P_1, P_2, P_3,$ and P_4 are in polar coordinates. Which one of the statements A-E below is true?

   $P_1 \left(\frac{3\sqrt{3}}{2}, \frac{\pi}{3} \right)$ $P_2 \left(-\frac{3\sqrt{3}}{2}, \frac{4\pi}{3} \right)$ $P_3 \ (0, \pi)$ $P_4 \left(-\frac{3\sqrt{3}}{2}, \frac{\pi}{3} \right)$

   A. P_1 and P_2 lie on both curves; P_3 and P_4 do not lie on either curve.
   B. P_1 and P_4 lie on both curves; P_2 and P_3 do not lie on either curve.
   C. P_1, P_2 and P_3 lie on both curves; P_4 does not lie on either curve.
   D. P_2 and P_3 lie on both curves; P_1 and P_4 do not lie on either curve.
   E. P_2 and P_4 lie on both curves; P_1 and P_3 do not lie on either curve.

4. [6.2/35] The volume of the solid generated when the region bounded by $y = x^2$ and $x = y^2$ is revolved about the *y*-axis is given by

A. $2\pi\int_0^1 x\left(\sqrt{x} - x^2\right)dx$ B. $2\pi\int_0^1 \left(\sqrt{x} - x^2\right)dx$ C. $\pi\int_0^1 x\left(\sqrt{x} - x^2\right)dx$

D. $2\pi\int_0^1 x\left(x^2 - \sqrt{x}\right)dx$ E. $2\pi\int_0^1 \left(x^2 - \sqrt{x}\right)dx$

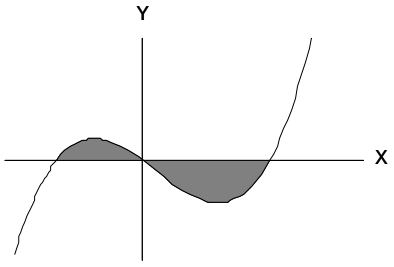
5. [Lab2] Find the number *k* so that the line $x = k$ bisects the area of the region bounded by $y = \frac{1}{x^2}$, $x = 1,$ and $x = 6.$

A. $\frac{5}{6}$ B. 1.258 C. $\frac{7}{2}$ D. $\frac{12}{7}$ E. $\frac{5}{12}$

6. [6.4/20] The length of the polar curve $r = e^{1-\theta}$, $0 \le \theta \le 1$ is

A. $\sqrt{2}\left(e+1\right)$ B. $\sqrt{2}(e-1)$ C. $-\sqrt{2}\left(e+1\right)$ D. $2\pi\left(e+1\right)$
E. $2\pi\left(e-1\right)$

7. [6.1/~17] Below is the graph of $y = x^3 - x^2 - 6x.$ The area of the shaded region is given by



A. $\int_{-1}^{5}\left(x^3 - x^2 - 6x\right)dx$ B. $\int_{-2}^{3}\left(x^3 - x^2 - 6x\right)dx$ C. $\int_{-3}^{2}\left(x^3 - x^2 - 6x\right)dx$

D. $\int_{-3}^{0}\left(x^3 - x^2 - 6x\right)dx - \int_{0}^{2}\left(x^3 - x^2 - 6x\right)dx$

E. $\int_{-2}^{0}\left(x^3 - x^2 - 6x\right)dx - \int_{0}^{3}\left(x^3 - x^2 - 6x\right)dx$

8. [Chapter 6 Review,~1] Convert the polar equation $r = \dfrac{a}{b\cos\theta + c\sin\theta}$ to a rectangular equation (a, b, and c are nonzero constants).

A. $r^2 = x^2 + y^2$ B. $cy + bx = a$ C. $a^2 = bx + cy$
D. $bx - cy = a$ E. $ax + by = c$

INSTRUCTIONS FOR PART II: For these questions, you **must** write down **all** steps in your solutions as if you did not have a calculator. Write legibly and carefully label any graphs or pictures. **Draw a box around your solution.** Partial credit will be given for those parts of your solution that are correct. Each of the questions in this part counts 10 points, for a total possible score of 50 points.

9. [7.2/3] $\int x \ln x \, dx$

10. (6.1/18) Find the area of the regions bounded by graphs of $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \pi$. Draw a carefully labeled graph and shade the region.

11. [6.4/~9] Compute the arc length of the curve $x = \dfrac{1}{16}y^4 + \dfrac{1}{2y^2}$ between $y = 2$ and $y = 3$.

12. [6.4/~13] Compute the surface area of a solid of revolution generated by revolving $y = 6x, 0 \le x \le 1$ about the x-axis.

13. (6.2/44) Find the volume of the solid when the region bounded by $y = x$, $y = 2x$, and $y = 1$ is revolved about the line $x = 1$.