MATH 2425

SPRING 2004 **MIDTERM 2A**

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the 8 questions in this part counts 6 points each, for a total possible score of 48 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. [7.7] Which of the following is NOT an improper integral?

   (a) $\int_0^1 (\ln x)^2\,dx$ (b) $\int_1^\infty (\ln x)^2\,dx$ (c) $\int_0^1 \frac{1}{(\ln x)^2}\,dx$ (d) $\int_1^e \frac{1}{(\ln x)^2}\,dx$

   (e) $\int_1^e (\ln x)^2\,dx$

2. [6.3/~22] Which of the following points

   $P_1\left(0,\frac{\pi}{4}\right)$ $P_2(1,0)$ $P_3\left(\frac{\sqrt{2}}{2},\frac{\pi}{4}\right)$ $P_4\left(-\frac{\sqrt{2}}{2},\frac{\pi}{4}\right)$

   lie on the intersection of the polar curves $r=\cos\theta$ and $r=\sin\theta$?

   (a) P_1 & P_2, not P_3, P_4 (b) P_1 & P_3, not P_2, P_4 (c) P_3 only
   (d) P_1, P_3, P_4, not P_2 (e) none of the above

3. [6.4/~30] Which of the following integrals represents the surface area of the surface of revolution created when the curve $y=\sin x, 0\le x\le\frac{\pi}{4}$, is revolved about the *y*-axis?

   (a) $\int_0^{\sqrt{2}/2} 2\pi x\sqrt{1+\cos^2 x}\,dx$ (b) $\int_0^{\sqrt{2}/2} 2\pi y\sqrt{1+\cos^2 y}\,dy$

   (c) $\int_0^{\sqrt{2}/2} 2\pi \sin x\sqrt{1+\cos^2 x}\,dx$ (d) $\int_0^{\pi/4} 2\pi x\sqrt{1+\cos^2 x}\,dx$

   (e) $\int_0^{\sqrt{2}/2} 2\pi y\sqrt{\frac{2-y^2}{1-y^2}}\,dx$

4. [Ch.6 Review/20] Which of the following integrals represent volumes of solids of revolution?

I. $\pi\int_a^b \left[f(x)\right]^2 dx$ II. $\pi\int_a^b \left[f(x)-g(x)\right]dx$ III. $\pi\int_a^b \left\{\left[f(x)\right]^2-\left[g(x)\right]^2\right\}dx$

(a) I only (b) I and II (c) I and III (d) II and III (e) I, II, and III

5. [8.5, Ex. 3] The series $\sum_{k=2}^{\infty}\frac{1}{2k-3}$

(a) converges because the ratio test yields 1
(b) diverges because the ratio test yields 1
(c) converges because $\lim_{k\to\infty}\frac{1}{2k-3}=0$
(d) diverges because $\int_2^{\infty}\frac{dx}{2x-3}$ diverges
(e) diverges because $\lim_{k\to\infty}\frac{\frac{1}{2k-3}}{\frac{1}{k^2}}=+\infty$

6. [8.2/45] If $\sum a_k^2=\sum b_k^2=4$ and $\sum a_k b_k=3$, find the value of $\sum\left(a_k-b_k\right)^2$.

(a) 2 (b) 5 (c) 0 (d) 14 (e) the series diverges

7. [8.1/47 and ~Lab 8] A drug is administered into the body. At the end of each hour, the amount of drug present is half what is was at the end of the previous hour. What percent of the drug is present at the end of *n* hours?
(a) $100\left(\frac{1}{2}\right)^n$ (b) $100\left(\frac{1}{n}\right)^2$ (c) $100n^{1/2}$ (d) 0 (e) not enough information

8. [8.1/35] Compute the limit of the convergent sequence $\left\{\left(an+b\right)^{1/n}\right\}$ where *a* and *b* are positive constants.
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) e (e) $e^{1/2}$

INSTRUCTIONS FOR PART II: For these questions, you **must** write down **all** steps in your solutions as if you did not have a calculator. Write legibly and carefully label any graphs or pictures. **Draw a box around your solution.** Partial credit will be given for those parts of your solution that are correct. Each of the questions in this part counts 10 points, for a total possible score of 50 points.

9. [7.3/~Ex. 7] $\int \frac{dx}{x^2\sqrt{x^2+9}}$

10. [7.4/~32] $\int \frac{x+1}{x^3+x}\,dx$

11. [7.7/51] Let $f(x)=\begin{cases} e^{x+1}, & x\le -1 \\ 1, & -1<x<1 \\ \frac{1}{x^2}, & x\ge 1 \end{cases}$. Evaluate $\int_{-\infty}^{+\infty} f(x)\,dx.$

12. [8.2/41] Evaluate $\sum_{k=0}^{\infty}\left[\frac{1}{2^k}+\frac{1}{3^k}\right]^2$.

13. (a)[8.5/55] Does $\sum_{k=1}^{\infty}\frac{2^k}{k!}$ converge?

(b) Show $\lim_{k\to\infty}\frac{2^k}{k!}=0.$