MATH 2425, Calculus II, Fall 2006

Final Exam: Multiple Choice Review Questions

(1) Compute the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $x = 9$ and $y = 0$ about the x-axis.

(a) $81\pi/2$ (b) 36π (c) $75\pi/2$ (d) 18π (e) $243\pi/5$

(2) Find the volume of the solid that results when the region enclosed by the curves $y = \sqrt{25 - x^2}$ and $y = 3$ is revolved about the x-axis.

(a) $472\pi/3$ (b) $216\pi/3$ (c) $364\pi/3$ (d) $256\pi/3$ (e) $128\pi/3$

(3) Compute the arc length of the curve $y = \frac{2x^{3/2}}{3}$ on the interval $0 \le x \le 3$.

(a) 8 (b) 14/3 (c) $4\sqrt{2}/3$ (d) $7\sqrt{3}/2$ (e) 11/2

(4) What is the form of the partial fraction decomposition for the rational function $\frac{2}{x^3+x}$

(a) $\frac{A}{x} + \frac{B}{x^2+1}$ (b) $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-2}$ (c) $\frac{Ax^2+Bx+C}{x^3+x}$ (d) $\frac{2}{x^3+x}$ (e) $\frac{A}{x} + \frac{Bx+C}{x^2+1}$

(5) Evaluate the improper integral $\int_3^4 \frac{1}{(x-3)^2} dx$

(a) It is divergent (b) 1/2 (c) 3/2 (d) 2 (e) 1

(6) Evaluate the improper integral $\int_4^\infty \frac{2}{x^2-1} dx$

(a) It is divergent (b) $\ln 4 - \ln 5$ (c) $\ln 5 - \ln 4$ (d) $\ln 3 - \ln 5$ (e) $\ln 5 - \ln 3$

(7) The geometric series $\sum_{k=1}^\infty (-1)^k 2^k 5^{1-k}$ has an r-value of

(a) $r = 10$ (b) $r = 5/2$ (c) $r = 2/5$ (d) $r = 2/25$ (e) $r = -2/5$

(8) Find the radius of convergence of the series $\sum_{k=1}^\infty \frac{(-1)^k x^{2k}}{\sqrt{k}4^k}$

(a) ∞ (b) 0 (c) 1 (d) 2 (e) 4

(9) Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be vectors. Then $[(\mathbf{u} \times \mathbf{v}) \times (3\mathbf{w})] \cdot \mathbf{u}$ is

(a) a scalar (b) a vector (c) both (a) and (b)

(d) neither (a) nor (b) (e) undefined

(10) Find an equation of the plane whose points are equidistant from the points $(2, -1, 1)$ and $(3, 1, 5)$

(a) $x - 2y + 4z = 29/2$ (b) $x + 2y - 4z = 29/2$ (c) $x + 2y + 4z = 29/2$

(d) $x - 27 - 4z = 29/2$ (e) $x + 2y + 4z = -29/2$

(11) Find the center of the sphere: $x^2 - 2x + y^2 + z^2 + 4z = -2$

(a) $(1, 0, -2)$ (b) $(1, 0, 2)$ (c) $(-1, 0, 2)$ (d) $(-1, 0, -2)$ (e) $(0, 0, 0)$

(12) Which equation represents a sphere of radius 1?

(a) $x^2 + y^2 + 2x - 4y + (z-1)^2 + 4 = 0$ (b) $x^2 + y^2 + 2x - 4y + (z-1)^2 + 6 = 0$

(c) $x^2 - y^2 + (z-1)^2 = 1$ (d) $(x+1)^2 + (y-2)^2 = 1$ (e) $x + y + z = 1$

(13) The area of the triangle with vertices $P(-1, 1, 0)$, $Q(3, 2, 0)$, and $R(7, -4, 0)$ can be calculated as

(a) $\frac{1}{2}||(-\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})||$ (b) $\frac{1}{2}||(4\mathbf{i} + \mathbf{j}) \times (-8\mathbf{i} + 5\mathbf{j})||$

(c) $\frac{1}{2}(-\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$ (d) $\frac{1}{2}(4\mathbf{i} + \mathbf{j}) \cdot (-8\mathbf{i} + 5\mathbf{j})$

(e) $\frac{1}{2}[(-\mathbf{i} + \mathbf{j}) \times (3\mathbf{i} + 2\mathbf{j})] \cdot (7\mathbf{i} - 4\mathbf{j})$

(14) Which of the following is a vector *not* perpendicular to $\mathbf{v} \times \mathbf{w}$?

(a) $\mathbf{v} - \mathbf{w}$ (b) $\mathbf{v} \cdot \mathbf{w}$ (c) $\mathbf{v} + \mathbf{w}$ (d) $\mathbf{v} \times (\mathbf{v} \times \mathbf{w})$ (e) $\mathbf{v} + (\mathbf{v} \times \mathbf{w})$

(15) The series $\sum_{k=2}^{\infty} \frac{\ln(\ln k)}{k \ln k + 1}$

(a) Converges Absolutely (b) Converges Conditionally (c) Diverges

(d) None of (a),(b),(c) (e) (a) and (b) are both true

(16) Which one of the following is the approximation of $\sqrt{4.5}$ using the 2nd degree Taylor polynomial for $f(x) = \sqrt{x}$ at $c = 4$?

(a) $\frac{17}{8}$ (b) $\frac{543}{256}$ (c) $\frac{53033}{25000}$ (d) 2.12 (e) $\frac{271}{128}$

(17) Which one of the following is the Maclaurin series for $f(x) = e^{2x} + e^{3x}$?

(a) $\sum_{k=0}^{\infty} \frac{2x^k + 3x^k}{k!}$ (b) $\sum_{k=0}^{\infty} \frac{(5x)^k}{k!}$ (c) $\sum_{k=0}^{\infty} \frac{(6x)^k}{k!}$

(d) $\sum_{k=0}^{\infty} \frac{(2^k + 3^k)x^k}{k!}$ (e) None of the above

(18) The series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ for $p > 1$

(a) Diverges by the divergence test. (b) Converges by the ratio test.

(c) Converges by the integral test. (d) Is a telescoping series.

(e) None of the above.

(19) The series $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$ is a convergent alternating series. Let S denote its value. The second partial sum of this series is $S_2 = 0$. The upper bound for $|S - S_2|$ using the fact that it is an alternating series is

(a) 1/6 (b) 1/4 (c) 1/2 (d) 4/5 (e) 1

(20) Using the direct comparison test, which one of the following series converges because the p-series $\sum_{k=4}^{\infty} \frac{1}{k^2}$ converges?

(a) $\sum_{k=4}^{\infty} \frac{1}{(k-3)^3}$ (b) $\sum_{k=4}^{\infty} \frac{1}{(k-3)}$ (c) $\sum_{k=4}^{\infty} \frac{1}{(k-3)^2}$

(d) $\sum_{k=4}^{\infty} \frac{1}{(k-3)^{3/2}}$ (e) $\sum_{k=4}^{\infty} \frac{1}{(k-3)^{1/2}}$

(21) Suppose that you use the partial fraction decomposition to evaluate $\int \frac{x}{(x+1)(x+2)^2} dx$. The numerator of the term with $(x+2)^2$ in the denominator will be

(a) $x+3$ (b) $2x-1$ (c) $x-1$ (d) x (e) None of the above

(22) Find the value of the integral $\int_2^{\infty} \frac{dx}{x(\ln x)^p}$ when it exists:

(a) $\frac{1}{p-1}$ (b) $\frac{(\ln 2)^{1-p}}{p-1}$ (c) $(p-1)(\ln 2)$ (d) $\ln 2^p$ (e) None of the above

(23) Find the area of the polar curve $r = \theta$ for θ between 0 and $3\pi/4$

(a) $9\pi^3/64$ (b) $9\pi/128$ (c) $9\pi^3/128$ (d) $\pi/128$ (e) $27\pi^3/128$

(24) Which of the following integrals gives the surface area when the region bounded by the equations $y = 1+\cos x$, $y = 0$ for x between 0 and π is revolved about the x-axis.

(a) $\int_0^{\pi} 2\pi x\sqrt{1+\sin^2 x}dx$ (b)$\int_0^{\pi} 2\pi(1+\cos x)\sqrt{1+\sin^2 x}dx$

(c) $\int_0^{\pi} \pi^2(1+\cos x)\sqrt{1+\sin^2 x}dx$ (d) $\int_0^{\pi} 2\pi y\sqrt{1+\sin^2 y}dy$

(e) $\int_0^{\pi} \sqrt{1+\sin^2 x}dx$

(25) Which of the following is true for the series $\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{k^2+1}$

(a) It is conditionally convergent. (b) It is absolutely convergent.

(c) It is a geometric series, therefore convergent.

(d) It is a telescoping series, therefore divergent.

(e) It is divergent.

(26) Which of the following is true for the series $\sum_{k=1}^{\infty} \frac{(-1)^k k}{2^k}$

(a) It is conditionally convergent. (b) It is absolutely convergent.

(c) It is a geometric series, therefore convergent.

(d) It is a telescoping series, therefore divergent.

(e) It is divergent.

(27) Let R be the region between $y = \cos x$ and $y = \sin x$, for $-3\pi/4 \leq x \leq \pi/4$. Which of the following integrals represents the volume of the solid obtained by rotating R about the line $x = \pi$?

(a) $\pi \int_{-3\pi/4}^{\pi/4} ((\cos x - \pi)^2 - (\sin x - \pi)^2) dx$ (b) $2\pi \int_{-3\pi/4}^{\pi/4} (x - \pi)(\cos x - \sin x) dx$

(c) $\pi \int_{-3\pi/4}^{\pi/4} (x - \pi)(\sin x - \cos x) dx$ (d) $2\pi \int_{-3\pi/4}^{\pi/4} (\pi - x)(\cos x - \sin x) dx$

(e) $\pi \int_{-3\pi/4}^{\pi/4} (\cos x - \sin x - \pi)^2 dx$

(28) All of the following tests are valid for proving the convergence of $\sum_{k=1}^{\infty} \frac{\ln k}{k^3}$ EXCEPT

(a) Ratio Test (b) Direct Comparison Test (c) Integral Test

(d) Limit Comparison Test (e) None of the Above: The series is divergent.

(29) Find all values of p such that the series $\sum_{k=1}^{\infty} \frac{k^2+k+1}{k^p(k^4+10k^3+4)}$ converges.

(a) $p \geq 1$ (b) $p > 0$ (c) $p \geq 0$ (d) $p > 1$ (e) $p > -1$

(30) Find the radius of convergence R for $\sum_{k=1}^{\infty} \frac{k! x^k}{k^k}$

(a) 0 (b) 1/2 (c) 1 (d) 2 (e) e

(31) A river 2.1 miles wide flows south with a current of 3.1 miles per hour. What speed should a motorboat assume to travel across the river from east to west in 30 minutes?

(a) 3.21 mph (b) 4.10 mph (c) 5.22 mph (d) 7.56 mph (e) 8.89 mph

(32) Find the angle between the vector $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and the plane determined by the points $P(1, -2, 3)$, $Q(-1, 2, 3)$ and $R(1, 2, -3)$.

(a) 30 degrees (b) 40 degrees (c) 50 degrees (d) 60 degrees (e) 90 degrees

(33) Find all values of p for which $\int_0^1 \frac{1}{x^p} dx$ converges

(a) $p \leq 0$ (b) $p \geq 0$ (c) $p < 1$ (d) $p > 1$ (e) None of the above

(34) Which answer is the closest to the angle between the vector $\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and the vector $\mathbf{w} = -2\mathbf{i} + \mathbf{j} - \mathbf{k}$?

(a) $2\pi/3$ (b) 2.09 (c) $-2\pi/3$ (d) -2.09 $5\pi/6$

(35) Which one of the following is is a correct statement one half of the 0-∞ case of the Limit Comparison Test. Let $a_k > 0$ and $b_k > 0$ for sufficiently large k.

(a)If $\lim_{k\to\infty} \frac{a_k}{b_k} = 0$ and $\sum a_k$ converges then $\sum b_k$ converges.

(b) If $\lim_{k\to\infty} \frac{a_k}{b_k} = 0$ and $\sum b_k$ diverges then $\sum a_k$ diverges.

(c) If $\lim_{k\to\infty} \frac{a_k}{b_k} = 0$ and $\sum b_k$ converges then $\sum a_k$ converges.

(d) If $\lim_{k\to\infty} \frac{a_k}{b_k} = \infty$ and $\sum b_k$ converges then $\sum a_k$ converges.

(e) If $\lim_{k\to\infty} \frac{a_k}{b_k} = \infty$ and $\sum a_k$ diverges then $\sum b_k$ diverges.