PRINT YOUR NAME LEGIBLY AS IT APPEARS ON CLASS ROLL

LAST name:______________________________ FIRST name:______________________________

ID NUMBER: XXX–XX– __ __ __ __

CHECK THE APPROPRIATE SECTION

☐ Dr. Jorgensen Section 206

☐ Dr. Krueger Section 105

☐ Dr. Lin Section 107

☐ Dr. Liu Section 583

ON YOUR SCANTRON FORM, FILL IN THE TABLE:

NAME	last, first		
SUBJECT	MATH 2425-____	TEST NO.	VERS A

TURN OFF ALL CELL PHONES AND BEEPERS

& PUT THEM OUT OF SIGHT

DO NOT WRITE BELOW THIS LINE — DO NOT START UNTIL SO INSTRUCTED

	Points Earned
Part I (50 points)	$\cdot 5 =$
11 (10 points)	
12 (10 points)	
13 (10 points)	
14 (10 points)	
15 (10 points)	
PART II (50 points)	
TOTAL SCORE (100 points)	

The square brackets following an exam-question number refer to a section/problem number in the text or a lab worksheet. Problem numbers preceded by the symbol $\sim$ are modeled on that problem from the text or lab, but are not identical to it; problem numbers without the symbol are identical to, or very close to, the problem from the text or lab.

INSTRUCTIONS FOR PART I Write your answers for these questions on a scantron form (882-ES or 882-E) and mark only one answer per question.
Each of the questions in this part counts 5 points each (no partial credit), for a total possible score of 50 points. You may use an approved calculator. You may write on this exam or request scratch paper, if needed.

1. [§6.1] The integral which represents the area of the region bounded by $x = y^2 - 4y$ and the y-axis is

   (a) $2\int_0^4 (\sqrt{x+4}+2)\,dx$ (b) $\int_0^4 (y^2-4y)\,dy$ (c) $\int_0^4 (\sqrt{x+4}+2)\,dx$

   (d) $\int_0^4 (2-\sqrt{x+4})\,dx$ (e) $\int_0^4 (4y-y^2)\,dy$.

2. [§8.2: 15] The series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k}$

   (a) is an alternating series which converges because the terms are increasing

   (b) is a geometric series which converges because $r = \frac{1}{2}$

   (c) is a geometric series which converges because $r = -\frac{1}{2}$

   (d) is an alternating series which converges because the terms have limit $\frac{1}{2}$ (e) diverges.

3. [§8.2] The series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(1+k)^2+1+k}$ is

   (a) a convergent telescoping series with general term $\dfrac{1}{k+2} - \dfrac{1}{k+1}$

   (b) a convergent telescoping series with general term $\dfrac{1}{k+1} - \dfrac{1}{k+2}$

   (c) a convergent geometric series with $r = 1+k$

   (d) a convergent geometric series with $r = \dfrac{1}{1+k}$

   (e) convergent because $\displaystyle\lim_{k\to\infty}\left(\frac{1}{(1+k)^2+1+k}\right) = 0$.

4. [§8.3] Determine all the values of the constant a such that the series $\displaystyle\sum_{k=5}^{\infty} 7k^{-(1+a)}$ converges.

   (a) $(-1, 1)$ (b) $[0, \infty)$ (c) $(-\infty, 7)$ (d) $(0, \infty)$ (e) always divergent.

5. [§6.4] Find the arc length of the curve defined by $x = \frac{2}{3}y^{3/2}$ between the points $(0, 0)$ and $(2\sqrt{3}, 3)$.

   (a) $\dfrac{16\pi}{3}$ (b) $\dfrac{14}{3}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{4}$ (e) $\dfrac{16}{3}$.

6. [§8.4] The series $\sum_{k=1}^{\infty} a_k$, where $a_k = \dfrac{k+1}{k^2+1}$

(a) diverges because $\lim_{k\to\infty} \left(\dfrac{a_k}{b_k}\right) = 1$, where $b_k = \dfrac{1}{k}$

(b) diverges because $\lim_{k\to\infty} \left(\dfrac{a_k}{b_k}\right) = +\infty$, where $b_k = \dfrac{1}{k^2}$

(c) diverges because $\lim_{k\to\infty} \left(\dfrac{k+1}{k^2+1}\right) = 0$

(d) diverges because $\dfrac{k+1}{k^2+1} > \dfrac{1}{k^2+1}$ and $\sum_{k=1}^{\infty} \dfrac{1}{k^2+1}$ converges (e) converges to 1.

7. [§6.4: 14] Find the surface area generated by the graph of $y = 2\sqrt{x}$ revolved about the x-axis on $[0, 3]$.

(a) 23π (b) $\dfrac{49\pi}{3}$ (c) $\pi - 1$ (d) $\dfrac{56}{3}$ (e) $\dfrac{56\pi}{3}$.

8. [§7.4: 61, §7.5:∼Example 1(e),(f)] Which of the following methods can be used to evaluate the integral $\displaystyle\int \frac{x}{x^2-9}\,dx$?

I. Use the substitution $u = x^2 - 9$ II. Use the substitution $x = 3\sec\theta$

III. Use a partial-fraction decomposition IV. Multiply the integrand by $x^2 - 9$

(a) I only (b) I, II & III only (c) I & II only (d) IV only (e) none of these methods work.

9. [§7.4: Example 5] The first step in the partial fraction decomposition of $\dfrac{x^4+2x^3-4x^2+x-3}{x^2-x-2}$ is

(a) long division (b) guess $\dfrac{x^4+2x^3-4x^2+x-3}{x^2-x-2} = \dfrac{A}{x-2} + \dfrac{B}{x+1}$, then solve for A and B

(c) guess $\dfrac{x^4+2x^3-4x^2+x-3}{x^2-x-2} = \dfrac{Ax}{x-2} + \dfrac{Bx}{x+1}$, then solve for A and B

(d) guess $\dfrac{x^4+2x^3-4x^2+x-3}{x^2-x-2} = \dfrac{Ax}{x+2} + \dfrac{Bx}{x-1}$, then solve for A and B

(e) guess $\dfrac{x^4+2x^3-4x^2+x-3}{x^2-x-2} = \dfrac{A}{x+2} + \dfrac{B}{x-1}$, then solve for A and B.

10. [§7.7] Which of the following are improper integrals?

I. $\displaystyle\int_{-\infty}^{+\infty} \frac{dx}{1+x^2}$ II. $\displaystyle\int_0^2 \frac{dx}{(x-1)^2}$

III. $\displaystyle\int_0^r \frac{dx}{\sqrt{r^2-x^2}}$, where r is a positive constant IV. $\displaystyle\int_0^{\frac{\pi}{4}} \tan x\,dx$.

(a) I & IV only (b) I only (c) I & III only

(d) I, II & III only (e) none of these integrals are improper.

INSTRUCTIONS FOR PART II For these questions, you must write down **all** steps in your solutions **as if you do not have a calculator**. Write legibly, and **label any graphs or pictures**. Draw a box around your solution. Partial credit will be given for those parts of your solution that are correct. Total possible score for this part is 50 points.

11. [§8.2] [10 points] Show that the series $\sum_{k=1}^{\infty}(-1)^k\,2^k\,3^{1-k}$ is convergent and find its sum.

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12. [§8.6: 11] [10 points]
Determine whether the series $\sum_{k=1}^{\infty}\frac{(-1)^k}{\sqrt{k}}$ converges absolutely, converges conditionally, or diverges.

13. [§8.5:∼10] [10 points] Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{2^k}$ converges or diverges. In your explanation, explain which test you are using.

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14. [§8.1:∼27] [10 points] Compute the limit of the convergent sequence $\{n^{1/(n+2)}\}_{n=1}^{\infty}$.

15. [§7.4: 30] [10 points] Show that $\displaystyle\int \frac{x+2}{x(x-1)^2}\,dx = 2\ln\left|\frac{x}{x-1}\right| - \frac{3}{x-1} + C$, where C is a constant, by using integration techniques.

Have you shown all work in Part II? Write version A on scantron form.

Indicate instructor on front of exam.