HONR 1426 FALL 2008

Problem Solving: Worksheet 12

NAME:________________________

Study Technique 4 Try this technique to help you prepare for a test. A couple/few days before the test, look over all the homework. The day/night before the test, look over the homework again, and ask yourself what the key concepts and methods are per question (WITHOUT reworking the questions). Jot this down on a sheet of paper and, when you are done, compare with your solutions to see if you were correct. Any question in which you were on the wrong track, look over your solution to it to make sure you see what the main idea is. This work the day/night before will help get your brain to think faster and bring the material to the front of your brain to help you think faster during the actual test.

Part A is due at the start of class on Mon Nov 24, 2008. Part B is not due.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown $\Rightarrow$ partial/full credit not possible, so show as much work as possible.

PART A (Due Mon Nov 24)

Many students incorrectly evaluate the indeterminate forms of type 0^0, ∞^0 and 1^∞ as 1, since they think that "anything to the zero power is 1" and "1 to any power is 1". In this worksheet, we will see that these indeterminate forms can produce limits that are nonnegative real numbers or limits that are infinite.

1. In class last week, we saw (without using L'Hôpital's Rule) that $\lim_{x\to 0}\left[\left(e^{-\frac{1}{x^2}}\right)^{x^2}\right] = \frac{1}{e}$, even though its type is 0^0. Let $a \in (0,1)$. Show that the indeterminate form of $\lim_{x\to 0}\left[\left(e^{-\frac{\ln(1/a)}{x^2}}\right)^{x^2}\right]$ is 0^0 and, withOUT using L'Hôpital's Rule, show that this limit equals a.
2. An indeterminate form of type ∞^0 can be any positive real number. Let $a \in (0,\infty)$. Show that the indeterminate form of $\lim_{x\to\infty} x^{(\ln a)/(1+\ln x)}$ is ∞^0 and, using L'Hôpital's Rule, show that this limit equals a.
3. An indeterminate form of type 1^∞ can be any positive real number. Let $a \in (0,\infty)$. Show that the indeterminate form of $\lim_{x\to 0^+}(x+1)^{(\ln a)/x}$ is 1^∞ and, using L'Hôpital's Rule, show that this limit equals a.
4. An indeterminate form of type 1^∞ can be infinite. Show that the indeterminate form of $\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{e^x}\right]$ is 1^∞ and, using L'Hôpital's Rule, show that this limit is infinite.
5. For the limits in Questions 2-4, verify that the hypotheses of L'Hôpital's Rule are satisfied wherever you have used it.

PART B (Not due)

Test 3 will be Wed Nov 19 during lab time. Below are practice questions for Test 3 (in no particular order). Test 3 will cover college algebra and material in Secs 2.1-4.8 inclusive. The homework is available from my website: www.uta.edu/math/vancliff/T/F08 .

1. Rework the practice questions provided for Tests 1 & 2 on worksheets 3 & 7, and rework Tests 1 & 2. Go over all quizzes, assigned homework and worksheet questions.

2. If $f(x) = x^{\sin x}$ for $x > 0$, then $\left.\dfrac{df}{dx}\right|_{x=\pi/2}$ is

   (a) 0 (b) 1 (c) $\dfrac{\pi}{2}$ (d) does not exist (e) not enough information given.

3. If $g(t) = t^{e^t}$, then $\left.\dfrac{dg}{dt}\right|_{t=1}$ is

   (a) 0 (b) 1 (c) e (d) does not exist (e) not enough information given.

4. Given that $x^{\cos y} = y^{\sin x}$ and that $\cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} = \sin\dfrac{\pi}{4}$, compute $\dfrac{dy}{dx}$ at the point $\left(\dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$.

   (a) 0.681 (b) 1.468 (c) $\dfrac{4+\pi\ln\pi-\pi\ln 4}{4-\pi\ln\pi+\pi\ln 4}$ (d) $\dfrac{4-\pi\ln\pi+\pi\ln 4}{4+\pi\ln\pi-\pi\ln 4}$ (e) none of these.

5. A block of ice, in the shape of a cube, originally having volume 2,000 cm^3, is melting in such a way that the length of each of its edges is decreasing at the rate of 3 cm/hr. Assuming that the block of ice maintains its cubical shape, find the rate of change of the surface area of the cube at the time the volume is 1728 cm^3. In so doing, sketch a picture of the situation, labelling relevant items, and lay out your work very clearly (as is usually requested).

6. A balloon is rising vertically at the rate of 10 ft/s. An observer is standing on the ground 300 ft horizontally from the point where the balloon was released.

   (a) Sketch and label a picture of this situation.

   (b) At what rate is the distance between the observer and the balloon changing when the balloon is 400 ft high?

7. A balloon is rising vertically at a constant speed of 5 meters per second. A dog is running along a straight line at 15 meters per second, chasing the balloon, and overshoots it. When the dog passes under the balloon, the balloon is 45 meters above the dog. How fast is the distance between the dog and the balloon increasing three seconds after the dog passes under the balloon?

   (a) 12.5 m/s (b) 13 m/s (c) 13.5 m/s (d) $\dfrac{22}{\sqrt{3}}$ m/s (e) none of these.

8. If $f(x) = 3 + x + e^x$ and $g = f^{-1}$, then $\left.\dfrac{dg}{dx}\right|_{x=4}$ is

   (a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) does not exist (e) not enough information given.

9. Given that $g(x) = \sqrt{x^2+7}$ and $\dfrac{dg}{dx} = \dfrac{x}{\sqrt{x^2+7}}$, there is a function f with the property that $\dfrac{df}{dx} = \dfrac{dg}{dx}$ for all x and $f(3) = 14$. Find $f(\sqrt{29})$. (a) 1 (b) 6 (c) 10 (d) 16 (e) 4.

10. A function f has derivative $f'(x) = 24x^5 + 2$. Given that $f(1) = 8$, find $f(-1)$.

(a) 4.3 (b) 4 (c) -4 (d) not enough information given (e) none of these.

11. All the critical numbers of $g(t) = 5t^{\frac{2}{3}} + t^{\frac{5}{3}}$ are (a) -2 & 0 (b) -2 & 1 (c) 0 & 1 (d) -2 (e) 1.

12. Find the smallest and largest values of $x^4 - 2x^5 + 5$ on $[0, 1]$.

(a) 4, 5.00512 (b) 4, 5 (c) 3.9981, 5.01 (d) 2/5, 1 (e) 2/5, 5.

13. If a function f has derivative $f'(x) = (1-x)^{\frac{2}{5}} - \frac{2x}{5}(1-x)^{-\frac{3}{5}}$, then f has a local maximum at

(a) 0 (b) 1 (c) $\dfrac{5}{7}$ (d) does not exist (e) not enough information given.

14. If a function f has derivative $f'(x) = x^2(x-2)(5x-6)$, then f has a local minimum at

(a) 0 (b) $\dfrac{6}{5}$ (c) 2 (d) does not exist (e) not enough information given.

15. If $s(t) = 1 - 2t - t^2$, then the absolute minimum of s over the interval $[-4\ , 1]$ is

(a) -7 (b) -2 (c) -1 (d) 2 (e) does not exist.

16. If $h(y) = 2y^3 + 3y^2 + 4$, then the absolute minimum of h over the interval $[-2\ , 1]$ is

(a) 0 (b) 4 (c) 5 (d) 9 (e) does not exist.

17. All the critical numbers of $g(t) = \sqrt{t}(1-t)$ are (a) 0 (b) 0 & $\dfrac{1}{3}$ (c) 0 & 1 (d) $\dfrac{1}{3}$ (e) 1.

18. If a function f has derivative $f'(x) = (1-x)^{-\frac{3}{5}}(5-7x)$, then f has a local maximum at

(a) 0 (b) 1 (c) $\frac{5}{7}$ (d) does not exist (e) not enough information given.

19. A function f has derivative $f'(x) = (x-2)^3(5x-2)$ and second derivative

$f''(x) = 4(x-2)^2(5x-4)$. Find the x-coordinate(s) of the inflection point(s) of the original function f.

(a) 0.8, 2 (b) $\sqrt{\frac{3}{5}}$ (c) 2, $\sqrt{\frac{3}{5}}$ (d) $\frac{2}{5}$, 2 (e) 0.8.

20. A box with an open top and a square base is to be built so that the height of the box plus the length of one of the sides is to be 21 meters. Find the EXACT dimensions for such a box that yield the maximal volume; justify your reasoning, showing all steps in your solution.

21. A piece of wire 30 cm long is cut into two pieces. One piece is bent into a square and the other piece is shaped into a circle. To maximize the total area enclosed, the length of wire for the circle should be

(a) 0 cm (b) $\dfrac{30\pi}{4+\pi}$ cm (c) 30 cm (d) does not exist (e) not enough information given.

22. A piece of wire 30 cm long is cut into two pieces. One piece is bent into a square and the other piece is shaped into an equilateral triangle. To minimize the total area enclosed, the length of wire for the triangle should be

(a) 0 cm (b) $\dfrac{270}{9+4\sqrt{3}}$ cm (c) 30 cm (d) does not exist (e) not enough information given.

23. Sketch the graph of one function f with all the following properties:

$$\begin{array}{llll} f'(x) < 0 & \text{for } x < -1, & f''(x) > 0 & \text{for } x < 2, \\ f'(x) < 0 & \text{for } x > 3, & f''(x) < 0 & \text{for } x > 2. \\ f'(x) > 0 & \text{for } x \in (-1, 3), & & \end{array}$$

(Note: you are NOT asked to find a formula for f, and there could be more than one correct answer to this question, or there could be no such f.)

If you claim that there is no such f, then you should justify your claim.

Such a function described above could have relative extrema and/or inflection points; some forced by the stated conditions, and possibly some additional ones put in by you.

Give the x-coordinates where such a function **must** have relative extrema (not ones added in by you).

Give the x-coordinates where such a function **must** have inflection points (not ones added in by you).

24. One piece of the River Soda is 10 miles wide and flows from west to east. Missy Smith is at a point A on the north bank of the river. Directly across the river from Missy, on the south bank, is a point B, and Missy wishes to reach a cabin C located 5 miles down the river from B. Missy can row at 5 mph (including the effect of the current), and run at 13 mph on ground. Find the route that will take Missy the least amount of time to get from A to C.

(a) row to a point on the south bank that is east of B and within a mile of B and then run along the south bank to C

(b) row directly from A to C

(c) row directly from A to B and then run along the south bank to C

(d) row to a point on the south bank that is west of C and within a mile of C and then run along the south bank to C

(e) not enough information given.

25. (a) Find $\lim_{x\to\infty}\left(x\ln\left(1+x^{-1}\right)\right)$. Justify all steps in your work. (b) Using (a), find $\lim_{x\to\infty}(1+x^{-1})^x$.

26. Evaluate $\lim_{x\to 0}\left(\dfrac{\tan x}{x+\sin x}\right)$. (a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) does not exist (e) not enough information given.

27. Compute $\lim_{x\to 0}\left(\dfrac{1-\cos kx}{x^2}\right)$, where $k\in\mathbb{R}$, $k\neq 0$. (a) $2k^2$ (b) $\dfrac{2}{k^2}$ (c) k^2 (d) 0 (e) $\dfrac{k^2}{2}$.

28. Here is an example that L'Hôpital used in his 1696 book (the first calculus textbook ever published) to illustrate the method we now call L'Hôpital's Rule. Let $a\in(0,\infty)$. Using one application of L'Hôpital's Rule, find $\lim_{x\to a}\dfrac{\sqrt{2a^3x-x^4}-a\sqrt[3]{a^2x}}{a-\sqrt[4]{ax^3}}$.

Attached is the 3rd test given in Honr 1426 in Fall 2007.

You may use a simple calculator (as described on first-day handout). Request paper for rough work.

You have 30 minutes. Total number of points = 40. Keep your eyes on your own work!!

No partial credit will be awarded for questions 1-5; on these questions, only one choice is correct, and you should circle the correct answer. Each of questions 1-5 is worth 2 points. Partial credit will be awarded for questions 6-7, so show as much work & clear reasoning as possible on those questions.

1. [§3.2,§3.5] Let f be a function for which $f(4) = 3$ and $\left.\dfrac{df}{dx}\right|_{x=4} = 5$. If $g(x) = x^2 f(x^2)$, find $\left.\dfrac{dg}{dx}\right|_{x=2}$.

   (a) 92 (b) 52 (c) 20 (d) $12 + 16\sqrt{3}$ (e) 32 (f) 80.

2. [§3.6,§3.9] Let x and y denote differentiable functions of t. If $5x^2 - y = 100$ and $\dfrac{dx}{dt} = 2$, find $\left.\dfrac{dy}{dt}\right|_{x=7}$.

   (a) 70 (b) $\dfrac{1}{35}$ (c) 0 (d) -900 (e) 10 (f) 140.

3. [§3.10] Find the linearization of $f(x) = \dfrac{1}{\sqrt{1+2x}}$ at $x = 0$. (a) $L(x) = 0$ (b) $L(x) = 1$

   (c) $L(x) = 1 - x$ (d) $L(x) = 1 + 2x$ (e) $L(x) = \sqrt{1+2x}$ (f) $L(x) = 1 - \dfrac{x}{2}$.

4. [§§4.1-4.4] Which of the following statements is (are) TRUE for all twice-differentiable functions f and g?

   I. If $\dfrac{df}{dx} = \dfrac{dg}{dx}$ for all x, then $f(x) = g(x)$ for all x.

   II. If $f(c) = 0$, then $x = c$ is a critical number of f.

   III. If $\left.\dfrac{df}{dx}\right|_{x=e} = 0$ and $\left.\dfrac{d^2f}{dx^2}\right|_{x=e} = \pi$, then f has a relative minimum at $x = e$.

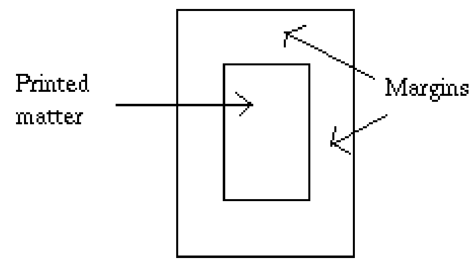
   (a) I & III only (b) III only (c) I, II & III (d) I only (e) II only (f) none of them.

5. [§4.8] A function f has derivative $f'(x) = 24x^5 + 2$. Given that $f(1) = 8$, find $f(-1)$.

   (a) 4.3 (b) -22 (c) -4 (d) 4 (e) -18 (f) no such f.

6. [§4.5:∼11] [15 points]

A rectangular poster is to contain 108 cm^2 of rectangular printed matter, with margins of 6 cm each at the top and bottom and 2 cm on the sides (shown right). What is the least cost to make the poster if it costs 5 cents/cm^2 to make the part consisting of printed matter and 1 cent/cm^2 to make the part consisting of the margins?



7. [§4.6]

(a) [10 points] Find the indeterminate form of the limit, $\lim_{x\to 0}\left[\dfrac{\ln(e^x+x)}{x}\right]$, and show that the limit equals 2.

(b) [5 points] Find the indeterminate form of the limit, $\lim_{x\to 0}\left[(e^x+x)^{1/x}\right]$, and use (a) to find the limit.

END TEST