

Study Technique 2 While working through homework, it is very tempting to use a solution manual. Solution manuals can be very helpful and very effective, but only if used correctly. The correct way to use a solution manual is, if possible, to read the first one or two lines of a solution and then try to continue the problem from there without looking at the manual’s solution again. If that is not possible, then read the solution all the way through, but then close the manual and try to reproduce the solution (or at least the main ideas of the solution) without the aid of the manual.

Study Technique 3 After you complete your homework, read the section(s) in the textbook that will be covered in the next lecture. It is likely that you will not understand everything you read; however, reading the sections before lecture will help you understand what is presented in the lecture.

Due at the start of class on Mon Sept 15, 2008.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown \Rightarrow partial/full credit not possible, so show as much work as possible.

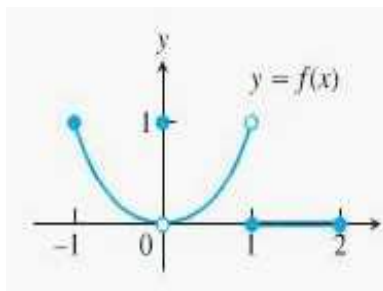
PART A

Our determination of whether or not a limit of a function f exists as $x \rightarrow c$ entails study of the behavior of f as $x \rightarrow c$ from the left and also study of the behavior of f as $x \rightarrow c$ from the right. One can define these one-sided limits formally (pg 98) as was done for a two-sided limit, and the limit from the left is denoted $\lim_{x \rightarrow c^-} f(x)$ and the limit from the right is denoted $\lim_{x \rightarrow c^+} f(x)$. These one-sided limits are related to $\lim_{x \rightarrow c} f(x)$ via Theorem 6 (pg 97) which says:

THEOREM 6
A function $f(x)$ has a limit as x approaches c if and only if it has left-hand and right-hand limits there and these one-sided limits are equal:

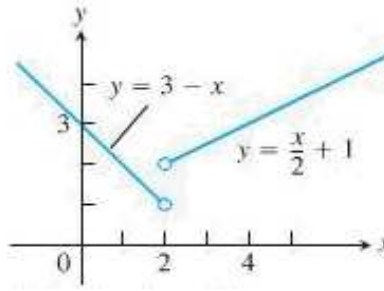
$$\lim_{x \rightarrow c} f(x) = L \iff \lim_{x \rightarrow c^-} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c^+} f(x) = L.$$

1. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



- | | |
|---|---|
| (a) $\lim_{x \rightarrow -1^+} f(x) = 1$ | (b) $\lim_{x \rightarrow 0^-} f(x) = 0$ |
| (c) $\lim_{x \rightarrow 0^-} f(x) = 1$ | (d) $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$ |
| (e) $\lim_{x \rightarrow 0} f(x)$ exists | (f) $\lim_{x \rightarrow 0} f(x) = 0$ |
| (g) $\lim_{x \rightarrow 0} f(x) = 1$ | (h) $\lim_{x \rightarrow 1} f(x) = 1$ |
| (i) $\lim_{x \rightarrow 1} f(x) = 0$ | (j) $\lim_{x \rightarrow 2^-} f(x) = 2$ |
| (k) $\lim_{x \rightarrow -1^-} f(x)$ does not exist | (l) $\lim_{x \rightarrow 2^+} f(x) = 0.$ |

2. Let $f(x) = \begin{cases} 3 - x & \text{if } x < 2 \\ \frac{x}{2} + 1 & \text{if } x > 2 \end{cases}$

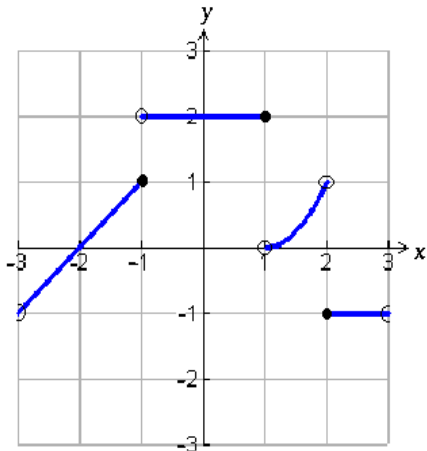


- (a) Find $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$.
 (b) Does $\lim_{x \rightarrow 2} f(x)$ exist? If so, what is it? If not, why not?
 (c) Find $\lim_{x \rightarrow 4^-} f(x)$ and $\lim_{x \rightarrow 4^+} f(x)$.
 (d) Does $\lim_{x \rightarrow 4} f(x)$ exist? If so, what is it? If not, why not?

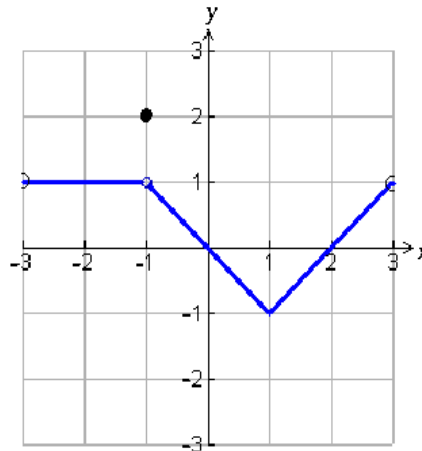
3. (a) Sketch a graph of $f(x) = \begin{cases} x^3 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

- (b) Find $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^+} f(x)$.
 (c) Does $\lim_{x \rightarrow 1} f(x)$ exist? If so, what is it? If not, why not?

4. The limit rules for two-sided limits (pg 78/9 Theorem 1) also apply to one-sided limits (pg 102 Theorem 8). Use the graphs below, the limit rules and Theorem 6 applied to one-sided limits to determine whether the following limits exist; if the limit exists, then find the limit, but if the limit does not exist, then explain why not.



Graph of $y = f(x)$.



Graph of $y = g(x)$.

- (a) $\lim_{x \rightarrow -1} [f(x) + g(x)]$
 (b) $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$
 (c) $\lim_{x \rightarrow 2} [f(x)g(x)]$
 (d) $\lim_{x \rightarrow -2} [g(f(x))]$
 (e) $\lim_{x \rightarrow -1} [f(g(x))]$.

5. Use the limit rules to find $\lim_{h \rightarrow 0^+} \frac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$

PART B

6. For each of the following, sketch a graph of a function, with domain \mathbb{R} , that satisfies all of the given properties.

(a) $\lim_{x \rightarrow 0} f(x) = 0$, $f(0) = 10$, $\lim_{x \rightarrow 1^+} f(x) = -1$, $\lim_{x \rightarrow 1^-} f(x) = 1$, $f(1) = 0$.

(b) $\lim_{x \rightarrow n^-} g(x) = n$, $g(n) = n + 1$ for every integer n .

PART C

7. The interior of a typical 1-L measuring cup is a right circular cylinder of radius $r = 6$ cm (see figure). The volume V of water we put in the cup is a function of the level h up to which the cup is filled, the formula being

$$V = (\pi r^2)h = \pi 6^2 h = 36\pi h.$$

Find, to two decimal places in cm (or mm), how closely we must measure h to measure out 1 L (1000 cm^3) of water with an error of no more than 1% (10 cm^3)? Sketch a graph of V against h , and indicate on your graph the maximum error of 1% allowed for V and the least accurate measurements of h corresponding to that error.

