

Due at the start of class on Mon Oct 6, 2008.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown \Rightarrow partial/full credit not possible, so show as much work as possible.

As stated at the end of Section 2.7, the following concepts are the same:

- the slope of $y = f(x)$ at $x = a$;
- the slope of the tangent line to the graph $y = f(x)$ at $x = a$;
- the rate of change of $f(x)$, with respect to x , at $x = a$;
- the derivative of f at $x = a$;
- the limit of the difference quotient $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.

PART A

1. An object is dropped from the top of a 98-meter-high tower. Its height above the ground after t seconds is $98 - 4.9t^2$ meters.
 - (a) How fast is the object falling 2 seconds after it is dropped?
 - (b) Does the speed of the object reach 45 m/sec? Explain.
2. Let $f(x) = 2x^3 + 6x^2 - 51x$.
 - (a) Find the slope of the graph of $y = f(x)$ at the point $(a, f(a))$.
 - (b) The graph $y = f(x)$ has some tangent lines parallel to the line $y = -3x + 100$. Find the equations of these lines.
3. Let b , c and d denote constants and let $f(x) = 2x^2 + bx + c$ and let $g(x) = dx - x^2$. The graphs $y = f(x)$ and $y = g(x)$ share a common tangent line at the point $(1, 0)$. Find b , c and d . (Hint: what are $f(1)$, $f'(1)$ etc?)

PART B

The Product Rule

If f and g are differentiable functions at x , then so is their product, and

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x).$$

The Quotient Rule

If f and g are differentiable functions at x , then so is the quotient $\frac{f(x)}{g(x)}$, and

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

4. Functions f and g , their first derivatives, f' and g' , and their second derivatives, f'' and g'' , are defined on \mathbb{R} ; their values at $x = 1$ and at $x = 2$ are given in the table:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$f''(x)$	$g''(x)$
1	1	4	-2	3	-1	6
2	-4	8	-2	12	-2	12

- (a) If $Q(x) = \frac{f(x)}{x^2}$, compute $Q'(2)$.
- (b) If $H(x) = 3f(x) + 2x^2$, compute $H''(1)$. (Hint: first find $H''(x)$.)
- (c) If $T(x) = x^2f(x)g(x)$, find an equation for the tangent line to the graph of $y = T(x)$ at $x = 2$.
- (d) If $G(x) = e^xg(x)$, compute $G'(1)$.
- (e) If $F(x) = e^{\ln(f(x))}$, compute $F'(1)$, if it exists; if it does not exist, explain why not.
- (f) If $F(x) = e^{\ln(f(x))}$, compute $F'(2)$, if it exists; if it does not exist, explain why not.
5. What was the first study technique (given on Worksheet 1)?