

Study Technique 4 Try this technique to help you prepare for a test. A couple/few days before the test, look over all the homework. The day/night before the test, look over the homework again, and ask yourself what the key concepts and methods are per question (WITHOUT reworking the questions). Jot this down on a sheet of paper and, when you are done, compare with your solutions to see if you were correct. Any question in which you were on the wrong track, look over your solution to it to make sure you see what the main idea is. This work the day/night before will help get your brain to think faster and bring the material to the front of your brain to help you think faster during the actual test.

These questions are not due. Test 2 will be Wed Oct 15 during lab time.

Below are practice questions for Test 2 (in no particular order). Test 2 will cover college algebra and material in Secs 2.1-3.6 inclusive. The homework is available from my website: www.uta.edu/math/vancliff/T/F08 .

1. Rework the practice questions provided for Test 1 on worksheet 3, and rework Test 1. Go over all quizzes, assigned homework and worksheet questions.
2. Functions f and g and their first and second derivatives, f' , g' , f'' , g'' , are defined on \mathbb{R} , and, at 0, 1, 2, they take on the values given in the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$f''(x)$	$g''(x)$
0	5	3	2	-10	7	3
1	0	3	2	-4	-3	4
2	-8	7	6	3	4	3

- (a) Let $G(x) = x^4 f(x)$. Compute $G'(2)$ if it exists; if it does not exist, explain why not.
 - (b) Let $H(x) = (g \circ f)(x) = g(f(x))$. Compute $H'(1)$ if it exists; if it does not exist, explain why not.
 - (c) Let $F(x) = e^{g(x)}$. Compute $F'(0)$ if it exists; if it does not exist, explain why not.
 - (d) Let $F(x) = e^{g(x)}$. Compute $F''(0)$ if it exists; if it does not exist, explain why not.
 - (e) Let $J(x) = \sin(x + g(x) - 3)$. Compute $J'(0)$ if it exists; if it does not exist, explain why not.
3. The functions f and g and their first and second derivatives, f' , g' , f'' , g'' , are defined on \mathbb{R} , and, at 0, 1, 2, they take on the values given in the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$	$f''(x)$	$g''(x)$
0	2	-3	-4	7	-2	0
1	-4	1	10	-2	-4	-5
2	6	5	5	3	3	4

(a) If $F(x) = \frac{f(x)}{x}$, compute $F'(2)$.

(b) If $G(x) = (g(x))^3$, compute $G''(2)$.

(c) If $H(x) = f(x)e^{3x}$, compute $H'(0)$.

4. (a) Find $\frac{d}{dx}(\sqrt{e^{\sqrt{x}}})$. (b) Find $\frac{d}{dx}(\sqrt{e^{\sqrt{t}}})$.

5. Some functions and their corresponding derivatives are shown. Give the steps needed to compute the derivatives (NOT by using a limit) and state which rules you use.

(a) $f(x) = e^{\sin x}$ $f'(x) = (\cos x)e^{\sin x}$

(b) $f(x) = \frac{5x + 2}{7 \tan x}$ $f'(x) = \frac{5 \cos x \sin x - 5x - 2}{7 \sin^2 x}$

(c) $4 \cos x \sin y = e^y$ $\frac{dy}{dx} = \frac{4 \sin x \sin y}{4 \cos x \cos y - e^y}$

6. Suppose f is a function with the property that $f'(x) = \cos(x^2)$. Find $g'(x)$, where $g(x) = f(x^2)$.

(a) $g'(x) = 2x \cos(x^4)$ (b) $g'(x) = \cos(x^4)$ (c) $g'(x) = \sin(x^4)$ (d) undefined (e) none of these.

7. Suppose f is a function with the property that $f'(x) = \cos(x^2)$. Find $h'(x)$, where $h(x) = f(\frac{1}{x})$.

(a) $h'(x) = -\frac{1}{x^2}$ (b) $h'(x) = \cos\left(\frac{1}{x^2}\right)$ (c) $h'(x) = -\frac{1}{x^2} \cos\left(\frac{1}{x^2}\right)$ (d) undefined (e) none of these.

8. Interpret $\lim_{h \rightarrow 0} \left(\frac{e^h - 1}{h} \right)$ as a derivative.

(a) $\frac{d}{dx}(e^0)$ (b) $\frac{d}{dx}(e^x)$ (c) $\frac{d}{dh}(e^h)$ (d) $\frac{d}{dx}(e^x)|_1$ (e) $\frac{d}{dx}(e^x)|_0$.

9. Interpret $\lim_{h \rightarrow 0} \left(\frac{\cos h - 1}{h} \right)$ as a derivative.

(a) $\frac{d}{dx}(\cos 0)$ (b) $\cos'(x)$ (c) $\cos'(h)$ (d) $\cos'(0)$ (e) $\cos'(1)$.

10. If $f(x) = \begin{cases} x^2 \cos(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, then $f'(0)$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) does not exist (e) not enough information given.

11. If $s(t) = \begin{cases} (t-1)^2 \cos(\frac{1}{t-1}) & \text{if } t \neq 1 \\ 0 & \text{if } t = 1 \end{cases}$, then $s'(1)$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) does not exist (e) not enough information given.

12. If $x \sin y + \cos 2y = \cos y$, then the value of $\frac{dy}{dx}$ at $\left(1, \frac{\pi}{2}\right)$ is

(a) 0 (b) -1 (c) 1 (d) does not exist (e) not enough information given.

13. Let $x = 1 + \frac{1}{t^2}$ and $y = 1 - \frac{3}{t}$. Find an equation for the line in the xy -plane that is tangent to the curve at $t = 2$. Also find the value of $\frac{d^2y}{dx^2}$ at $t = 2$.

14. A dog swims back and forth in a swimming pool for 6 minutes, with position given by $s(t) = t^3 - 10t^2 + 25t$ feet after t minutes. What is the total distance swum by the dog?

(a) 10 ft (b) 12 ft (c) roughly 39 ft (d) roughly 43 ft (e) 0 ft (f) 6 ft.

Attached is the 2nd test given in Honr 1426 in Fall 2007, but that test was a week later than our test is going to be and so contained some topics that will not be on our Test 2.

You may use a simple calculator (as described on first-day handout). Request paper for rough work.

You have 50 minutes. Total number of points = 50. Keep your eyes on your own work!!

No partial credit will be awarded for questions 1-10; on these questions, only one choice is correct, and you should circle the correct answer. Each of questions 1-10 is worth 2 points. Partial credit will be awarded for questions 11-13, so show as much work & clear reasoning as possible on those questions.

1. [§3.2:~35] Let $f(x) = 5x^3e^x$. Find the slope of the graph $y = f(x)$ at $x = -2$.

- (a) 0 (b) $\frac{52}{e^2}$ (c) $\frac{20(3e+4)}{e^3}$ (d) $\frac{60}{e^2}$ (e) $-\frac{120}{e^3}$ (f) $\frac{20}{e^2}$.

2. [§3.5:~9] Let $f(x) = (7x^2 + 2x - 1)^9$. Find the slope of the graph $y = f(x)$ at $x = 0$.

- (a) -9 (b) 256 (c) 9 (d) 2304 (e) 18 (f) 0.

3. [§2.7:~11] A function f and its derivative, f' , are defined on \mathbb{R} , and, at 0, 1, they take on the values

given in the table:

x	$f(x)$	$f'(x)$
0	3	-3
1	-3	11

 Find the equation of the tangent line to the graph of f at $x = 1$.

- (a) $y = 11x - 14$ (b) $y = 11x - 4$ (c) $y = 8 - 11x$ (d) $y = -11x - 4$ (e) $y = x - 4$ (f) $y = 11x - 8$.

4. [§3.7:~11-15] Let $H(x) = \ln[f(x)]$, where f is the function in Question 3; compute the derivative

- $H'(0)$. (a) $\ln 3$ (b) $\frac{1}{3}$ (c) 0 (d) -1 (e) 1 (f) does not exist.

5. [§3.8] Let $K(x) = \tan^{-1}(f(x)) = \arctan(f(x))$, where f is the function in Question 3; compute

- the derivative $K'(1)$. (a) 1.1 (b) 1 (c) π (d) $-\frac{\pi}{11}$ (e) $\frac{3}{11}$ (f) does not exist.

6. [§3.1:~1] For all functions f , the slope of the tangent line to the graph of f at the point $(2, f(2))$ is defined to be

(a) $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$ (b) $\lim_{h \rightarrow 0} \frac{f(2) + h - 2}{h}$ (c) $\lim_{h \rightarrow 2} \frac{f(h) - f(2)}{h}$

(d) $\lim_{h \rightarrow 0} \frac{f(2+x) - f(2)}{h}$ (e) $\lim_{h \rightarrow 0} \frac{f(h)}{h}$ (f) $\lim_{h \rightarrow 0} \frac{f(2)}{h}$.

7. [§2.6,~Q4] Let a & b be nonzero constants. Determine a condition on a and b so that the function

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & \text{if } x \neq a \\ -2b & \text{if } x = a \end{cases} \quad \text{will be continuous on } \mathbb{R}. \quad \begin{array}{lll} \text{(a) } ab = 7 & \text{(b) } a = b & \text{(c) } a = -b \\ \text{(d) } a + b = 2 & \text{(e) } ab = x & \text{(f) } a^b = 1. \end{array}$$

8. [§3.5:~103] Let $x(t) = 4t^3 + 7t^2 - 2t + 1$ and $y(t) = 8t^6 - 3t^2 + 5t$, find $\frac{dy}{dx}$ at $t = 0$.

(a) 0 (b) 1 (c) -2.5 (d) $\sqrt{3}$ (e) -4 (f) x^2 .

9. [§3.9:1] Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of time t .

Write an equation that relates $\frac{dA}{dt}$ to $\frac{dr}{dt}$. (a) $\frac{dA}{dt} = 2\pi r$ (b) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

(c) $\frac{dA}{dt} = 2\pi \frac{dr}{dt}$ (d) $\frac{dA}{dt} = 2\pi r^2$ (e) $\frac{dA}{dt} = \pi \frac{dr}{dt}$ (f) $\frac{dA}{dt} = 2\pi \left(\frac{dr}{dt}\right)^2$.

10. [§3.7:~7,8] Let $f(x) = 7 + 2x + e^x$. Find the value of $\frac{df^{-1}}{dx}$ at $x = 8 = f(0)$.

(a) $\frac{1}{3}$ (b) 1 (c) 0 (d) $\frac{1}{2}$ (e) nonexistent (f) indeterminable without extra information.

11. [§3.6:~19-32] [10 points] If $5y^2 + 10xy + \cos x = 3e^y - 4$, find the exact value of $\frac{dy}{dx} = y'$ at $(\pi, 0)$.

12. [§3.3:~7] [10 points] A person is standing on a cliff with a yoyo. As a function of time $t \in [0, 3]$ seconds, the position of the yoyo is given by $s(t) = -2t^3 + 9t^2 - 12t + 9$ meters. Find the total distance through which the yoyo moves in the given time interval.

13. [§3.7:~89-96;W8] Suppose that f is a function that is differentiable for all real x , with derivative denoted f' .

Let $y = x^{f(x)}$ for all $x > 0$.

(a) [7 points] Find $\frac{dy}{dx}$ as a function of f , f' and x .

(b) [3 points] If $f(e) = \ln 7$ and $f'(e) = 1 - \frac{\ln 7}{e}$, find $\frac{dy}{dx}$ at $x = e$.

BONUS QUESTION (no partial credit) [2 points] Suppose $p(x) = \begin{cases} 2 - e^{-x} & \text{if } x \leq 0 \\ e^x & \text{if } x > 0. \end{cases}$

Compute $\lim_{h \rightarrow 0^+} \left(\frac{p(h) - p(0)}{h} \right)$.