HONR 1426 FALL 2008

Problem Solving: Worksheet 9

NAME:_____

Due at the start of class on Mon Nov 3, 2008.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown $\Rightarrow$ partial/full credit not possible, so show as much work as possible.

1. The goal of this question is to show that there is no number a for which the equation $x^3 - 3x + a = 0$ has *two* (or more) distinct solutions on $[-1,\ 1]$.

   (a) Suppose a is some number and assume that $x^3 - 3x + a$ has two distinct solutions, say s and t in $[-1,\ 1]$; i.e., $s^3 - 3s + a = 0$ and $t^3 - 3t + a = 0$. Write $f(x) = x^3 - 3x + a$. What can be said about $f(s)$ and $f(t)$?

   (b) What does Rolle's Theorem tell you about the derivative of f on the interval $(s,\ t)$?

   (c) Find the derivative of f.

   (d) Combine the information from (b) and (c) and derive a contradiction.

   (e) Your contradiction in (d) means that a prior statement is false. Which statement is false? What can you conclude from that?

2. The goal of this question is to show that for all numbers a and b, where $a > 0$, and for all positive integers n, the polynomial $p(x) = x^{2n+1} + ax + b$ has at most one real root.

   (a) Suppose a, b and n are numbers with $a > 0$ and with n a positive integer, and assume that p has two real roots, say s and t. What does this imply about $p(s)$ and $p(t)$?

   (b) What does Rolle's Theorem tell you about the derivative of p on the interval $(s,\ t)$?

   (c) Find the derivative of p.

   (d) Combine the information from (b) and (c) and derive a contradiction.

   (e) Your contradiction in (d) means that a prior statement is false. Which statement is false? What can you conclude from that?

Recall that the essay assignment is due Monday Nov 24, 2008.