MATH 1426 SPRING 2009

Problem Solving: Worksheet 10

NAME:_____

Due at the start of lecture (not lab) on Thurs April 23, 2009.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown ⇒ partial/full credit not possible, so show as much work as possible.

PART A

1. A bus company will charter a bus that holds 50 people to groups of 35 or more. If a group contains exactly 35 people, each person pays $60. In larger groups, everyone's fare is reduced by $1 for each person in excess of 35.

   (a) What is the revenue, if the bus is chartered to 35 people? 36 people? 37 people?

   (b) Find a formula for the revenue in terms of the number of people chartering the bus.

   (c) Determine the size of the group(s) for which the bus company's revenue will be the greatest. (Note: you need to check that your answer makes physical sense and that it is correct as the physical constraints allow.)

2. Of all triangles that pass through the point $(2,\ 1)$ and have two sides on the coordinate axes, find the dimensions of the one having the smallest area.

3. The number of boxes of the breakfast cereal Nutty Fruity sold in a month is $5000e^{-0.5p}$ boxes, where p is the price per box in dollars. Find a formula for the total consumer expenditure per month (i.e., the amount of money consumers will spend on Nutty Fruity in a month). Determine the price that will result in the greatest consumer expenditure.

4. Two towns A and B are 13 miles apart and are located 8 miles and 3 miles, respectively, from a long, straight highway. A construction company has a contract to build a road from A to the highway and then to B. Determine the length (to the nearest tenth of a mile) of the shortest road that meets these requirements.

PART B

5. In class, we saw (without using L'Hôpital's Rule) that $\lim_{x\to 0}\left[\left(e^{-\frac{1}{x^2}}\right)^{x^2}\right] = \frac{1}{e}$, even though its type is 0^0. Let $a \in (0, 1)$. Show that the indeterminate form of $\lim_{x\to 0}\left[\left(e^{-\frac{\ln(1/a)}{x^2}}\right)^{x^2}\right]$ is 0^0 and, withOUT using L'Hôpital's Rule, show that this limit equals a.

6. An indeterminate form of type ∞^0 can be any positive real number. Let $a \in (0, \infty)$. Show that the indeterminate form of $\lim_{x\to\infty} x^{(\ln a)/(1+\ln x)}$ is ∞^0 and, using L'Hôpital's Rule, show that this limit equals a.

7. An indeterminate form of type 1^∞ can be any positive real number. Let $a \in (0, \infty)$. Show that the indeterminate form of $\lim_{x\to 0^+} (x+1)^{(\ln a)/x}$ is 1^∞ and, using L'Hôpital's Rule, show that this limit equals a.

8. An indeterminate form of type 1^∞ can be infinite. Show that the indeterminate form of $\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{e^x}\right]$ is 1^∞ and, using L'Hôpital's Rule, show that this limit is infinite.

9. For the limits in Questions 6-8, verify that the hypotheses of L'Hôpital's Rule are satisfied wherever you have used it.