MATH 1426 SPRING 2009

Problem Solving: Worksheet 11

NAME:________________________

Due at the start of lecture (not lab) on Thurs April 30, 2009.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown ⇒ partial/full credit not possible, so show as much work as possible.

PART A

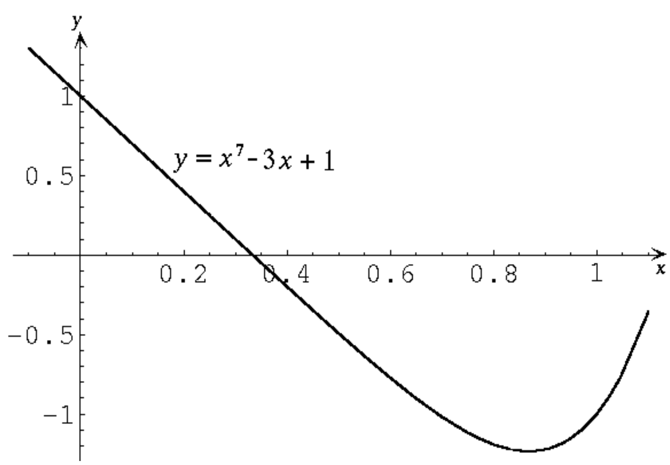
1. Here is an example that L'Hôpital used in his 1696 book (the first calculus textbook ever published) to illustrate the method we now call L'Hôpital's Rule. Let $a \in (0, \infty)$. Using one application of L'Hôpital's Rule, find $\displaystyle\lim_{x\to a} \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$.

PART B

In this part, our goal is to find a method that estimates solutions to equations of the form $f(x) = 0$ where f is a differentiable function.

2. Suppose $f(x) = x^7 - 3x + 1$. We seek an $x \in [0,\ 1]$ where $f(x) = 0$.

   (a) The graph of $y = f(x)$ is given below on $[0,\ 1]$. The graph crosses the x-axis at the solution we seek (why?). The tangent line L_1 at $(0,\ 1)$ crosses the x-axis very close to the solution. Find an equation for the tangent line L_1 at $(0,\ 1)$.



   (b) Find where L_1 crosses the x-axis. Call that x-coordinate x_1. Note that we may view x_1 as an estimate of the solution.

   (c) The tangent line L_2 at $(x_1,\ f(x_1))$ crosses the x-axis even closer to the solution than did x_1. Find an equation for L_2.

(d) Find where L_2 crosses the x-axis. Call that x-coordinate x_2. Note that we may view x_2 as an estimate of the solution, and that it is a better estimate than was x_1. In fact, this x_2 approximates the solution correctly to 8 decimal places.

(e) In summary, we have a sequence of numbers x_0, x_1, x_2 where we chose to use $x_0 = 0$, and used tangent lines L_1 and L_2 to find x_1 and x_2. L_1 crossed the x-axis closer to the solution than did x_0, and L_2 crossed the x-axis closer to the solution than did x_1. We could continue this process as long as we wish and thereby obtain better and better approximations each time to the solution. Imagine you do continue, and that, at some stage, you have an approximation x_n to the solution. The tangent line L_{n+1} at $(x_n, f(x_n))$ crosses the x-axis even closer to the solution than did x_n. Find an equation for L_{n+1}. (You should find that x_n appears in your equation since you do not know its value.) This equation generalizes the equations you found in (a) and (c).

(f) Use your equation in (e) to find where L_{n+1} crosses the x-axis. Your solution is x_{n+1}. Your formula for x_{n+1} should agree with that in (1) on page 326 of your textbook; if not, edit your work so that it does.

3. Read Section 4.7 in the textbook.
4. Do question 23 on page 330.
5. Do question 8 on page 329.
6. Do question 11 on page 330.