Problem Solving: Worksheet 2

NAME:________________________

Study Technique 2 While working through homework, it is very tempting to use a solution manual. Solution manuals can be very helpful and very effective, but only if used correctly. The correct way to use a solution manual is, if possible, to read the first one or two lines of a solution and then try to continue the problem from there without looking at the manual's solution again. If that is not possible, then read the solution all the way through, but then close the manual and try to reproduce the solution (or at least the main ideas of the solution) without the aid of the manual.

Due at the start of lecture (not lab) on Thursday Feb 5, 2009.

Answer the following questions in groups of two or three; turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown $\Rightarrow$ partial/full credit not possible, so show as much work as possible.

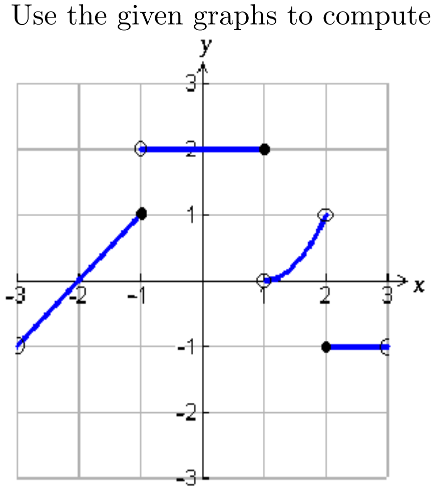
PART A

Our determination of whether or not $\lim_{x\to c} f(x)$ exists entails study of the behavior of f as $x \to c$ from the left and also study of the behavior of f as $x \to c$ from the right. We may use notation for these one-sided limits: $\lim_{x\to c^-} f(x)$ and $\lim_{x\to c^+} f(x)$ (as was done in lecture). These one-sided limits are related to $\lim_{x\to c} f(x)$ via Theorem 6 (pg 97) which says:

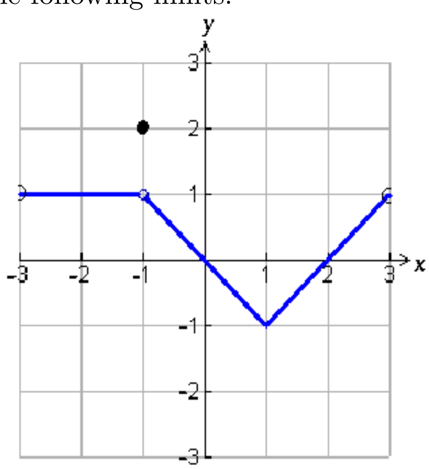
THEOREM 6
A function $f(x)$ has a limit as x approaches c if and only if it has left-hand and right-hand limits there and these one-sided limits are equal:

$$\lim_{x\to c} f(x) = L \quad \Leftrightarrow \quad \lim_{x\to c^-} f(x) = L \quad \text{and} \quad \lim_{x\to c^+} f(x) = L.$$

1. Use the given graphs to compute the following limits:



Graph of $y = f(x)$.



Graph of $y = g(x)$.

(a) $\lim_{x\to 1^-} f(x)$

(b) $\lim_{x\to 1^+} f(x)$

(c) $\lim_{x\to 1} f(x)$

(d) $\lim_{x\to -1^-} g(x)$

(e) $\lim_{x\to -1^+} g(x)$

(f) $\lim_{x\to -1} g(x)$.

2. If $\lim_{x \to c} f(x)$ does not exist, then any limit rule (pg 78/9 Theorem 1) that involves f **canNOT be used**; why not?

3. In some of the following questions concerning the above graphs, $\lim_{x \to c} f(x)$ does not exist or $\lim_{x \to c} g(x)$ does not exist, so (as explained in the previous question) any limit rule (pg 78/9 Theorem 1) that involves f or g in those questions **canNOT be used**. **Instead**, we use one-sided limits and apply the limit rules to the one-sided limits (pg 102 Theorem 8). Use the limit rules for one-sided limits together with Theorem 6 (stated above) to determine whether the following limits exist for the above given graphs; if the limit exists, then find the limit, but if the limit does not exist, then explain why not. (Note: if you simply apply the limit rules directly to each question without using a one-sided limit, you will obtain erroneous answers.)

   (a) $\lim_{x \to -1} [f(x) + g(x)]$

   (b) $\lim_{x \to 0} \left[\frac{f(x)}{g(x)}\right]$

   (c) $\lim_{x \to 2} [f(x)g(x)]$

   (d) $\lim_{x \to -2} [g(f(x))]$

   (e) $\lim_{x \to -1} [f(g(x))]$.

PART B

4. Use the limit rules to find the following limits. At any place where you use a limit rule or a theorem, state which one you use.

   (a) $\lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

   (b) $\lim_{x \to 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3}$

   (c) $\lim_{x \to 0} \frac{x^2}{1 - \sqrt{1 - x^2}}$

   (d) $\lim_{x \to a^+} \frac{|x - a|}{x - a}$

   (e) $\lim_{x \to 0} \frac{\sin^2 2x}{x^2}$

   (f) $\lim_{x \to 2^-} f(x)$, $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2. \end{cases}$