

Study Technique While writing solutions to your homework, practice the mathematical notation that is used in lecture and in your book; use the notation in your solutions. This way, you will become familiar with the notation and how to use it before the next big test, so that using the notation in that test will feel natural and you will be more likely to use the notation correctly.

Due at the start of lecture (not lab) on Thurs April 9, 2009.

Answer the following questions in groups of two, but turn in one solution sheet per student. Write neatly and orderly as points will be deducted for messy work. No work shown \Rightarrow partial/full credit not possible, so show as much work as possible.

1. Decide if the statements (a)-(e) are true or false. Give an explanation for each answer (i.e., if true, reference a theorem/definition in the text; if false, give an example to show the statement is false).
 - (a) If a function is continuous on a closed interval, then its absolute maximum can occur at two different values in the interval.
 - (b) If $x = c$ is a critical number of a function f , then it is also a critical number of the function $g(x) = f(x) + k$, where k is a constant.
 - (c) A critical point of a function f must give a local maximum or local minimum of f .
 - (d) Since the function $f(x) = 1/x$ is continuous for all $x > 0$, f has an absolute maximum on the interval $(0, 1)$.
 - (e) The Extreme-Value Theorem says that only continuous functions have absolute extrema on closed intervals of the form $[a, b]$, where $a \neq b$.

2. A rectangle is bounded by the x -axis, the y -axis and the graph of $y = (6 - x)/2$. What length and width should the rectangle have so that its area is a maximum?

3. A piece of wire, 100 cm long, is to be cut into several pieces and used to construct the skeleton of a rectangular box with a square base.
 - (a) What are the dimensions of the box with the largest volume?
 - (b) What are the dimensions of the box with the largest surface area?

4. Sketch the graph of one function f with all the following properties:

$$\begin{array}{lll} f'(x) < 0 & \text{for } x \in (-\infty, -6) \cup (2, \infty), & f(-6) = 5, \\ f'(x) > 0 & \text{for } x \in (-6, 2), & f(2) = -1. \end{array}$$

(Note: you are NOT asked to find a formula for f , and there could be more than one correct answer to this question, or there could be no such f .)

If you claim that there is no such f , then you should justify your claim.

5. Sketch the graph of one function f with all the following properties:

$$\begin{array}{llll} f'(x) < 0 & \text{for } x < 1, & f'(x) > 0 & \text{for } x > 1, \\ f''(x) < 0 & \text{for } x < 1, & f''(x) < 0 & \text{for } x > 1. \end{array}$$

(Note: you are NOT asked to find a formula for f , and there could be more than one correct answer to this question, or there could be no such f .)

If you claim that there is no such f , then you should justify your claim.

What can you say, if anything, about the derivative/slope of f at $x = 1$?

6. Sketch the graph of one function f with all the following properties:

$$\begin{array}{ll} f'(x) < 0 & \text{for } x < -1, \\ f'(x) < 0 & \text{for } x > 3, \\ f'(x) > 0 & \text{for } x \in (-1, 3), \\ f''(x) > 0 & \text{for } x < 2, \\ f''(x) < 0 & \text{for } x > 2. \end{array}$$

(Note: you are NOT asked to find a formula for f , and there could be more than one correct answer to this question, or there could be no such f .)

If you claim that there is no such f , then you should justify your claim.