

Elegant and Inelegant Proofs

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Other things being equal (in Latin, *ceteris paribus*), a shorter proof is a better proof. Let me illustrate this by giving two proofs, one of which is shorter—and more elegant—than the other. (Elegance, like height, is a matter of degree.) Suppose I set out to prove that

$$(N \cdot P) \supset O$$

follows from

$$N \supset O.$$

This is exercise 6 from part V on page 380 of our book. Here is the first of two proofs:

1. $N \supset O$	assumption (i.e., premise)
2. $(N \supset O) \vee \sim P$	1, Add.
3. $(\sim N \vee O) \vee \sim P$	2, Impl. (or MI)
4. $\sim N \vee (O \vee \sim P)$	3, Assoc.
5. $\sim N \vee (\sim P \vee O)$	4, Com.
6. $(\sim N \vee \sim P) \vee O$	5, Assoc.
7. $\sim(N \cdot P) \vee O$	6, DeM. (or DM)
8. $(N \cdot P) \supset O$	7, Impl. (or MI)

Here is the proof given on page 135 of the *Solutions Manual*:

1. $N \supset O$	assumption (i.e., premise)
2. $(N \supset O) \vee \sim P$	1, Add.
3. $\sim P \vee (N \supset O)$	2, Com.
4. $P \supset (N \supset O)$	3, Impl. (or MI) ¹
5. $(P \cdot N) \supset O$	4, Exp.
6. $(N \cdot P) \supset O$	5, Com.

As you can see, the first proof is longer (by two steps) than the second. Does this matter? It depends. If the question is *whether it constitutes a proof*, the answer is “No; it doesn’t matter.” Both are proofs

¹The manual has “2, Impl.,” but it should be “3, Impl.”

that the conclusion,

$$(N \cdot P) \supset O,$$

follows from the premise,

$$N \supset O.$$

Often there is more than one path to a given destination. Suppose you want to get to Denton from the UTA campus. One way to get there is to go east to I-35E and drive north. Another way is to go west to I-35W and drive north. Both routes will get you there, but one route may take longer. (There is nothing wrong, *per se*, with taking the long route to a destination. It may be more scenic, less stressful, or safer. Sometimes it's just fun to take the road less traveled, whatever the distance.)

Logicians strive to produce elegant proofs. The shorter the proof, the more elegant it is. By this standard, the first proof is inferior to the manual's proof. It gets the job done, just not as elegantly. Do you find it odd that elegance or simplicity should be a desideratum to logicians? It is to scientists. Other things (such as explanatory power) being equal, scientists prefer simpler theories to more complex theories. A simpler theory is one that postulates fewer types of entity or makes fewer assumptions. The aim of a prudent consumer is to get a lot of bang for his or her buck. The aim of a scientist is to *explain* a lot with a little. The aim of a logician is to *prove* a lot with a little.

By the way, in case it seems strange that

$$(N \cdot P) \supset O$$

follows from

$$N \supset O,$$

think of it this way. The latter says that N, *all by itself*, implies O. But if N all by itself implies O, then N conjoined with any other proposition implies O. Example: My being a bachelor is sufficient for

my being a male; therefore, my being *both* a bachelor *and* a homeowner (or a conservative, or a bicyclist, or a native Michigander) is sufficient for my being a male.