Truth Tables
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Truth tables, like Venn diagrams and squares of opposition, are tools. They help us perform certain tasks. Venn diagrams are used primarily to test the validity of standard-form categorical syllogisms (e.g., EAO-3). Squares of opposition are used primarily to show the logical relations between standard-form categorical propositions (All S is P; No S is P; Some S is P; Some S is not P). Truth tables have at least *four* uses. (This should not be surprising. A claw hammer—a different tool—can be used to pound a nail, pull a nail, stop a door, keep paper from blowing away, and kill people.) Here is a summary of the four uses of truth tables:

1. To define truth-functional connectives. A truth table shows how the truth value of a truth-functional compound statement depends on the truth values of its component (simple) statements. See Copi and Cohen, *Introduction to Logic,* 12th ed., 311. We can think of the truth table as *defining* the various truth-functional connectives (the dot, the curl, the wedge, the horseshoe, and the three-bar sign).

2. To classify statement forms. Every statement form is either tautologous (i.e., a tautology), self-contradictory (i.e., a self-contradiction), or contingent (i.e., neither a tautology nor a self-contradiction).

a. Statement form p is tautologous.

i. Definition. Statement form p is tautologous (i.e., logically true) if and only if (abbreviated "iff") p has only true substitution instances.

ii. Indication. The column under the main connective contains all "Ts."

iii. Example. p v ~p.

b. Statement form p is self-contradictory.

i. Definition. Statement form p is self-contradictory (i.e., logically false) iff p has only false substitution instances.

ii. Indication. The column under the main connective contains all "Fs."

iii. Example. p • ~p.

c. Statement form p is contingent.

i. Definition. Statement form p is contingent (i.e., neither logically true nor logically false) iff p has at least one true substitution instance and at least one false substitution instance.

ii. Indication. The column under the main connective contains at least one "T" and at least one "F."

iii. Example. p • q.

3. To compare statement forms. Every pair of statement forms is either logically equivalent, contradictory, or neither.

a. Statement forms p and q are logically equivalent.

i. Definition. Statement forms p and q are logically equivalent iff the statement of their material equivalence is a tautology (see above for the meaning of "tautology").

ii. Indication. The column under the main connective (the biconditional sign) contains all "Ts."

iii. Example. (p ⊃ q) is logically equivalent to (~p v q).

b. Statement forms p and q are contradictory.

i. Definition. Statement forms p and q are contradictory iff the statement of their material equivalence is a self-contradiction (see above for the meaning of "self-contradiction”).

ii. Indication. The column under the main connective (the biconditional sign) contains all “Fs.”

iii. Example. (p ⊃ q) is the contradictory of (p • ~q).

c. Statement forms p and q are neither logically equivalent nor contradictory.

i. Definition. Statement forms p and q are neither logically equivalent nor contradictory iff the statement of their material equivalence is a contingent statement (see above for the meaning of “contingent statement”).

ii. Indication. The column under the main connective (the biconditional sign) contains at least one “T” and at least one “F.”

iii. Example. (p v q) is neither logically equivalent to nor the contradictory of (p • q).

4. To classify argument forms. Every argument form is either valid (i.e., truth-preserving) or invalid (i.e., not truth-preserving).

a. Argument form A is valid.

i. Definition. Argument form A is valid iff A has only valid arguments as substitution instances.

ii. Indication. There is no row in which A’s premises are true and A’s conclusion is false.

iii. Example. p ⊃ q; p; therefore, q.

b. Argument form A is invalid.

i. Definition. Argument form A is invalid iff A has at least one invalid argument as a substitution instance.

ii. Indication. There is at least one row in which A's premises are true and A's conclusion is false.

iii. Example. p ⊃ q; q; therefore, p.

You will be tested on each concept as well as on your ability to use truth tables for each of the four stated purposes.