PHYS 1443

Ch. 5 Applications of Newton's Laws

Resistive Forces and Terminal Velocity

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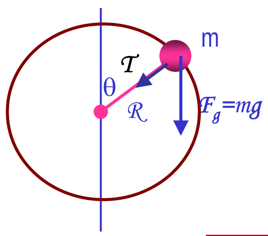
Example for Non-Uniform Circular Motion

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle. Determine the tension of the cord at any instant when the speed of the ball is v and the cord makes an angle θ with vertical.



- The gravitational force F_g
- The radial force, T, providing tension.

What will be the best coordinate ?

tangential comp.

$$\sum F_t = mg\sin\theta = ma_t$$

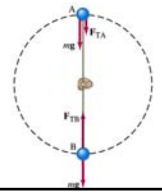
$$a_t = g\sin\theta$$

Radial comp.

$$\sum F_r = T + mg\cos\theta = ma_r = m\frac{v^2}{R}$$

$$T = m\left(\frac{v^2}{R} - g\cos\theta\right)$$

At what angles the tension becomes maximum and minimum. What are the tensions?



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Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional property of the medium.

Some examples? Air resistance, viscous force of liquid, etc

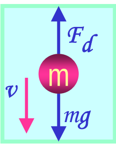
These forces are exerted on moving objects in opposite direction of the movement.

These forces are proportional to such factors as speed and viscosity.

Assuming there is no dragging force, what will be the speed of rainfall?

Resistive Force Proportional to Speed

Let's consider that a ball (*m*) is falling through a Fluid.



$$\sum \vec{F} = \vec{F}_g + \vec{F}_d = m\vec{a} \begin{cases} when...|\vec{F}_g| > |\vec{F}_d|, then |a| > 0 \\ when...|\vec{F}_g| = |\vec{F}_d|, then |a| = 0 \\ when...|\vec{F}_g| < |\vec{F}_d|, then |a| < 0 \end{cases}$$

If drag force is $F_d = bv$.
When $a = 0$?

$$\sum F_y = mg - bv = ma = m\frac{dv}{dt} \qquad \frac{dv}{dt} = a = 0 = g - \frac{b}{m}v$$

$$v = \frac{mg}{b} = v_T$$

Terminal velocity

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Ch. 6 Gravitation and Newton's Synthesis

- Gravity
- Weightless
- Kepler's Laws

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Newton's Law of Universal Gravitation

People have been very curious about stars in the sky, making observations for a long time. But the data people collected have not been explained until Newton has discovered the law of gravitation.

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

A long statement to a short formula

$$F_g \propto \frac{m_1 m_2}{r_{12}^2} \quad \xrightarrow{\text{With } G} \quad F_g = G\frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11} \quad N \cdot m^2 / kg^2$$

This constant is not given by the theory but must be measured by experiments.

This form of forces is known as the inverse-square law, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

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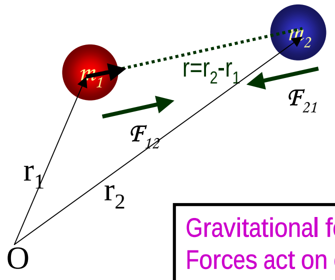
More on Law of Universal Gravitation

Consider two particles exerting gravitational forces to each other.



$$\vec{F}_{12} = G\frac{m_1 m_2}{r^2}\hat{r} = -\vec{F}_{21}$$

$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| \text{, but opposite directions.}$$

Gravitational force is a field force:
Forces act on object without physical contact between the objects at all times, independent of medium between them.

Point mass: The gravitational force exerted by a finite size, spherically symmetric mass distribution on an object outside of it is the same as when the entire mass of the distributions is concentrated at the center of the object.

What do you think the gravitational force on the surface of the earth look?

$$F_g = G\frac{M_E m}{R_E^2}$$

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Example for Gravitation

Using the fact that g=9.80m/s^2 at the Earth's surface, **find the average density of the Earth.**

Since the gravitational acceleration is

$$mg = G\frac{mM_E}{R_E^{\ 2}} = 6.67\times 10^{-11}\frac{M_E}{R_E^{\ 2}}$$

$$M_E = \frac{R_E^{\ 2} g}{G} \longrightarrow$$

Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E^{\ 2} g}{G}}{\frac{4\pi}{3}R_E^{\ 3}} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times 9.80}{4\pi\times 6.67\times 10^{-11}\times 6.37\times 10^{6}} = 5.50\times 10^{3} kg/m^3$$

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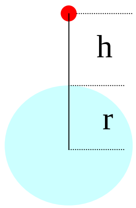
Free Fall Acceleration & Gravitational Force

What would the gravitational acceleration be if the object is at an altitude *h* above the surface of the Earth?



$$F_g = mg' = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E + h)^2}$$

$$g' = G\frac{M_E}{(R_E + h)^2}$$

Distance from the center of the Earth to the object at the altitude h.

What do these tell us about the gravitational acceleration?

- The gravitational acceleration is independent of the mass of the object
- The gravitational acceleration decreases as the altitude increases
- If the distance from the surface of the Earth gets infinitely large, the weight of the object approaches 0.

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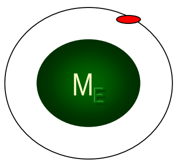
Example for Gravitational Force

The international space station is designed to operate at an altitude of 350km. When completed, it will have a weight (measured on the surface of the Earth) of 4.22x10[6]N. What is its weight when in its orbit?



The total weight of the station on the surface of the Earth is

$$F_{GE} = mg = G\frac{M_E m}{R_E^{\ 2}} = 4.22 \times 10^6 N$$

Since the orbit is at 350km above the surface of the Earth, the gravitational force at that height is

$$F_O = mg' = G\frac{M_E m}{(R_E + h)^2} = \frac{R_E^2}{(R_E + h)^2} F_{GE}$$

Therefore the weight in the orbit is

$$F_O = \frac{R_E^2}{(R_E + h)^2} F_{GE} = \frac{(6.37 \times 10^6)^2}{(6.37 \times 10^6 + 3.50 \times 10^5)^2} \times 4.22 \times 10^6 = 3.80 \times 10^6 N$$

What keeps the satellite up ?

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