PHYS 1443

Ch. 7 Work and energy

Work done by a constant force
Scalar Product of Vectors
Work done by a varying force
Work and Kinetic Energy Theorem
Potential Energy

Homework # 5 : solution of the 1st Exam (paper)
Homework #6 : Examples in Ch. 6 (paper)
→ change numbers in the examples by yourself

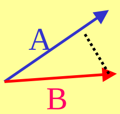
Scalar Product of two vectors

- Product of magnitude of the two vectors and the cosine of the angle between them

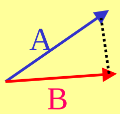
$$\vec{A} \bullet \vec{B} \equiv |\vec{A}||\vec{B}|\cos\theta$$ →Scalar product →Dot product

Vector Vector Scalar

$|\vec{A}|(|\vec{B}|\cos\theta)$



or



$|\vec{B}|(|\vec{A}|\cos\theta)$

$$\vec{A}\bullet\vec{B} = |\vec{A}||\vec{B}|\cos\theta = \begin{cases} 0 & \text{Minimum} \quad \text{When } \theta = 90^o \\ |\vec{A}||\vec{B}| & \textbf{Maximum} \quad \text{When } \theta = 0^o \end{cases}$$

Scalar Product of Vectors

- Operation is commutative $\vec{A}\bullet\vec{B} = |\vec{A}||\vec{B}|\cos\theta = |\vec{B}||\vec{A}|\cos\theta = \vec{B}\bullet\vec{A}$
- Operation follows distribution law of multiplication $\vec{A}\bullet(\vec{B}+\vec{C}) = \vec{A}\bullet\vec{B} + \vec{A}\bullet\vec{C}$
- Scalar products of Unit Vectors

$$\hat{i}\bullet\hat{i} = \hat{j}\bullet\hat{j} = \hat{k}\bullet\hat{k} = 1 \qquad \hat{i}\bullet\hat{j} = \hat{j}\bullet\hat{k} = \hat{k}\bullet\hat{i} = 0$$

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k} \qquad \vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$$

$$\vec{A}\bullet\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\bullet\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = \left(A_xB_x\,\hat{i}\bullet\hat{i} + A_yB_y\,\hat{j}\bullet\hat{j} + A_zB_z\,\hat{k}\bullet\hat{k}\right) + \text{cross terms}$$

cross terms =0

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

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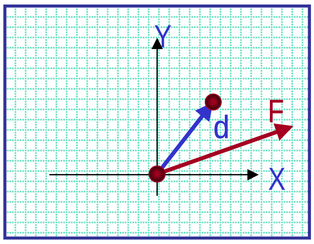
Example of Work by Scalar Product

A particle moving in the xy plane undergoes a displacement **d**=(2.0**i**+3.0**j**)m as a constant force **F**=(5.0**i**+2.0**j**) N acts on the particle.



a) Calculate the magnitude of the displacement and that of the force.

$$|\vec{d}| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6m$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4N$$

b) Calculate the work done by the force F.

$$W = \vec{F}\cdot\vec{d} = \left(2.0\,\hat{i} + 3.0\,\hat{j}\right)\bullet\left(5.0\,\hat{i} + 2.0\,\hat{j}\right) = 2.0\times5.0\,\hat{i}\cdot\hat{i} + 3.0\times2.0\,\hat{j}\cdot\hat{j} = 10 + 6 = 16(J)$$

or $W = \vec{F}\cdot\vec{d} = |\vec{F}||\vec{d}|\cos\theta$

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Examples

1. work done by the applied force F

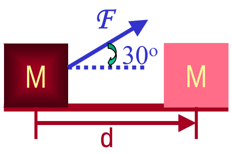
$$W_{app} = \left(\sum \vec{F}\right) \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta = \left|\vec{F}\right|\left|\vec{d}\right|$$

2. work done by the gravitational force F_g

$$W_g = \left(\sum \vec{F}_g\right) \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta = 0$$

Revisit the Example

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F=50.0N at an angle of 30.0º with East. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced by 3.00m to East.



$$W = \left(\sum \vec{F}\right) \cdot \vec{d} = \left|\left(\sum \vec{F}\right)\right|\left|\vec{d}\right| \cos\theta$$

or

$$W = \left(\sum \vec{F}\right) \cdot \vec{d} = \left|\vec{F}_x\right|\left|\vec{d}\right|$$

$$W = 50.0 \times 3.00 \times \cos 30^{\circ} = 130J$$

- Work done by the force along y-axis (F_y)

$$W_y = \left(\sum \vec{F}_y\right) \cdot \vec{d} = \left|\vec{F}_y\right|\left|\vec{d}\right| \cos 90^{o} = 0$$

Work Done by Varying Force

- If the force depends on position of the object through the motion
  - one must consider work in small segments of the position where the force can be considered constant

$$\Delta W = F_x \cdot \Delta x \quad \text{,where } F_x = F(x)$$

  - Then add all work-segments throughout the entire motion ($x_i \rightarrow x_f$)

$$W \approx \sum_{x_i}^{x_f} F_x \cdot \Delta x$$

In the limit where $\Delta x \rightarrow 0$

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \cdot \Delta x = \int_{x_i}^{x_f} F_x dx = W$$

- One of the forces depends on position is force by a spring

$$F_s = -kx$$

The work done by the spring force is

$$W = \int_{-x_{\max}}^{0} F_s dx = \int_{-x_{\max}}^{0} (-kx)\, dx = \frac{1}{2} kx^2$$

Other position dependent forces ?

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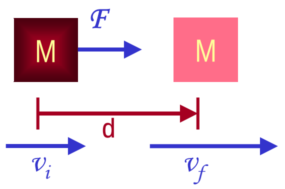
Kinetic Energy and Work-Kinetic Energy Theorem



Missing physical quantity in this cartoon.

$$W = \left(\sum \vec{F}\right) \cdot \vec{d} = Fd \cos 0 = Fd$$

$$W = \vec{F} \cdot \vec{d} = (ma)d \cos 0 = (ma)d$$

$$v_f^{\,2} - v_i^{\,2} = 2ad \;\rightarrow\; ad = \frac{1}{2}\left(v_f^{\,2} - v_i^{\,2}\right)$$

Work $W = mad = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$

Kinetic Energy $K = \frac{1}{2} mv^2$

Work $W = K_f - K_i = \Delta K$

The work done by the net force caused change of object's kinetic energy.

Work-Kinetic Energy Theorem

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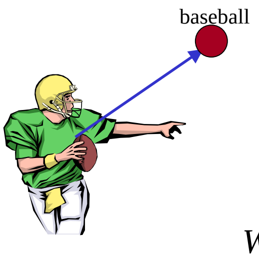
Example: KE

A 145g baseball is thrown with a speed of 25 m/s.

(a) What is kinetic energy ?

(b) How much work was done to reach this speed , starting from rest?



Kinetic Energy = KE = K

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.145kg)(25m/s)^2$$

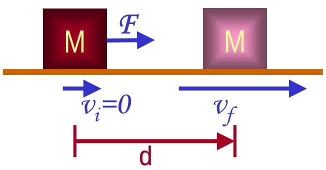
$$= 45kg \bullet m^2/s^2 = 45N \bullet m = 45J$$

$$W = K_f - K_i = 45J - 0J = 45J$$

Work, Kinetic energy are scalar quantities !!

Example of Work-KE Theorem

A 6.0kg block initially at rest is pulled along a horizontal, frictionless surface by a constant horizontal force of 12N. **Find the speed** of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12 \times 3.0 \cos 0^o = 36(J)$$

From the work-kinetic energy theorem,

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Since initial speed is 0, $W = \frac{1}{2}mv_f^2$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{6.0}} = 3.5m/s$$

Final velocity has been calculated without acceleration !!

Based on Newton's equation of motion

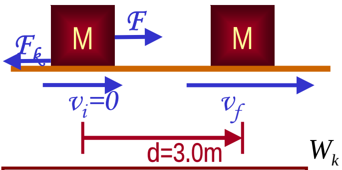
$$F = ma \qquad a = \frac{F}{m}$$

$$v_f^2 - v_i^2 = 2ad = \frac{2dF}{m}$$

$$v_f = \sqrt{2ad}$$

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force *F* is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36(J)$$

Work done by friction F_k is

$$W_k = \vec{F}_k \bullet \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k mg\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26(J)$$

Thus the net work is $W = W_F + W_k = 36 - 26 = 10(J)$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$

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Homework # 5 : solution of the 1st Exam (paper)

Homework #6 : Examples in Ch. 6 (paper)

→ change numbers in the examples by yourself

; Due 10/19 (this Thursday)

Homework #7: Examples in Ch. 7 (paper)

→ change numbers in the examples by yourself

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