Homework #7: Examples in Ch. 7 (paper) : Due 10/26

→ change numbers in the examples by yourself

Homework #8: Examples in Ch. 8 (paper) : Due 10/31

(1, 2, 3, 4, 6, 8, 10, 13, 14(moon-> other planet), 15)

→ change numbers in the examples by yourself

Reading assignment : Appendix B, 7-3, 7-4

Quiz # 3 : 10/26 (Ch 6 – 7)

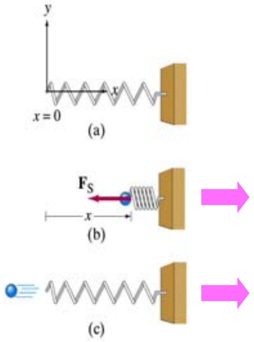
PHYS 1443

Ch. 8 Conservation of energy

1. More examples
2. Gravitational Potential Energy
   - Escape Speed
3. Power

Example of Spring Potential Energy

Find out the speed of the steel ball released from a spring gun.



Spring constant k = 250 N/m
Compressed length x = 6.0 cm
Mass of the ball m = 0.10 kg

$$U_i = \frac{1}{2}kx^2 = \frac{1}{2}\bullet 250 \bullet (0.06)^2 = 0.45J$$

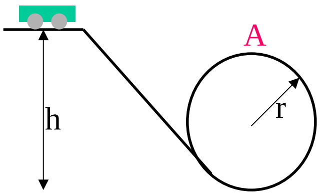
$$K_f = \frac{1}{2}mv^2$$

$$\text{Since } E = U_i + K_i = U_f + K_f$$

$$v = \sqrt{\frac{2}{m}U} = \sqrt{\frac{2}{0.10}0.45} = 3.0m/s$$

Example : Roller-coaster

Find out the minimum height of the Roller-coaster !! (Assuming no friction)



The question is what you expect at A ?

$$F = ma_c$$

$$mg = m\frac{v^2}{r}$$

$$v \geq \sqrt{rg}$$

Let's use Energy argument.

$$U_i = U_A + K_A$$

$$mgh = mg(2r) + \frac{1}{2}mv^2$$

$$mgh \geq mg(2r) + \frac{1}{2}mrg = \frac{5}{2}mgr$$

$$h \geq \frac{5}{2}r$$

How are Conservative Forces Related to Potential Energy?

Work done by a force component on an object through a displacement Δx is

$$\Delta W = F_x \Delta x = -\Delta U$$

$$\lim_{\Delta x \to 0} \Delta U = -\lim_{\Delta x \to 0} F_x \Delta x$$

For an infinitesimal displacement Δx

$$dU = -F_x dx$$

Results in the conservative force-potential relationship

$$F_x = -\frac{dU}{dx}$$

1. spring-ball system: $F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$

2. Earth-ball system: $F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}(mgy) = -mg$

Or you can say …

$$\int dU = -\int F_x dx \qquad U = -\int F_x dx$$

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The Gravitational Potential Energy

Since the gravitational force is a radial force,

$$dW = \vec{F} \cdot d\vec{r} = F(r)dr$$ For the whole path $$W = \int_{r_i}^{r_f} F(r)dr$$

Potential energy is the negative change of work in the path

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

Since the Earth's gravitational force is $F(r) = -\frac{GM_E m}{r^2}$

So the potential energy function becomes

$$U_f - U_i = \int_{r_i}^{r_f} \frac{GM_E m}{r^2} dr = -GM_E m\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

Since only the potential energy difference has a meaning, by taking the infinite distance as the initial point of the potential energy (U_i)

$$U = -\frac{GM_E m}{r}$$

The energy needed to take the particles infinitely apart.

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Escape Speed

Consider an object of mass m is projected vertically from the surface of the Earth with an initial speed v_i and eventually comes to stop v_f=0 at the distance r_{max}.

v_f=0 at r_{max}

Since the total mechanical energy is conserved

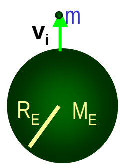
$$E = K + U = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{max}}$$

$$v_i = \sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)}$$

If r_{max} = infinite

$$= \sqrt{\frac{2GM_E}{R_E}}$$

Independent of the mass of the escaping object



$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 5.98\times 10^{24}}{6.37\times 10^{6}}}$$

$$= 1.12\times 10^{4}\, m/s = 11.2\, km/s$$

This is called the escape speed of The Earth.

Power

Rate at which the work is done or the energy is transferred

Average power $\overline{P} \equiv \frac{\Delta W}{\Delta t}$

Instantaneous power

$$P \equiv \lim_{\Delta t\to 0}\frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \lim_{\Delta t\to 0}\frac{\left(\sum\vec{F}\right)\bullet\Delta\vec{s}}{\Delta t} = \left(\sum\vec{F}\right)\bullet\lim_{\Delta t\to 0}\frac{\Delta\vec{s}}{\Delta t} = \left(\sum\vec{F}\right)\bullet\vec{v}$$

Unit of Power $J/s = Watts = W$

Traditionally… $1HP = 746\ Watts$

What do power companies sell? $1kWH = 1000Watts\times 3600s = 3.6\times 10^{6}\, J$

Energy

Example : Power

A 70 kg person climb up a 4.5 m stairs in 4.0 s.

Estimate power $\overline{P} \equiv \frac{\Delta W}{\Delta t}$ $\Delta W = mg\Delta y$

$$\overline{P} = \frac{mg\Delta y}{\Delta t} = \frac{(70kg)(9.8m/s^2)(4.5m)}{4s} = 770Watts$$

Calculate total energy

Since there isn't any kinetic energy involved in here,

$|\Delta W| = |\Delta U|$ $\Delta W = E$

$$E = \overline{P}t = (770Watts)(4.0s) = 3100J$$

You need to generate at least 3100J of energy to do this.

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General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new forms of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one place to another. ***Total energy of universe is constant!!***

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$E = mc^2$$

How many joules does your body correspond to?

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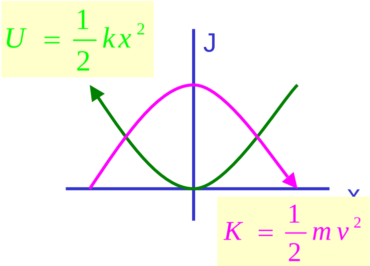
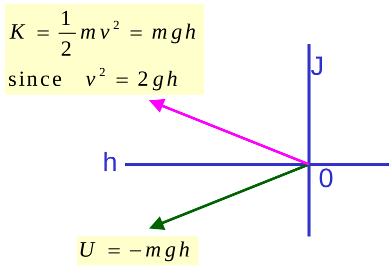
Energy Diagram of a System

One can draw potential energy as a function of position → *Energy Diagram*

spring-ball system

Under Gravitation





$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

$$E = K + U = \frac{1}{2}mv^2 + mgh = \text{constant}$$

The Gravitational Field

The gravitational force is a field force. The force exists everywhere in the universe.

If one were to place a test object of mass m at any point in the space in the existence of another object of mass M, the test object will feel the gravitational force exerted by M, $\vec{F}_g = m\vec{g}$.

Therefore the gravitational field g is defined as $\vec{g} \equiv \frac{\vec{F}_g}{m}$

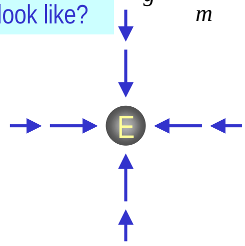
In other words, the gravitational field at a point in the space is the gravitational force experienced by a test particle placed at the point divided by the mass of the test particle.

So how does the Earth's gravitational field look like?

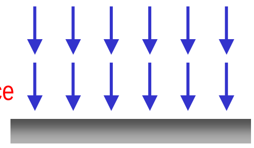
$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{R_E^2}\hat{r}$$

Where $\hat{r}$ is the unit vector pointing outward from the center of the Earth

Far away from the Earth's surface



Close to the Earth's surface



The Gravitational Potential Energy

What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

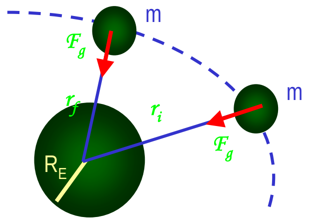
Do you think this would work in general cases?

No, it would not.

Why not?

Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth and the generalized gravitational force is inversely proportional to the square of the distance.

OK. Then how would we generalize the potential energy in the gravitational field?



Because gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be considered as consisting of many tangential and radial motions.
Tangential motions do not contribute to work!!!

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