PHYS 1443

Ch. 13 Fluids

1. Density and Specific Gravity
2. Fluid and Pressure
3. Absolute and Relative Pressure
4. Pascal's Law
5. Buoyant Force and Archimedes' Principle
6. Flow Rate and Continuity Equation
7. Bernoulli's Equation

Fluid and Density

Three states of matter — Solid, Liquid, and Gas

Fluid — Do not maintain a fixed shape. Have the ability to flow — Liquid and Gas

More scientifically: A collection of molecules that are randomly arranged and loosely bound by forces between them or by the external container.

Density, ρ (rho), of an object is defined as mass per unit volume

$$\rho \equiv \frac{M}{V}$$

Unit kg/m^3

Dimension $[ML^{-3}]$

Specific Gravity of a substance is defined as the ratio of the density of the substance to that of water at 4.0 °C (ρ_{H2O}=1.00g/cm^3).

$$SG \equiv \frac{\rho_{substance}}{\rho_{H_2O}}$$

Unit None

Dimension None

See Tab. 13.1

Pressure

Definition of the pressure is…

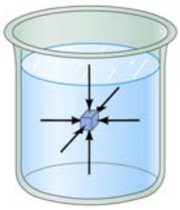
$$\text{Pressure} \equiv P = \frac{F}{A}$$

Unit: N/m^2=Pascal (Pa)

Dim.: $[M][L^{-1}][T^{-2}]$

In what way do you think fluid exerts force on the object submerged in it?

The only force the fluid exerts on an object immersed in it is the forces *perpendicular* to the surfaces of the object.



Therefore, pressure is a useful physical quantity to deal with fluids and objects system.

Experiment tells a fluid exerts a pressure in all directions.

Imagine when you were in the swimming pool.

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Example for Pressure

The mattress of a water bed is 2.00m long by 2.00m wide and 30.0cm deep. a) Find the weight of the water in the mattress.

The volume density of water at the normal condition (0°C and 1 atm) is $1000kg/m^3$. So the total mass of the water in the mattress is

$$m = \rho_W V_M = 1000 \times 2.00 \times 2.00 \times 0.300 = 1.20 \times 10^3 kg$$

Therefore the weight of the water in the mattress is

$$W = mg = 1.20 \times 10^3 \times 9.8 = 1.18 \times 10^4 N$$

b) Find the pressure exerted by the water on the floor when the bed rests in its normal position, assuming the entire lower surface of the mattress makes contact with the floor.

Since the surface area of the mattress is $4.00\ m^2$, the pressure exerted on the floor is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{1.18 \times 10^4}{4.00} = 2.95 \times 10^3$$

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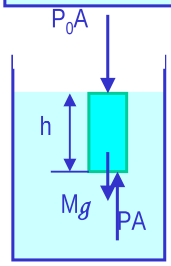
Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude.



It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's imagine a virtual liquid cylinder with height *h* and cross sectional area *A* in a fluid of density ρ at rest (equilibrium).

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is $M = \rho V = \rho A h$

Let's think about the forces in the cylinder: Force = Pressure x Area

Since the system is in its equilibrium $\sum F = PA - P_0A - Mg = PA - P_0A - \rho A h g = 0$

Therefore, $P = P_0 + \rho g h$ → $P \cong \rho g h$

Atmospheric pressure P_0 is $1.00\,atm = 1.013 \times 10^5\,Pa$

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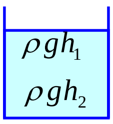
Pascal's Principle and Hydraulics

A change in the pressure applied to a fluid is transmitted to every point of the fluid and to the walls of the container. : Pascal's Principle

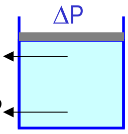
In fluid, $P = P_{air} + \rho g h \simeq \rho g h$

If pressure increased by ΔP ...



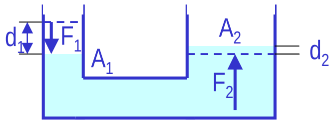
$P_1 = \rho g h_1 + \Delta P$

$P_2 = \rho g h_2 + \Delta P$



The resultant pressure P at any given depth h increases as much as the change in Pressure.

This is the principle behind hydraulic pressure.



Before apply force : $P_1 = P_2$

After apply force : $P_1' = P_1 + \Delta P_1 = P_1 + \frac{F_1}{A_1}$ $\quad P_2' = P_2 + \Delta P_2 = P_2 + \frac{F_2}{A_2}$

Because of Pascal's principle

$\Delta P_1 = \Delta P_2$

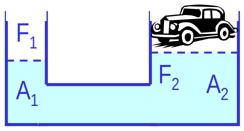
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{A_2}{A_1} F_1$$

$$d_2 = \frac{A_1}{A_2} d_1$$

Example for Pascal's Principle

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?



Using the Pascal's principle, one can deduce the relationship between the forces, the force exerted by the compressed air is

$$F_1 = \frac{A_1}{A_2}F_2 = \frac{\pi(0.05)^2}{\pi(0.15)^2}\times 1.33\times 10^4 = 1.48\times 10^3\,N$$

Therefore the necessary pressure of the compressed air is

$$P = \frac{F_1}{A_1} = \frac{1.48\times 10^3}{\pi(0.05)^2} = 1.88\times 10^5\,Pa$$

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Example for Pascal's Principle

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

$$P - P_0 = \rho_W gh = 1000\times 9.8\times 5.0 = 4.9\times 10^4\,Pa$$

Estimating the surface area of the eardrum at 1.0cm^2=1.0x10^{-4} m^2, we obtain

$$F = (P - P_0)A \approx 4.9\times 10^4\times 1.0\times 10^{-4} \approx 4.9\,N$$

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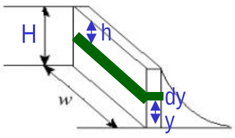
Example for Pascal's Principle

Water is filled to a height H behind a dam of width w. Determine the resultant force exerted by the water on the dam.



Since the water pressure varies as a function of depth, we will have to do some calculus to figure out the total force.

The pressure at the depth h is

$$P = \rho g h = \rho g (H - y)$$

The infinitesimal force dF exerting on a small strip of dam dy is

$$dF = PdA = \rho g (H - y) w dy$$

Therefore the total force exerted by the water on the dam is

$$F = \int_{y=0}^{y=H} \rho g (H - y) w dy = \rho g w \left[Hy - \frac{1}{2} y^2 \right]_{y=0}^{y=H} = \frac{1}{2} \rho g w H^2$$

Buoyant Forces and Archimedes' Principle

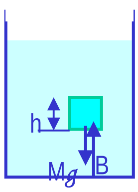
Why is it so hard to put an inflated beach ball under water while a small piece of steel sinks in the water?

The water exerts force on an object immersed in the water.
This force is called **Buoyant force**.

How does the Buoyant force work?

The magnitude of the buoyant force always equals the weight of the fluid in the volume displaced by the submerged object.

This is called, **Archimedes' principle**.



Let's consider a cube whose height is h and bottom area is A, it is in equilibrium so that its weight Mg is balanced by the buoyant force B.

$$B = F_g = Mg$$

The pressure at the bottom of the cube is larger than the top by ρgh.

Therefore, $\Delta P = B / A = \rho g h$

$$B = \Delta P A = \rho g h A = \rho g V$$

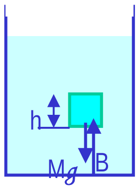
$$B = \rho g V = Mg = F_g$$

Where Mg is the weight of the fluid.

More Archimedes' Principle

Let's consider buoyant forces in two special cases.

Case 1: Totally submerged object



Let's consider an object of mass M, with density ρ_0, is immersed in the fluid with density ρ_f.

The magnitude of the buoyant force is $B = \rho_f V g$

The weight of the object is $F_g = Mg = \rho_0 V g$

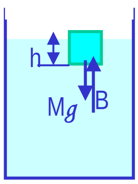
Total force of the system is $F = B - F_g = (\rho_f - \rho_0) V g$

The total force applies to different directions depending on the difference of the density between the object and the fluid.

1. If the density of the object is smaller than the density of the fluid, the buoyant force will push the object up to the surface.
2. If the density of the object is larger that the fluid's, the object will sink to the bottom of the fluid.

More Archimedes' Principle

Case 2: Floating object



Let's consider an object of mass M, with density ρ_0, is in static equilibrium floating on the surface of the fluid with density ρ_f, and the volume submerged in the fluid is V_f.

The magnitude of the buoyant force is $B = \rho_f V_f g$

The weight of the object is $F_g = Mg = \rho_0 V_0 g$

Therefore total force of the system is $F = B - F_g = \rho_f V_f g - \rho_0 V_0 g = 0$

Since the system is in static equilibrium $\rho_f V_f g = \rho_0 V_0 g$

$$\frac{\rho_0}{\rho_f} = \frac{V_f}{V_0}$$

What does this tell you?

Since the object is floating its density is always smaller than that of the fluid.

The ratio of the densities between the fluid and the object determines the submerged volume under the surface.

Example for Archimedes' Principle

Archimedes was asked to determine the purity of the gold used in the crown. The legend says that he solved this problem by weighing the crown in air and in water. Suppose the scale read 7.84N in air and 6.86N in water. What should he have to tell the king about the purity of the gold in the crown?

In the air the tension exerted by the scale on the object is the weight of the crown

$$T_{air} = mg = 7.84\,N$$

In the water the tension exerted by the scale on the object is

$$T_{water} = mg - B = 6.86N$$

Therefore the buoyant force B is

$$B = T_{air} - T_{water} = 0.98N$$

Since the buoyant force B is

$$B = \rho_w V_w g = \rho_w V_c g = 0.98\,N$$

The volume of the displaced water by the crown is

$$V_c = V_w = \frac{0.98\,N}{\rho_w g} = \frac{0.98}{1000 \times 9.8} = 1.0 \times 10^{-4} m^3$$

Therefore the density of the crown is

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84}{V_c g} = \frac{7.84}{1.0 \times 10^{-4} \times 9.8} = 8.3 \times 10^3 kg / m^3$$

Since the density of pure gold is $19.3 \times 10^3 kg/m^3$, this crown is not made of pure gold.

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Example for Buoyant Force

What fraction of an iceberg is submerged in the sea water?

Let's assume that the total volume of the iceberg is V_i. Then the weight of the iceberg F_{gi} is

$$F_{gi} = \rho_i V_i g$$

Let's then assume that the volume of the iceberg submerged in the sea water is V_w. The buoyant force B caused by the displaced water becomes

$$B = \rho_w V_w g$$

Since the whole system is at its static equilibrium, we obtain

$$\rho_i V_i g = \rho_w V_w g$$

Therefore the fraction of the volume of the iceberg submerged under the surface of the sea water is

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917\,kg / m^3}{1030\,kg / m^3} = 0.890$$

About 90% of the entire iceberg is submerged in the water!!!

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~~Home Work # 15~~: Reading Assignment:

Examples : Ch. 13 example 1,2,3,5,6, 7, 8, 9,10,11

Final exam : Dec 12 (Tuesday) 5:30 ~

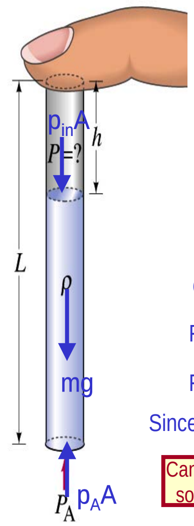
From Ch. 10 to What you learned today

+

Exam 2, quiz 4

Finger Holds Water in Straw

You insert a straw of length *L* into a tall glass of water. You place your finger over the top of the straw so that no air can get in or out, and then lift the straw from the liquid. You find that the straw strains the liquid such that the distance from the bottom of your finger to the top of the liquid is *h*. Does the air in the space between your finger and the top of the liquid have a pressure P that is (a) greater than, (b) equal to, or (c) less than, the atmospheric pressure P_A outside the straw?



Gravitational force on the mass of the liquid $F_g = mg = \rho A(L-h)g$

Force exerted on the top surface of the liquid by inside air pressure $F_{in} = p_{in}A$

Force exerted on the bottom surface of the liquid by outside air $F_{out} = -p_AA$

Since it is at equilibrium $F_{out} + F_g + F_{in} = 0$ ⇒ $-p_AA + \rho g(L-h)A + p_{in}A = 0$

Cancel A and solve for p_{in} ⇒ $p_{in} = p_A - \rho g(L-h)$

So p_{in} is less than P_A by ρgh.

Flow Rate and the Equation of Continuity

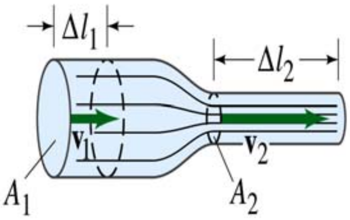
Study of fluid in motion: Fluid Dynamics

If the fluid is water: Water dynamics?? Hydro-dynamics

Two main types of flow

- **Streamline or Laminar flow**: Each particle of the fluid follows a smooth path, a streamline
- **Turbulent flow**: Erratic, small, whirlpool-like circles called eddy current or eddies which absorbs a lot of energy

Flow rate: the mass of fluid that passes a given point per unit time $\Delta m / \Delta t$



$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

since the total flow must be conserved

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Equation of Continuity

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