

Study Guide for Mechanics Lab Final

This study guide is provided to help you prepare for the lab final. The lab final consists of multiple-choice questions, usually two for each unit, and 4 work out problems that usually consist of multiple steps. The questions and problems are taken from the pre-lab questions, some of the reading material from the lab theory section and from the work performed in the lab exercise.

There are sample questions and problems. Answer to these questions and problems will be given at the end of this review.

Unit 1 – Uncertainties

1. Be familiar with the Standard Deviation equation, and use it to determine the standard deviation for a set of values.- Appendix II
2. Be able to combine uncertainties from different sources- Appendix II
3. Be able to use propagation of uncertainties to determine the uncertainty in calculated values such as Volume or density. Appendix II
4. Know the difference between a best-fit line and a worst fit line for plotted data Appendix II
5. Know how to determine the uncertainty in slope and y-intercept from a graph- Appendix II

Uncertainties: What they are and where do they come from

Unlike Math where $2+2 = 4$, Physics deals with values that are quantitative not exact. Gravity, the speed of sound or light , and any measured value are all quantitative and are prone to have some degree of inaccuracy or uncertainty. Using such values in calculations would propagate the error to the result, giving the result a degree of uncertainty as well. It is for us to determine where the uncertainties exist, and the degree of inaccuracy they give to the measured or calculated value.

In making length measurements the accuracy of the measuring device is determined by the device itself and the user making the measurements. A device marked in divisions of millimeters (mm) is more accurate than one marked in divisions of centimeters (cm). One person may judge that they can accurately determine down to one-quarter of a division while another one can only determine to one-half a division. The respected uncertainties of the two individuals would be ± 0.25 division and ± 0.5 division.

In the scope of our labs we set the uncertainty for a meter stick marked in mm. to be ± 0.5 mm.

Instrument uncertainty due to the user observations can be reduced by using a more accurate instrument such as the vernier caliper. The instrument used in the lab can make accurate measurements of ± 0.01 cm or 0.1 mm.

1.Reducing uncertainties and combining uncertainties

One method to reduce uncertainties of measured values is to take numerous measurements under the same conditions, then finding the average and standard deviation from the average. Below is the formula for the standard deviation.

$$s.d = \sqrt{\frac{\Sigma \Delta^2}{n-1}}$$

2 Combining Uncertainties

In this lab you measured 5 trials of the length, width, and height of a wooden block using a vernier caliper and found the average and standard deviation for each. Now we have an uncertainty of the average (standard deviation), but also the uncertainty of the vernier caliper as well. The formula below is used to combine uncertainties from different sources. In this case δ_{sd} is from the standard deviation and δ_{vc} is from the vernier caliper.

$$\delta_{total} = \sqrt{\delta_{sd}^2 + \delta_{vc}^2}$$

3 Propagation of uncertainties in calculations

When using values that have an uncertainty in calculations the resulting value also has some degree of uncertainty. There are certain rules which can be followed to determine the uncertainty in this value.

For values of $x \pm \delta x$ and $y \pm \delta y$	
Addition or Subtraction $z = x + y$ or $z = x - y$	$\delta z = \delta x + \delta y$
Multiplication or Division $z = x*y$ or $z = x/y$	$\delta z = z \left(\frac{\delta x}{x} + \frac{\delta y}{y} \right)$
Powers such as $z = x^2 + \sqrt{y}$	$\delta z = z \left(2 \frac{\delta x}{x} + 1/2 \frac{\delta y}{y} \right)$

Graphing Data

One way to represent sets of data points is to graph them. A graph should be presentable and should convey information on its own. To do this certain guide lines should be followed.

1. Label the axis with the quantity and its units with the x axis containing the independent quantity. The independent quantity is the quantity that is being controlled. If you are taking measurements of velocity at fixed time intervals the independent quantity (x-axis) is time and the dependent (y-axis) is velocity.
2. Select a coordinate scale for each axis that will allow the data to cover most of the page but also in such a way that the data can be extracted by someone who has no knowledge of the original data. Also keep in mind what information will be extracted from the graph. Example: If the y-intercept is going to be determined from the graph the x-axis must have zero on the x-axis. Also keep the coordinate scale linear (i.e. don't have the first major division a step of 0.3 then from then on each subsequent major division in steps of 0.1)
3. Title the graph as dependent vs. independent.
4. As you plot the data, represent each data point distinctly. If more than one set of data is to be represent you should make the data points for each data set a different symbol. A legend should be included with each symbol and what it represents. For example you may plot a velocity vs. time graph for two different vehicles, with a circle represents car 1 and a square represents car 2.
5. If your data has uncertainties, error bars in the direction of the uncertainty should be drawn to scale. Example: If the measured velocity has an uncertainty of 0.2m/s a vertical line should be drawn above and below the data point. The length of the line drawn depends on the scale of the graph and the amount of uncertainty. The line should be capped off with a small horizontal line making the error bar look like a capital I.

Graphical Analysis with uncertainties

Once data has been graphed information can be extracted from the graph. In this lab the slope of a straight line fitted to the data and its y-intercept was to be determined.

A best fit line is a straight line drawn through as many data points as possible while also trying to keep an equal amount of points which do not fit on the line, above and below the line. If the y-intercept is to be found the line needs to cross the y-axis at $x = 0$.

A worst fit line is used to help determine the uncertainty in the slope and y-intercept of the plotted data. A worst fit line is drawn from the top of the error bar of the largest y-axis value to the bottom of the error bar of the smallest y-axis value. (see figure to the right.) The worst fit line is the dashed line.

Δx and Δy along with the drawn line forms a triangle. The larger the triangle is the more accurate the value of the slope of the plotted line. The slope is determined by using $(y_2 - y_1)/(x_2 - x_1)$.

The y-intercept is read directly from the graph where the line crosses the y-axis at $x = 0$.

The uncertainty in the slope of a plotted line can be found by finding the slope of the worst fit line and then subtracting it from the best fit line.

slope = best fit slope \pm | best fit line - worst fit line |

y-intercept = best fit y-intercept \pm | best y-intercept - worst y-intercept |

Unit 2 Newton's 2nd Law of Motion

1. If given a table like table 1 be able to use the information to determine the slope and y-intercept using equations 7 and 8 (least squares fit).
2. From a graph of \mathbf{W}_2 vs. acceleration, know what the slope and y-intercept represents.
3. Understand why this lab proved the validity of Newton's 2nd Law.
4. Given information about a graph for \mathbf{W}_2 vs. acceleration such as points on a best-fit line for an accelerating cart, be able to determine the slope. Also, be able to use this information to determine the mass of the cart.

"The acceleration of an object is directly proportional to the net force applied to the object and inversely proportional to the mass of the object"

In short $\mathbf{F} = ma$

In this lab you should have verified that as the force applied to the object increased, the acceleration increased.

$$\mathbf{W}_2 = ma + \mathbf{F}_f \quad [4]$$

The mass, from equation [4], is the total mass which is moving. In the context of the lab this mass was the hanging mass, the mass of the cart and the mass added to the cart. The force \mathbf{W}_2 is applied to the cart by the hanging mass via the Tension in the string. To increase this force mass was moved from the cart to the hanging mass, and keeping the total mass constant.

The force applied to the cart, \mathbf{W}_2 was determined by multiplying the hanging mass and the acceleration of gravity.

Graphical and Mathematical Analysis of Data

Your data should have shown that as the applied force increased, the measured acceleration also increased. The data table included preliminary calculations needed for the Mathematical analysis of the data. By plotting the data of force \mathbf{W}_2 vs. acceleration a straight line could be fitted to the data and drawn through the y-axis.

The straight line gave you a slope of $\Delta F / \Delta a$ with units of $\text{N}/(\text{m}/\text{s}^2)$.

The units simplified to kg, therefore the slope represented the total mass in motion or the total mass of the system under observation.

We can relate this to the equation of a line $y = mx + b$. From equation [4], y is the applied force \mathbf{W}_2 , the slope m is the total mass m , x is the acceleration and the y-intercept b , is the force of friction.

The data plotted in the graph can also be mathematical analyzed to also find the slope and y-intercept this method is called a least squares fit for the data. The equations for the slope and y-intercept are shown below.

$$\text{slope} = \frac{n \left(\Sigma(a)(W_2) \right) - (\Sigma a)(\Sigma W_2)}{n(\Sigma a^2) - (\Sigma a)^2}$$

$$y - \text{intecept} = \frac{(\Sigma W_2)(\Sigma a^2) - (\Sigma a)\Sigma(a)(W_2)}{n(\Sigma a^2) - (\Sigma a)^2}$$

What the least square fit does is to take the data for the x and y coordinates and mathematically determines the slope and y-intercept for the best fit line to the data. A table similar to the one in the sample below would need to be generated to perform the least squares fit using the equations above; each column is summed and then their results entered into the equations for slope and y-intercept. However for those of you with graphing calculators the linreg(linear regression) function performs the same task for any given set of (x,y) data. When the computer determined the slope for the 20 data points of velocity vs. time to determine the acceleration this is the method it used.

Extracting information

The slope of the line gave you the total moving mass. The total mass included the unknown mass of the cart and the known mass, the mass on cart and the hanging mass, which was used to conduct the experiment. From this you were able to extract the mass of the cart and then compare it to the actual measured value of the cart. Also from the graph you can extract the y-intercept. The y-intercept has units of Newton's and therefore is a Force; examining equation [4] relates it to the force of friction acting on the cart.

Unit 3 – Composition and Resolution of Forces

1. Know the difference between a resultant and an equilibrant.
2. If asked, be able to graphically determine the **equilibrant** of a set of forces.
3. Know how to break a vector down into its x and y components.
4. Given a set of forces (i.e. $250@25^\circ$, $425@165^\circ$), be able to mathematically determine the magnitude and direction of either the resultant or equilibrant.
5. Be familiar with problems 1-3. Be able to determine the unknown tensions.
6. Be familiar with questions 3 and 4. Know how to determine if a set of forces can achieve a desired displacement.

Resultants and equilibrants

A number of different forces can act on a body causing it to accelerate according to Newton's 2nd law $\Sigma \mathbf{F} = m\mathbf{a}$. All of these forces can be replaced by a single force to create the same acceleration. This force is called the **resultant** where $\mathbf{R} = \Sigma \mathbf{F}$. The force \mathbf{R} is the resultant.

If the **lines of action** all the forces acting on a body passes through a common point there exist a force which when applied through this common point creates a condition of equilibrium($\mathbf{a} = \text{zero}$).

The force is named the **equilibrant, E**.

In this lab the strings were the lines of action passing through the center of the ring.

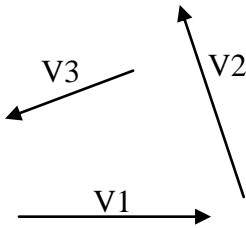
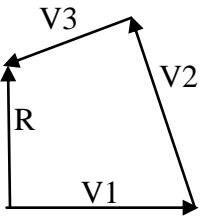
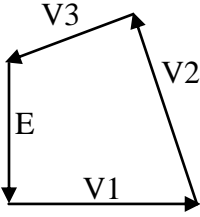
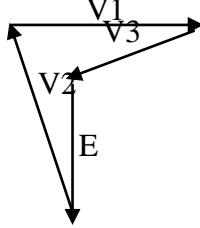
If \mathbf{R} is all the forces acting on the body then $\mathbf{E} = -\mathbf{R}$

\mathbf{R} and \mathbf{E} would each have the same magnitude but \mathbf{E} would be in opposite direction of \mathbf{R} (180° apart).

Graphical Analysis of Vector Forces

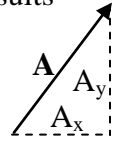
One reason to graphically analyze vectors is to quickly get an ideal of the magnitude and direction of a resultant or equilibrant for a set of vectors.

One method to graphically analysis vectors is the tip to tail method. Each vector is drawn to scale in its respected direction. The pictures below shows vectors being add graphically using the tip to tail method. The tail of the second vector is placed at the tip of the first, then the tail of the third at the tip of the second. This method would continue until all vectors that are to be added has been placed. The equilibrant is a vector which is drawn with its tail at the tip of the last vector to the tail of the first vector thus forming a complete polygon.

			
<p>A set of vectors $V_1=100@0^\circ$ $V_2=105@108^\circ$ $V_3=70@200^\circ$</p>	<p>Arranging the vectors by the tip to tail method. The resultant of these vectors is indicted in the left side panel and the equilibrant in the right side panel.</p>	<p>How the vectors are arranged will not affect the results of the magnitude or direction of the equilibrant or resultant.</p>	

Mathematical Analysis of Vector Forces

The graphical method requires accurate drawing and placement of the vectors to get accurate results for the magnitude and direction. A more practical method to get accurate results when adding vectors is mathematically. Each vector to be added is broken down into its horizontal (x) and vertical (y) components. Vector **A** has a magnitude of A and a direction of θ . The dash lines in the figure shows the x and y components of the vector **A**. To break the vector down into its components you can solve for the sides of the right triangle.



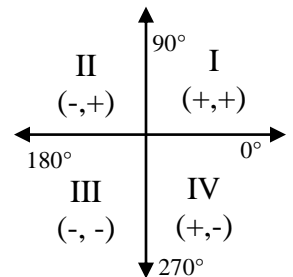
All angles are measured in reference to the + x axis as 0° .

The three vectors in the above graphical example can be broken down as follows.

Vector	$V_x = V \cos \theta$ $V_y = V \sin \theta$	V_x	V_y
$V_1 = 100 @ 0^\circ$	$V_{1x} = 100 \cos 0$ $V_{1y} = 100 \sin 0$	100	0
$V_2 = 105 @ 108^\circ$	$V_{2x} = 105 \cos 108$ $V_{2y} = 105 \sin 108$	-32.45	99.86
$V_3 = 70 @ 200^\circ$	$V_{3x} = 70 \cos 200$ $V_{3y} = 70 \sin 200$	-65.79	-23.94
Resultant	sum all V_x and sum all V_y	$R_x = 1.76$	$R_y = 75.92$

The magnitude of the resultant $R = \sqrt{R_x^2 + R_y^2}$ and the direction is $\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$

A few tidbit of caution when calculating the angle, if both R_x and R_y are negative a calculator will yield a result where the angle indicates it is in the 1st quadrant (i.e. between 0 and 90 degrees). Also if R_x or R_y has a negative value it can result with a negative angle. So pay attention to the signs to help determine the quadrant the vector points into to aid in determining the correct angle for the vector.



Remember the equilibrant is of the same magnitude but opposite direction of the resultant.

Review of the problems and question

Problems 1 and 2

The figure for problems one and two is shown to the left. And a free-body diagram for the forces is shown below it.

The tension T_2 is broken down to its x and y components T_{2x} and T_{2y} .

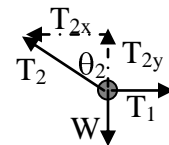
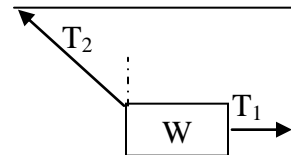
The body is at rest therefore the vector sum of the forces in x and y direction is equal to zero.

$$\Sigma F_x = T_1 - T_{2x} = 0 \quad \text{and} \quad \Sigma F_y = T_{2y} - W = 0$$

The angle θ_2 is measured from the vertical.

If the angle, θ_2 , and weight, W , are given and T_1 and T_2 are unknown

To solve for T_1 and T_2 the equations shown above must be satisfied.



The tension T_2 must be broken down into its x and y components using the simple right triangle.

$$\cos \theta_2 = \text{adj/hyp} = T_{2y}/T_2$$

$$\sin \theta_2 = \text{opp/hyp} = T_{2x}/T_2$$

solving for T_{2x} and T_{2y}

$$T_2 \cos \theta_2 = T_{2y}$$

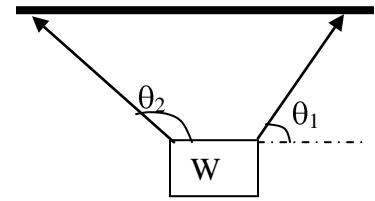
$$T_2 \sin \theta_2 = T_{2x}$$

Now the equations for F_x and F_y becomes $\Sigma F_x = T_1 - T_2 \sin \theta_2 = 0$ and $\Sigma F_y = T_2 \cos \theta_2 - W = 0$

Since both W and θ_2 is known T_2 can easily be determined and once T_2 is known it can be used to solve for T_1 in the equation for F_x .

A more complicated version of the problem is shown to the right along with its free body diagram.

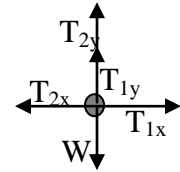
Now both T_1 and T_2 have angles in respect to the horizontal and needs to be broken down to their respective x and y components to facilitate a solution.



$$\Sigma F_x = T_{1x} - T_{2x} = 0 \text{ and } \Sigma F_y = T_{1y} + T_{2y} - W = 0$$

If T_1 and T_2 are unknown and the angles θ_1 , θ_2 and W are given. to solve for T_1 and T_2 , T_{1x} , T_{2x} , T_{1y} and T_{2y} will need to be found in terms of T_1 with its respective angle θ_1 and T_2 with its respective angle θ_2 .

Using the method shown in the first example and keeping in mind that the angles are given in respect to the horizontal T_{1x} , T_{2x} , T_{1y} and T_{2y} can be determined.



$$T_{1x} = T_1 \cos \theta_1, \quad T_{2x} = T_2 \cos \theta_2 \text{ and } \quad T_{1y} = T_1 \sin \theta_1, \quad T_{2y} = T_2 \sin \theta_2$$

The equations $\Sigma F_x = T_{1x} - T_{2x} = 0$ and $\Sigma F_y = T_{1y} + T_{2y} - W = 0$ becomes

$$\Sigma F_x = T_1 \cos \theta_1 - T_2 \cos \theta_2 = 0 \text{ and } \Sigma F_y = T_1 \sin \theta_1 + T_2 \sin \theta_2 - W = 0$$

First using the equation for F_x solve for either T_1 or T_2 in terms of the other ie $T_1 = \# T_2$

$$T_1 \cos \theta_1 = T_2 \cos \theta_2$$

$$T_1 = (\cos \theta_2 / \cos \theta_1) T_2$$

$$T_1 = \# T_2$$

Then in the equation for F_y substitute $\#T_2$ for T_1 and solve for T_2 .

$$\#T_2 \sin \theta_1 + T_2 \sin \theta_2 - W = 0$$

Once T_2 is known go back to $T_1 = \#T_2$ and solve for T_1 .

Review of Questions

If you have a group of vectors and were free to orient them in anyway you wished to achieve a given net resultant there is a simple way to test if the given group can achieve the desired net resultant.

For Instances when the group contains more than two vectors.

If the desired net resultant is zero.

Take the greatest value in the group and subtract the sum of the other values in the group.

If the result is greater than zero then the group of vectors cannot achieve a net result of zero.

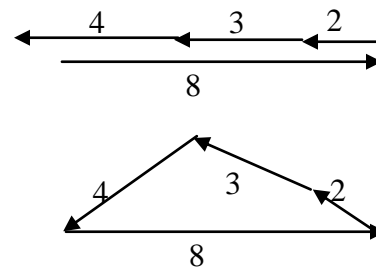
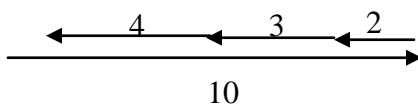
If the result is less than zero then the members of the group can be position in such a way to achieve a net result of zero.

Below there is two groups of vectors {2,3,4,10} and {2,3,4,8}

each group the lesser numbers has a sum of $2+3+4=9$

in the first group $10-9=1$

the second group $8-9=-1$



In the first example the shortest path back to zero falls short, the second group it exceeds zero and the vectors can be position in some manner to reach zero as illustrated in the above diagram.

In the first example the shortest path back to the origin cannot be achieved therefore those vectors cannot be reposition to reach zero but they can be in the second example.

If there are only two Vectors in the set then setting up a boundary condition may be easier.

Say the desire resultant is R .

And you have a set of two values $\{L,G\}$ where L is less than G

Then setting up this statement

$$(G - L) \leq R \leq (G + L)$$

If this statement is true then the resultant, R , can be achieved.

Unit 4 Uniform Circular Motion

1. Be familiar with equation [4] and its variants [5] and [6]. Under the given conditions for Parts A, B and C, know which variable represents the slope, y-axis and x-axis. Also if given numerical values for the variables, be able to determine the theoretical slope with it correct units.
2. Be familiar with the pattern of the velocity for the given conditions in parts A, B, and C. Know whether the velocity should be increasing or decreasing as one variable is incrementally increased as the other two remain constant.
3. Be familiar with centripetal acceleration and force.

An object undergoes Uniform Circular Motion when it travels at a constant velocity in a circular arc at a fixed radius about a point. Even though it is traveling at a constant velocity there is a change in direction of the velocity as it travels through the circular arc, therefore acceleration is also present. This acceleration points inwards toward the center of the circle and is named centripetal acceleration. If the object has mass there is centripetal force acting on the object. When two objects of different masses are traveling about a circle with a fixed radius at a constant velocity their centripetal acceleration would be the same, yet the object with the larger mass would have greater centripetal force acting on it.

Sometimes the origin of the centripetal force acting on the object is difficult to see. For this experiment the centripetal force was supplied by the spring connected to the shaft. For a car on a freeway turn the force of friction between the tires and the road. Gravitational force provides the centripetal force between earth and its satellites. In the Bohr atom the electron is attracted to the proton in the nucleus. These are a few examples for what force provides the centripetal force on an object.

Confirming theoretical expectations through experimentation results

Theory shows that $F_c = m(v^2/r)$
Solving for v^2 we find that $v^2 = (rF_c/m)$

This is a linear equation of the form $y = mx + b$ where the y-intercept passes through the origin.

In order to determine the velocity of the mass undergoing Uniform Circular Motion, the period of rotation is measured then divided by the radius of rotation.

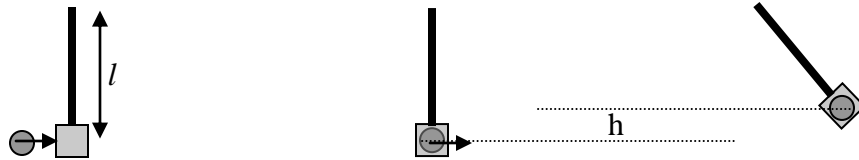
To experimentally prove that the equation is linear it should obey the fundamental equation $y = mx + b$. In this case the y-intercept will pass through the origin. The term y is represented by v^2 and the variables mx by (rF_c/m) . By varying one of the variables, (rF_c/m) , the varied variable would represent the x variable of mx and the remaining two the slope, m. The table on the next page will give an account of the relationships for each case of the varied variable.

Original equation	The fixed variables slope of line	The varied variable x value	The trend of velocity as the varied variable increases
(rF_c/m)	F_c /m	r	increases
(rF_c/m)	r /m	F_c	increases
(rF_c/m)	rF_c	m	decreases

Unit 5 Projectile motion

1. Be able to determine how long a projectile would be in flight from a given height.
2. Be able to determine the initial velocity of a projectile when given the necessary information using either the ballistic pendulum method or the ranging method.
3. Been given or having determine the velocity of a projectile fired horizontally, be able to determine the range the projectile will travel from an angle above the horizontal and a height above the ground.

Determining the initial velocity of the projectile using conservation of momentum and energy



The figures above are illustrations of a projectile being fired into a ballistic pendulum. The projectile has a mass, m , and is fired with an initial velocity of v_o . This gives the projectile an initial momentum of mv_o . As it is caught by the pendulum with mass, M , it imparts its momentum to the pendulum. Since momentum is conserved and the pendulum and projectile now moves as one its momentum is defined as $(m+M)V$, where V is the velocity of the pendulum-ball combination. Since momentum is conserved the equation below represents the conservation of energy for this system.

$$mv_o = (m+M)V$$

The pendulum and ball combination has a center of gravity at which the mass is concentrated and is free to rotate about a point near the top of the pendulum. The pendulum-ball combination has an initial velocity, V , with a kinetic energy of $KE = \frac{1}{2} (m+M)V^2$. As the pendulum-ball starts to rise it loses kinetic energy and gains potential energy. As the center of gravity of the pendulum-ball reaches a height, h , it momentarily comes to rest, the energy is now all potential energy and is equal to $PE = (m+M)gh$. Since energy is conserved the initial kinetic energy equals the final potential energy so $\frac{1}{2} (m+M)V^2 = (m+M)gh$.

The pendulum initially at rest, has a length, l , which is the distance from the pivot point to the center of gravity. As the pendulum rises an angle forms between the vertical and the pendulum. At the maximum height the pendulum-ball rises the angle which is formed can be used to determine the height the pendulum rose to from its initial resting position such that $h = l - l \cos \theta$.

During the experiment the masses and length, l , were measured and the angle θ , was determined experimentally. From these known quantities the initial velocity of the pendulum-ball, V , could be determine using conservation of energy. Then using conservation of momentum the velocity of the projectile could be determined.

Determining initial velocity using the range method

The range method is more straight forward than the previous method. The projectile is fired horizontally from a height h . Once it leaves the mechanism it starts to fall according to the kinematics equation

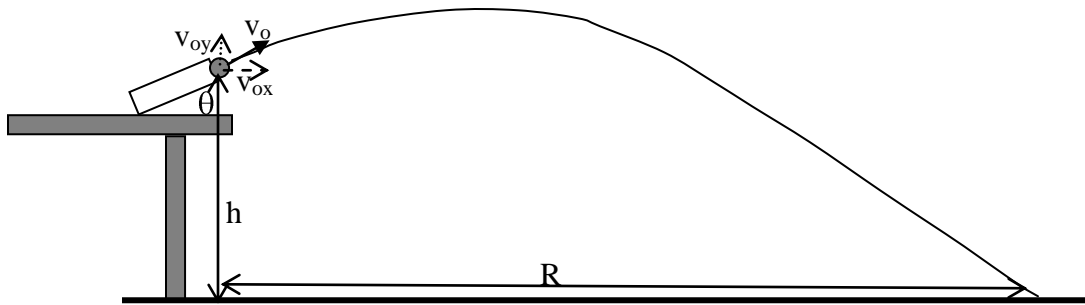
$$y = y_o + v_{oy}t + \frac{1}{2} gt^2$$

Since the initial velocity in the y direction, v_{oy} is zero and if you assign the variable y to be ground then $y = 0$ and $y_o = h$. The kinematics equation now becomes $0 = h - \frac{1}{2} gt^2$ then solving for t yields

$$t = \sqrt{\frac{2h}{g}} \text{ where } t \text{ is the time of flight for the projectile}$$

When the projectile is fired numerous times from the same position an average horizontal range, R , the projectile traveled during its flight can be determined. The range divided by the time will result in the velocity of the ball, $v_o = R/t$.

Determine the range of a projectile fired at angles other than the horizontal.



The picture depicts a projectile being fired at an angle θ above the horizontal. When the projectile leaves the mechanism it has a velocity, v at an angle θ . The velocity has a horizontal and vertical component, v_{ox} and v_{oy} respectively. The velocity v_{ox} remains constant throughout the travel of the projectile. The velocity v_{oy} changes during its travel; first decreasing until the maximum height is obtained where it is momentarily zero then increasing as it falls to the ground. The path of the projectile's travel is parabolic. The initial values for v_{ox} and v_{oy} are determined using

$$v_{ox} = v \cos \theta \text{ and } v_{oy} = v \sin \theta$$

To determine the range, R , of the projectile the time of flight needs to be determined.

Unlike the range method used previously the projectile now has an initial velocity in the vertical direction so the equation $t = \sqrt{2h/g}$ cannot be used. The kinematics equation $y = y_o + v_{oy}t + \frac{1}{2}gt^2$ is still the basis used but now v_{oy} instead of being zero it is $v \sin \theta$. Substituting the variables into the equation yields $0 = h + (v \sin \theta)t - \frac{1}{2}gt^2$.

This equation is a binomial in the form of $0 = c + bx + ax^2$ and t can be solved using the quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ yields } t = \frac{-v \sin \theta \pm \sqrt{(v \sin \theta)^2 - 4(g/2)h}}{2(g/2)}$$

The equation has both a positive and negative answer for time. Using the positive value and the horizontal velocity v_{ox} , the range can be determined $R = (v \cos \theta)t$.

Unit 6 Rotational Equilibrium

1. When a mass is placed on the x or y axis, be able to determine in what axis the torque is produced.
2. If given a mass at coordinates (x,0) or (0,y), be able to conceptually determine where a 2nd mass(either x times heavier or lighter) would need to be placed to achieve equilibrium.
3. Know the definition for center of gravity as it applies in this lab.
4. If given a group of masses placed on a balance table be able to determine the center of gravity for this group. Also be able to determine where a single mass, equal to the combined total of masses in the group, would need to be placed to achieve equilibrium.
5. Be able to determine the total torque produced about an axis.

Torque is a product of displacement and force. The units used in the lab and the calculation of the torque were modified slightly to prevent the necessity to make numerous conversions to determine a precise value for the torque. To accomplish this, the following modifications were made.

- 1) Force would be the weight of a mass, since weight is a product of mass and gravity and gravity is essential the same within the confines of the experiment it was omitted. Therefore the torque in the experiment was a product of displacement and the mass.
- 2) The balance table had holes drilled into it 1 inch apart. To reduce making numerous conversions for displacement it was decided for the scope of the lab to leave the units unchanged and allow the units of torque to be in gram-inches.

A mass placed directly upon the +x axis of a balance table produces a force perpendicular to the x-axis in the -z direction. This causes the table to tilt downward or rotate about the y-axis.

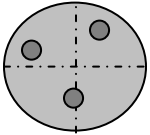
A right hand rule can be used to quickly determine the direction of torque. Your index finger represents the direction of displacement. Your middle finger represents the direction of the applied force. Lastly your thumb would represent the direction of the torque. Displacement, force and torque are perpendicular to each other. Position your thumb and the two fingers so that they create a corner of a cube. Point the index finger in the direction of displacement; rotate your wrist so that the middle finger points in the direction of the force. The thumb should now point in the direction of the torque. For the balance table the x and y axis are in the horizontal plane and the z axis in the vertical so generally the middle finger which represents force would be pointing in the downward direction.



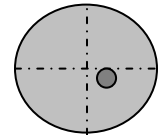
The mass placed on the + x axis produces a + torque about the y-axis. Another mass placed on the -x axis would produce a - torque also about the y-axis. The sum of the torques would be the net torque about the y-axis. When the sum of the torques is equal to zero then the system is in equilibrium.

$$\sum \mathbf{L}_y = m_1x_1 + m_2x_2 + \dots + m_nx_n$$

Center of gravity for a system of masses



An extended body can be made up of numerous masses as shown in the picture to the left. The total gravitational affect of the extended body is made up of the sum of the gravitational effect of each individual mass making up the extended body.



The **center of gravity** of the extend body of masses is a point where the accumulative torque of each mass can be replaced by a single mass equal to the sum of the masses and still produce the same net torque. The coordinates for the placement of the single mass can be determined by the following equations.

$$X_{c.g.} = \frac{\sum w_i x_i}{\sum w_i} = \frac{\sum m_i x_i}{m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$Y_{c.g.} = \frac{\sum w_i y_i}{\sum w_i} = \frac{\sum m_i y_i}{m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_n y_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

The weight, w was factored down to only the masses, m, because gravity is a constant.

To achieve equilibrium for the system of masses a single mass equal to the sum of all the masses can be placed at a point opposite in sign to the center of gravity for the system of masses. For example if a system of masses has a center of gravity at (-2,5) then a single mass equal to the sum of the masses placed at (2, -5) would bring the system into equilibrium.

Center of gravity for an irregular shaped object

A rigid irregular shaped object has a point within it where its center of mass or its center of gravity is located. This point can be experimentally found by placing it within a quadrant of a balance table. The center of mass of the irregular shaped object now would be concentrated at a point and have an x and y position on the table. If the table is brought into equilibrium by using two known masses position along the x and y axis the center of gravity for the system of masses would be located at the origin (0,0).

Therefore $x_{cg} = 0 = m_1 x_1 + m_2 x_2 + m_3 x_3$ and $y_{cg} = 0 = m_1 y_1 + m_2 y_2 + m_3 y_3$

If m_3 is the irregular shaped object and the positions and mass for m_1 and m_2 are determined then the position at which the center of mass for the irregular shape object can be found.

For example an 150 gram object is place on the table. Two 100 gram masses are position so that equilibrium is achieved and there positions are at (3.25,0) and (0,2.5). Then

$$\begin{aligned} 100(3.25) + 100(0) &= -150(x_3) & \text{and} & & 100(0) + 100(2.5) &= -150(y_3) \\ 325 &= -150 x_3 & & & 250 &= -150 y_3 \\ -2.16 &= x_3 & & & -1.67 &= y_3 \end{aligned}$$

Unit 7 Hooke's Law

1. From a Force vs. displacement graph for a stretched spring, know how to determine the spring constant.
2. Know what the area under this graph represents.
3. Be familiar with the comparison of the work require to stretch a spring and the work required to restore the spring back to its original position, also for a rubber band.
4. Be able to determine the spring constant for spring when given a period of a mass undergoing simple harmonic motion. Also, be able to determine the frequency of this motion.
5. Know how the mass affects the period and how the distance the spring is stretched affects the period.
6. Be able to do energy calculations (PE, KE, TE) for a spring-mass undergoing SHM, when given the necessary data.

What is Hooke's Law?

Not all materials that can be stretched or compress Hooke's Law. When a material is clamped on one end and a force is applied at the other end to either stretch or compress it, and if the distance the material stretches or compresses is directly proportional to the applied force this material obeys Hooke's Law.

The fundamental equation for Hooke's Law is $F = -kx$.

Where k is the spring constant of the material and x is the distance the material is compressed or stretched. The negative sign indicates that the force is in the opposite direction of the applied force.

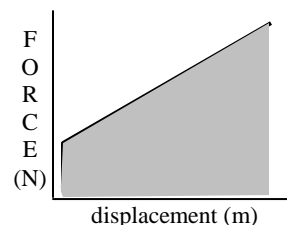
What information can be obtained from a Force vs. displacement graph for a stretched spring?

The picture to the right shows a typical force vs. displacement graph for a spring and can be used to extract information about the spring.

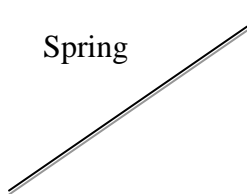
The slope of the slanted line is the spring constant, k with units of N/m.

The shaded area under the curve is the work to stretch the spring, with units of Joules, J.

Also if it obeys Hooke's Law the amount of work to stretch the spring will equal the amount of work to restore the spring to its original position.



Not all items that can be stretch obey Hooke's Law.



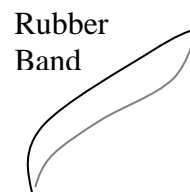
Spring

The dark line in both pictures is when the material is stretched and the gray line is when the material is restored to its original position.

In the spring there is no loss of energy, therefore the two curves are on top of one another.

For the rubber band there is some energy loss, the area of the gap

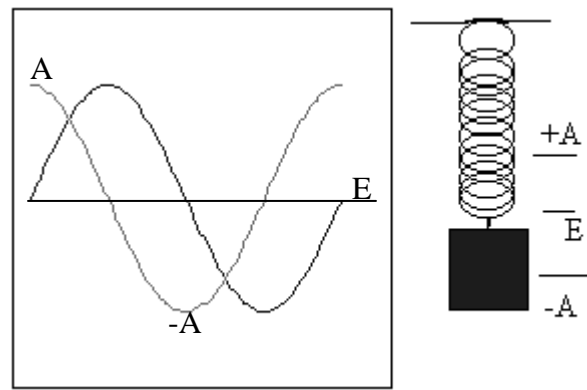
between the two curves is the amount of energy lost. Heat is on contributing factor to the loss of energy.



Rubber Band

Simple Harmonic Motion

The picture to the right represents an object undergoing simple harmonic motion. The lighter line is the displacement of the object from its equilibrium point, E and the darker line is the velocity of the object. When the object is displaced a distance $-A$ from its equilibrium position and released it will begin to accelerate towards the equilibrium position, because the force from the spring tries to restore itself to equilibrium. The acceleration of the object is $a = kx/m$ as it approaches the equilibrium position, where x is the distance the object is from its equilibrium. So as x approaches zero so does the acceleration. However when the object is at the equilibrium position its velocity is now at its maximum and as it passes through it begins to decelerate so that $a = -kx$, until it reaches the $+A$ position where it momentarily comes to rest. It then starts back toward the equilibrium position passing through it towards the $-A$ position. This process will continue until it is stopped by outside forces.



The period of the oscillation for the Simple Harmonic motion can be calculated from the equation

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where k is the spring constant and m is the mass of the object.

If any two of the variables are known then the third can be determined.

Energies within a spring

A spring obeys to the laws of conservation of energy. When the mass is attached to the spring and is attached a distance of A the total energy in the spring is potential energy and is calculated using $TE = PE = 1/2kA^2$. When the spring is released and at the moment it passes through the equilibrium position the total energy is kinetic and can be determined by $TE = KE = 1/2mv^2$.

Since energy is conserved $KE = PE$. At any position between the equilibrium and one of its maximums the total energy in the spring is partially kinetic and partially potential.

Therefore $TE = KE + PE = 1/2mv^2 + 1/2kx^2$. Where x and v are the position and velocity of the object at any given position.

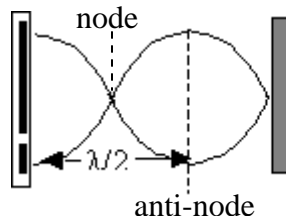
Unit 8 Archimedes Principle

1. Be familiar with equation [6] and manipulating it to solve for the density of an unknown material or liquid.
2. Be able to determine the density of a material when given its mass and dimensions. Eq. [1].
3. Be able to determine the volume of an irregular shaped object, its measured weight in air and in a known liquid.

Unit 9 Speed of Sound

1. If given the frequency and distance between Antinodes (maximums) or nodes (minimums) be able to determine the measured speed of sound
2. Be able to determine the speed of sound at a given temperature.
3. If given the dimensions of a metal rod or bar and its mass. Be able to determine its density and its expected speed of sound through the metal when given the Young's Modulus for the material Also be able to determine its velocity when given either the frequency of the fundamental it produces while clamped in its center and struck longitudinally along with the length of the bar/rod..
4. In air, know what happens to the wavelength between the nodes or antinodes as the frequency increases or decreases.
5. Know how the speed of sound for the metal used in this experiment compares to the speed of sound in air.

Node and anti-nodes



A node is where two standing waves come together and are at their minimum amplitude. An anti-node is where the two standing waves are each at the maximums.

The above diagram depicts the resonance tube used in the experiment. On the left side the speaker and microphone are located and are at a fixed point. As the plunger is moved to the right nodes and anti-nodes are formed. An anti-node is formed at the microphone when a node is formed at the plunger

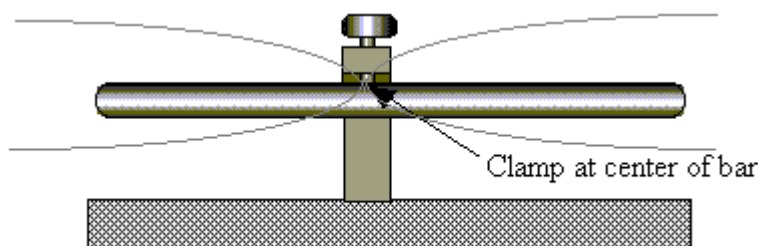
Speed of sound in air

Between each anti-node there is a distance of $\lambda/2$. To determine the speed of sound in air a known frequency is used. Since frequency has units of Hertz (1/seconds) and multiplying it by the measured wavelength λ the speed of sound in air is determined. $v = \lambda f$.

Theoretically the speed of sound can be determined from the equation $v = v_0 + (0.6 \text{ m/sec } ^\circ\text{C}) * T$.

Where $v_0 = 331.7 \text{ m/s}$ and T is the temperature.

Speed of sound in metal



The same principles can be used to determine the speed of sound in metal. By clamping a metal bar at the middle of its length and then tapping the bar on its end to send a longitudinal wave through the bar will create a node at the clamp section and anti-nodes on each end. The wavelength λ for the bar will be twice its length. In this lab the frequency was measured using the computer to give you the Fast Fourier Transform of the sound collect by a microphone. The product of the measured frequency and the wavelength will yield the speed of sound in the metal $v_m = \lambda f$.

Theoretically the speed of sound in the metal can be determine by first determining the volume, V , and mass, m , of the bar and then finding the density of the metal. $\rho = m/V$. Each material has a value called Young's Modulus, in the equation it is denoted by the letter Y .

$$v_m = \sqrt{\frac{Y}{\rho}}$$