Solving for multiply unknowns using Kirchoff's Rules

Rule 1:	$I_1 + I_2 - I_3 = 0$
Rule 2:	$I_1R_1 + I_3R_3 = V_1$ (loop 1)
	$I_2R_2 + I_3R_3 = V_2$ (loop 2)

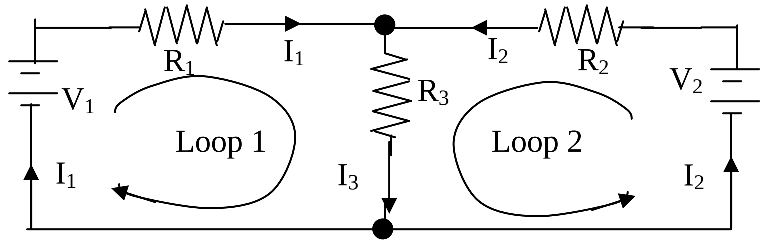

Figure 18-3

Method 1 Algebraically

Given R_1, R_2, R_3, V_1 and V_2

From Rule 1 we have $I_3 = I_1 + I_2$. Substitute this in for I_3 in rule 2 Loops 1 and 2.

We have

Loop1 $I_1R_1 + (I_1 + I_2)R_3 = V_1$ and

Loop2 $I_2R_2 + (I_1 + I_2)R_3 = V_2$ and

Grouping the unknown variables

$(R_1 + R_3)\, I_1 + R_3\, I_2 = V_1$ [1]

$I_1R_3 + I_2\,(R_2 + R_3) = V_2$ [2]

To find I_1 you must first eliminate I_2

Multiply [1] by $(R_2 + R_3)$ $\quad (R_2 + R_3)(R_1 + R_3)\, I_1 + (R_2 + R_3)\, R_3\, I_2 = V_1(R_2 + R_3)$

Multiply [2] by $- R_3$ $\quad -I_1(R_3)^2 - R_3(R_2 + R_3)\, I_2 = -V_2\, R_3$

Add the two equations $\quad I_1((R_2 + R_3)(R_1 + R_3) - (R_3)^2) + 0 = V_1(R_2 + R_3) - V_2R_3$

Solve for I_1

$$I_1 = \frac{V_1(R_2 + R_3) - V_2R_3}{(R_2 + R_3)(R_1 + R_3) - (R_3)^2}$$

To find I_2 you must first eliminate I_1

Multiply [1] by $- R_3$ $\quad - R_3\,(R_1 + R_3)\, I_1 - (R_3)^2\, I_2 = -V_1\, R_3$

Multiply [2] by $(R_1 + R_3)$ $\quad (R_3)\,(R_1 + R_3)\, I_1 + (R_2 + R_3)\,(R_1 + R_3)\, I_2 = V_2(R_1 + R_3)$

Add the two equations $\quad 0 + I_2((R_2 + R_3)\,(R_1 + R_3) - (R_3)^2) = V_2(R_2 + R_3) - V_1R_3$

Solve for I_2

$$I_2 = \frac{V_2(R_1 + R_3) - V_1R_3}{(R_2 + R_3)(R_1 + R_3) - (R_3)^2}$$

Method 2 Matrices

If your calculator is capable of creating matrices generate a 3 x 3 matrix for A

Where we will be solving for the unknown currents set the unknown currents to 1

$$A = \begin{bmatrix} 1 & 1 & -1 \\ R_1 & 0 & R_3 \\ 0 & R_2 & R_3 \end{bmatrix} \begin{matrix} \text{Rule 1} \\ \text{Loop 1} \\ \text{Loop} \end{matrix} \qquad \text{Also a 3 x 1 matrice } B = \begin{bmatrix} 0 \\ V_1 \\ V_2 \end{bmatrix}$$

Then A^{-1} B yields the three unknown currents and yield $\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$