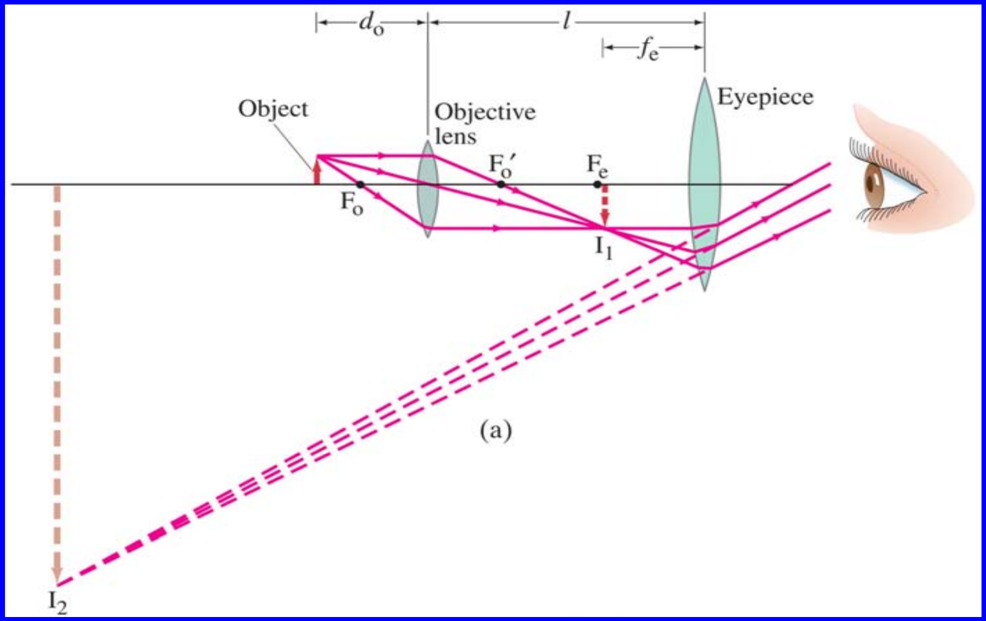
PHYSICS 1442-003

Fall 2011

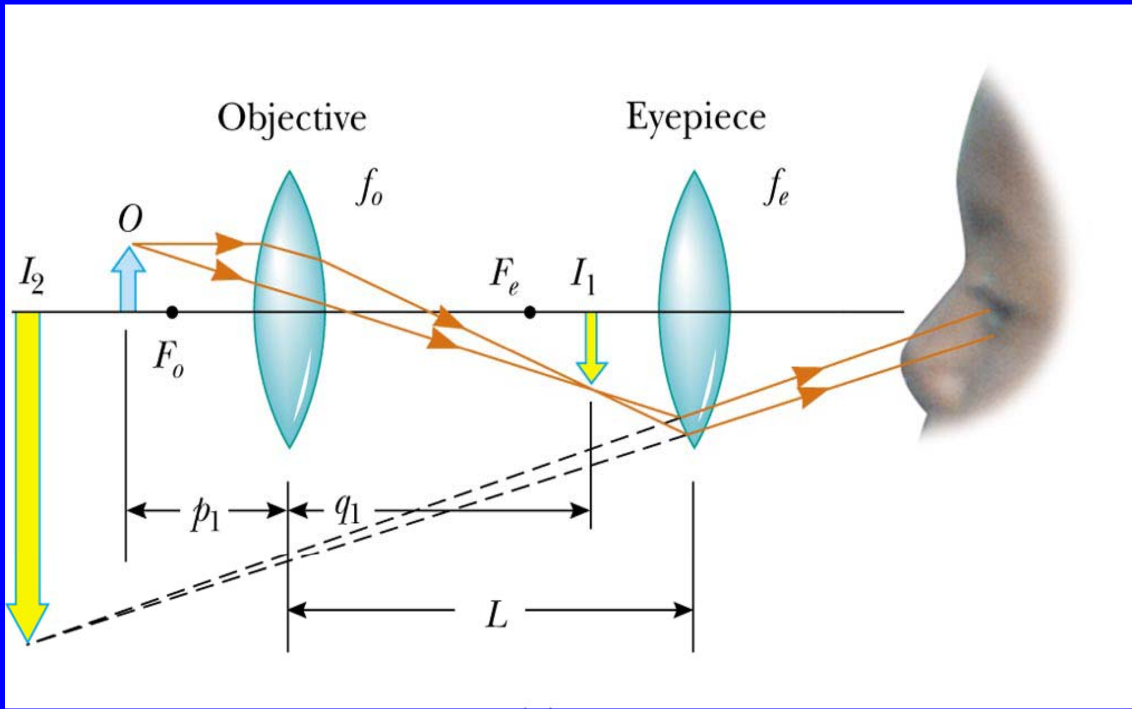
Lecture 24

Optical Instruments



Compound Microscope

- A compound microscope consists of two lenses
  - Gives greater magnification than a single lens
  - The objective lens has a short focal length, f_o<1 cm
  - The ocular lens (eyepiece) has a focal length, f_e of a few cm

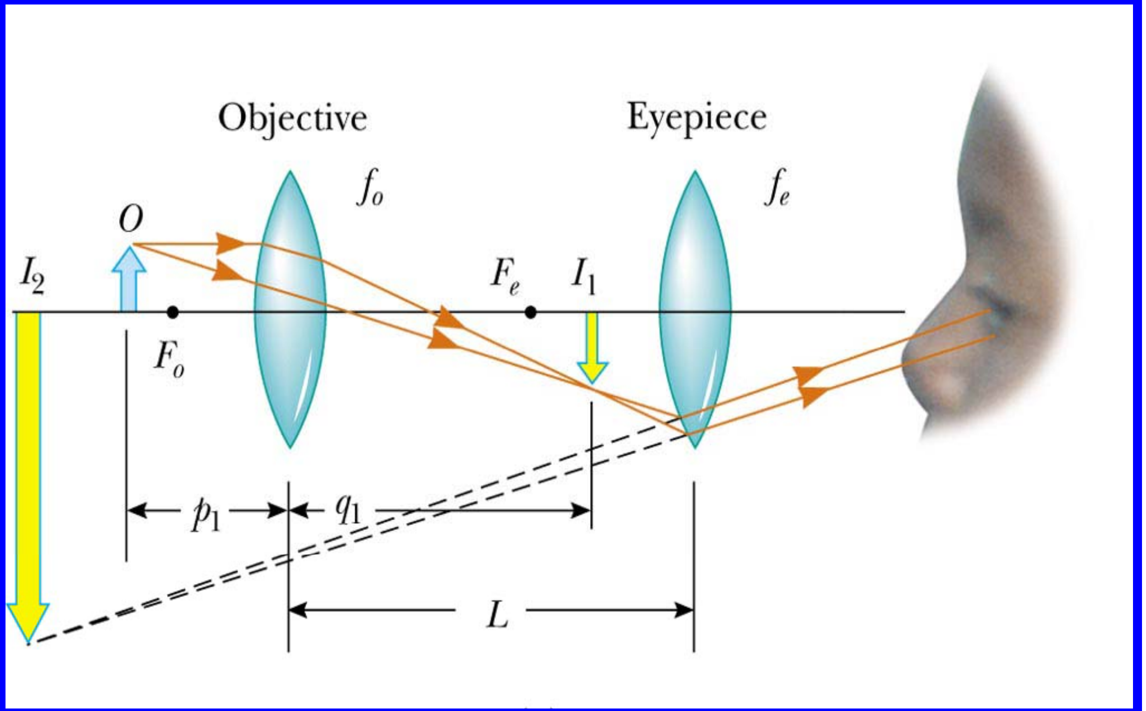


The lens are separated by a distance L

L is much greater than either focal length

Compound Microscope

- The approach to analysis is the same as for any two lenses in a row
  - The image formed by the first lens becomes the object for the second lens
- The image seen by the eye, I_2, is virtual, inverted and very much enlarged



Magnifications of the Compound Microscope

- The *lateral magnification* of the microscope is

$$M_l = -\frac{q_l}{p_l} \approx -\frac{L}{f_o}$$

- The *angular magnification* of the eyepiece of the microscope is

$$m_e = \frac{25\,cm}{f_e}$$

- The *overall magnification* of the microscope is the product of the individual magnifications

$$m = M_l\, m_e = -\frac{L}{f_o}\left(\frac{25\,cm}{f_e}\right)$$

Other Considerations with a Microscope

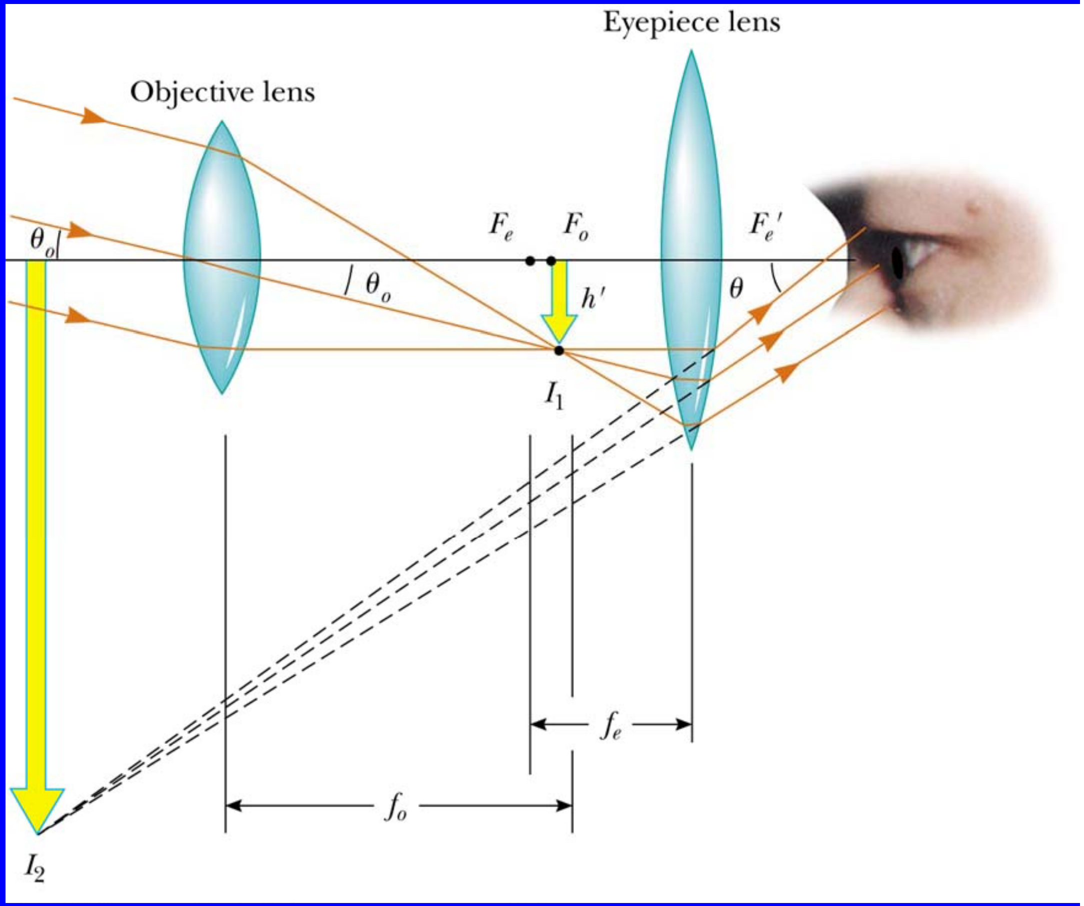
- The ability of an optical microscope to view an object depends on the size of the object relative to the wavelength of the light used to observe it
  - For example, you could not observe an atom ($d \approx 0.1$ nm) with visible light ($\lambda \approx 500$ nm)

Telescopes

- Two fundamental types of telescopes
  - Refracting telescope uses a combination of lens to form an image
  - Reflecting telescope uses a curved mirror and a lens to form an image
- Telescopes can be analyzed by considering them to be two optical elements in a row
  - The image of the first element becomes the object of the second element

Refracting Telescope

- The two lenses are arranged so that the objective forms a real, inverted image of a distance object
- The image is near the focal point of the eyepiece
- The two lenses are separated by the distance $f_o + f_e$ which corresponds to the length of the tube
- The eyepiece forms an enlarged, inverted image of the first image

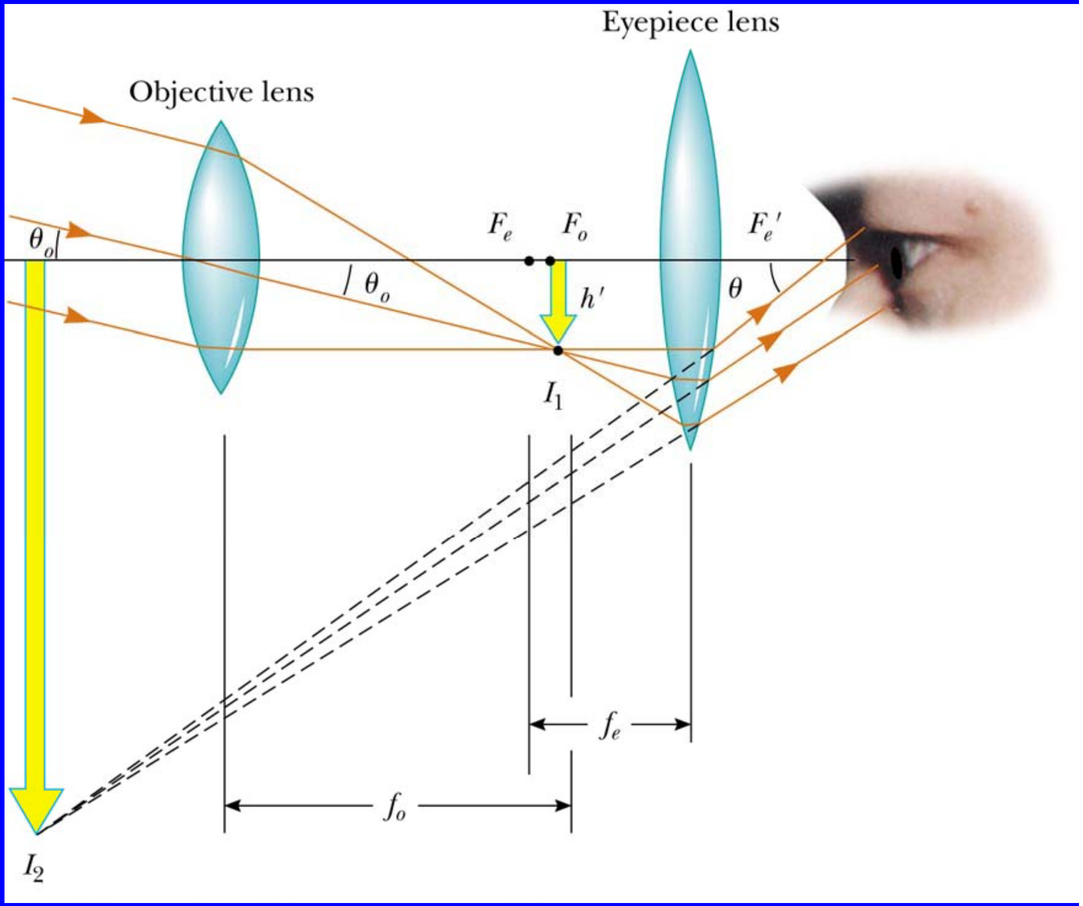


Angular Magnification of a Telescope

- The angular magnification depends on the focal lengths of the objective and eyepiece

$$m = \frac{\theta}{\theta_o} = \frac{f_o}{f_e}$$

- Angular magnification is particularly important for observing nearby objects
  - Very distance objects still appear as a small point of light

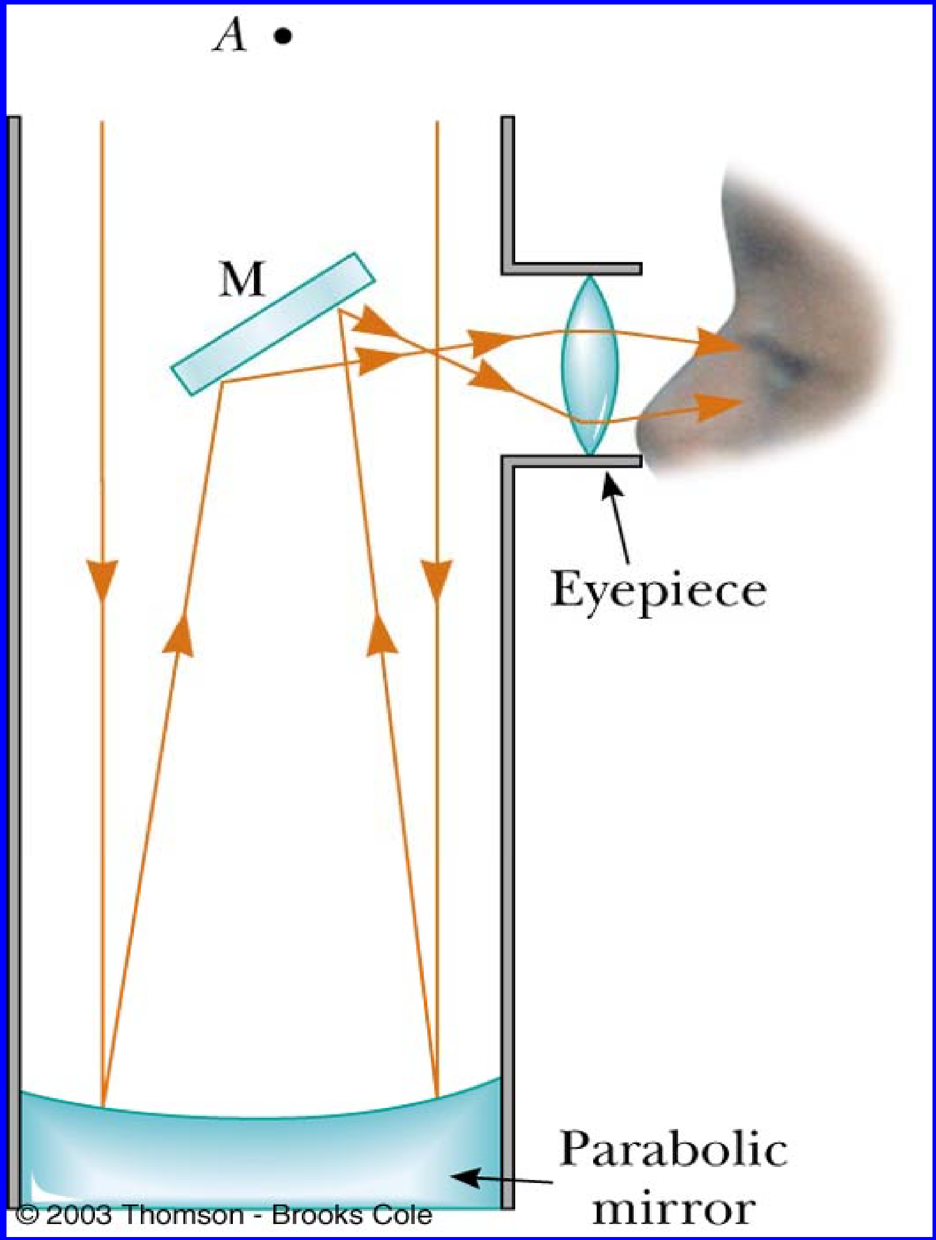


Reflecting Telescope

- Helps overcome some of the disadvantages of refracting telescopes
  - Replaces the objective lens with a mirror
  - The mirror is often parabolic to overcome spherical aberrations
- In addition, the light never passes through glass
  - Except the eyepiece
  - Reduced chromatic aberrations

Reflecting Telescope

- The incoming rays are reflected from the mirror and converge toward point A
  - At A, a photographic plate or other detector could be placed
- A small flat mirror, M, reflects the light toward an opening in the side and passes into an eyepiece



Examples of Telescopes

- Reflecting Telescopes
  - Largest in the world are 10 m diameter Keck telescopes on Mauna Kea in Hawaii
  - Largest single mirror in US is 5 m diameter on Mount Palomar in California
- Refracting Telescopes
  - Largest in the world is Yerkes Observatory in Wisconsin
    - Has a 1 m diameter

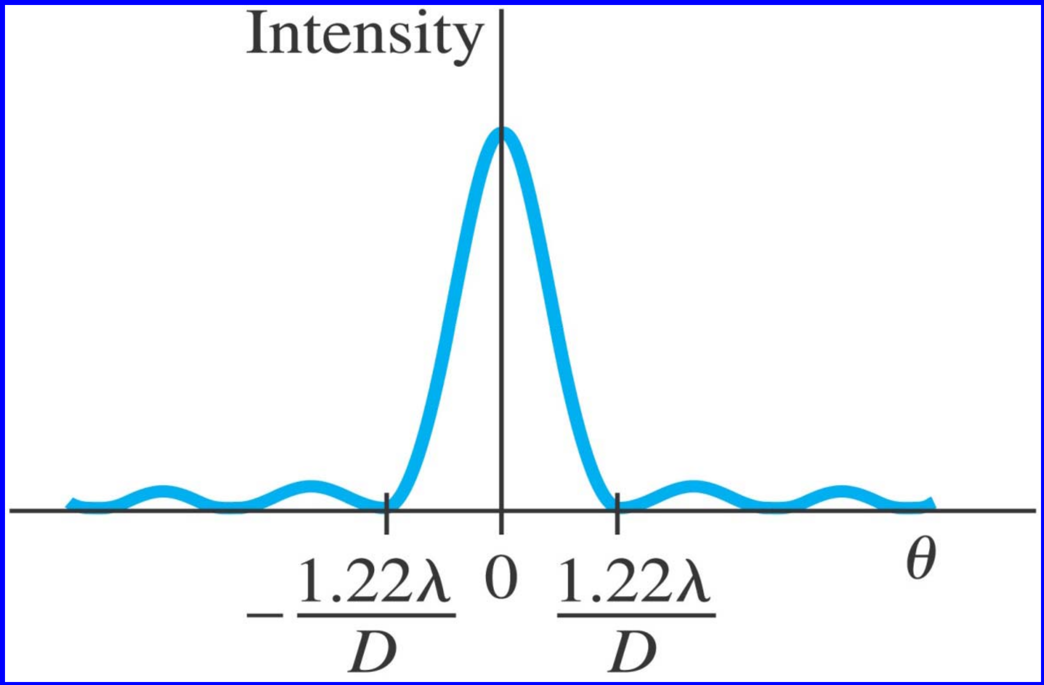
Resolution

- The ability of an optical system to distinguish between closely spaced objects is limited due to the wave nature of light
- If two sources of light are close together, they can be treated as non-coherent sources
- Because of diffraction, the images consist of bright central regions flanked by weaker bright and dark rings

Circular Apertures

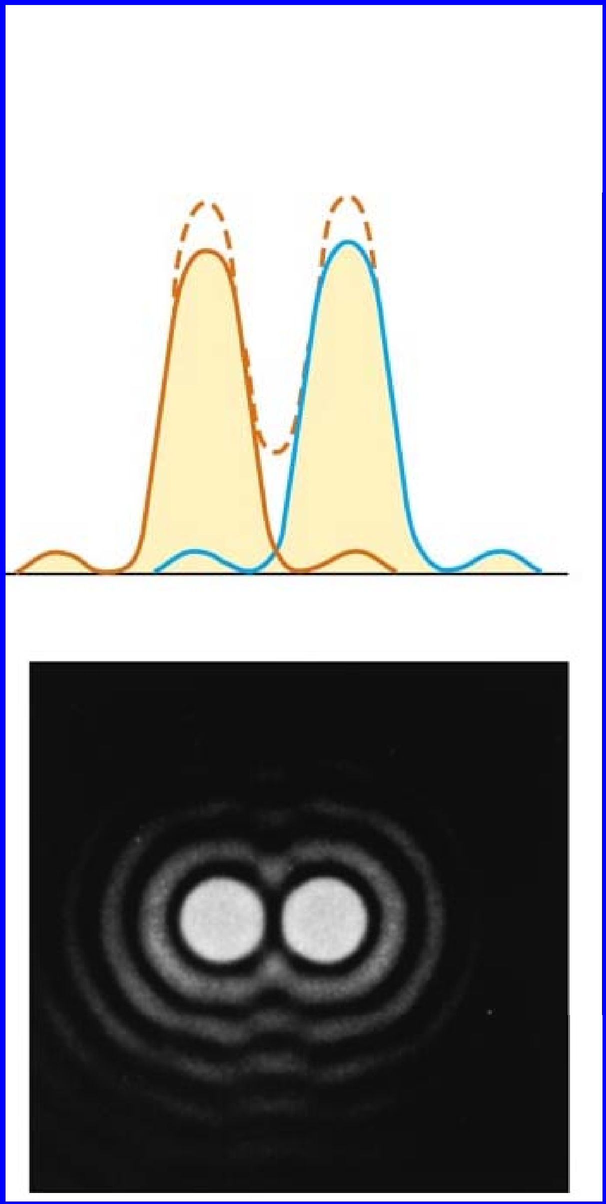
For a circular aperture of diameter D, the central maximum has an angular width:

$$\theta = \frac{1.22\lambda}{D}$$



Rayleigh's Criterion

- If the two sources are separated so that their central maxima do not overlap, their images are said to be resolved
- The limiting condition for resolution is *Rayleigh's Criterion*
  - When the central maximum of one image falls on the first minimum of another image, they images are said to be just resolved
  - The images are just resolved when their angular separation satisfies Rayleigh's criterion

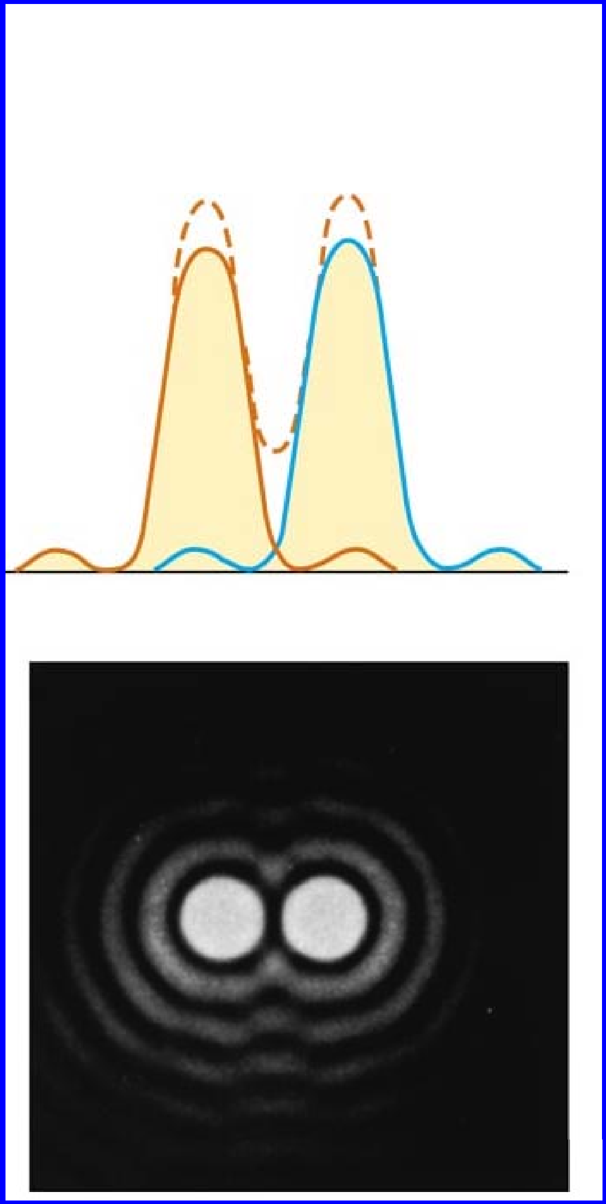


Just Resolved

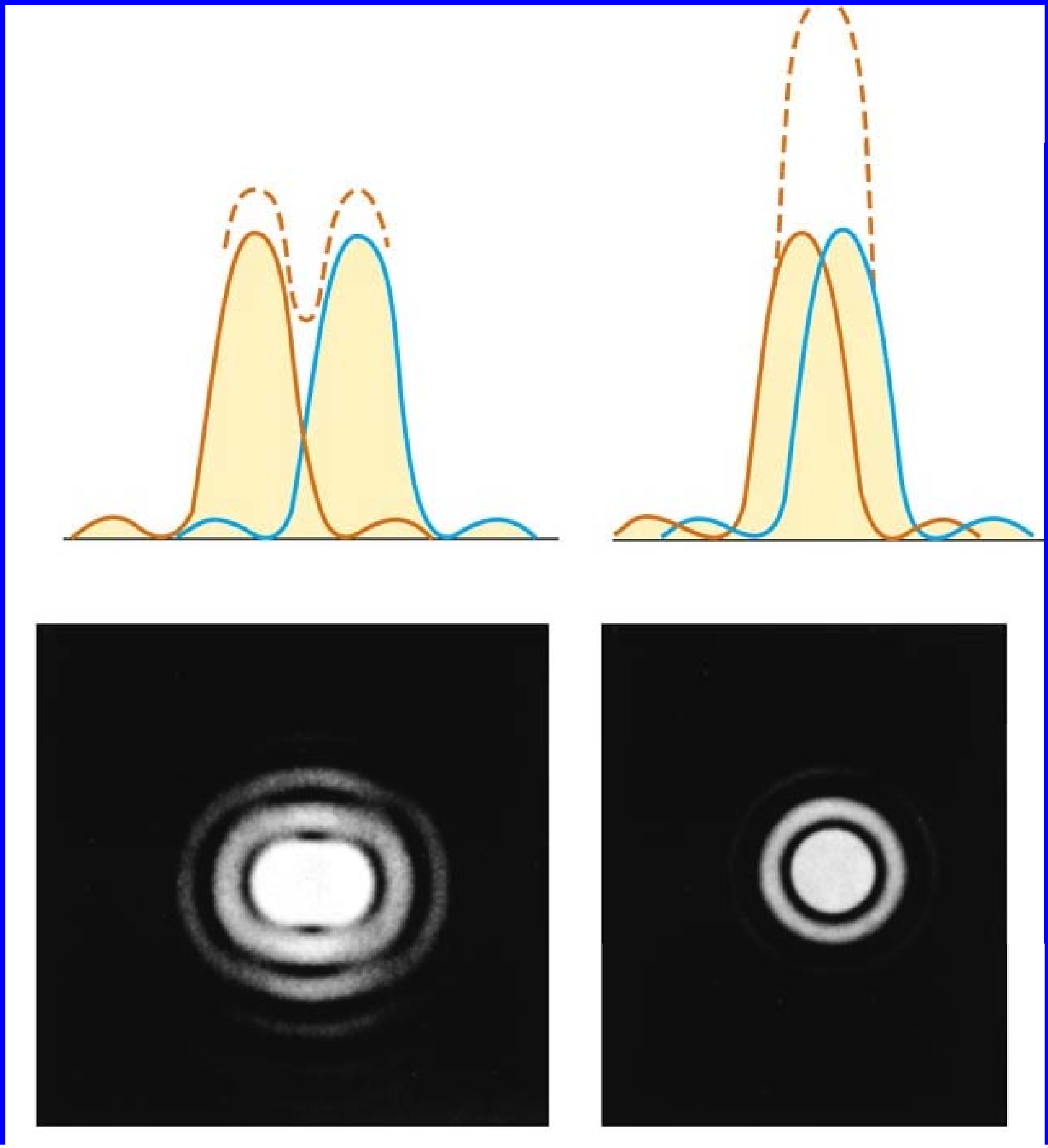
- If viewed through a slit of width a, and applying Rayleigh's criterion, the limiting angle of resolution is

$$\theta_{\min} = \frac{\lambda}{a}$$

- For the images to be resolved, the angle subtended by the two sources at the slit must greater than θ_{min}



Barely Resolved (Left) and Not Resolved (Right)



Resolution with Circular Apertures

- The diffraction pattern of a circular aperture consists of a central, circular bright region surrounded by progressively fainter rings
- The limiting angle of resolution depends on the diameter, D, of the aperture

$$\theta_{\min} = 1.22 \frac{\lambda}{D}$$

QUICK QUIZ 1

Suppose you are observing a binary star with a telescope and are having difficulty resolving the two stars. You decide to use a colored filter to help you. Should you choose a blue filter or a red filter?

QUICK QUIZ 1 ANSWER

We would like to reduce the minimum angular separation for two objects below the angle subtended by the two stars in the binary system. We can do that by reducing the wavelength of the light—this in essence makes the aperture larger, relative to the light wavelength, increasing the resolving power. Thus, we would choose a blue filter.

Resolving Power of a Diffraction Grating

- If λ_1 and λ_2 are nearly equal wavelengths between which the grating spectrometer can just barely distinguish, the *resolving power*, R, of the grating is

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$$

  - All the wavelengths are nearly the same

Resolving Power of a Diffraction Grating

- A grating with a high resolving power can distinguish small differences in wavelength
- The resolving power increases with order number
  - $R = Nm$
    - N is the number of lines illuminated
    - m is the order number
  - All wavelengths are indistinguishable for the zeroth-order maximum
    - $m = 0$ so $R = 0$

Resolution of Telescopes and Microscopes; the λ Limit

Since the resolution is directly proportional to the wavelength and inversely proportional to the diameter, radio telescopes are built to be very large.

Resolution of Telescopes and Microscopes; the λ Limit

For microscopes, assuming the object is at the focal point, the resolving power is given by:

$$\mathrm{RP} = s = f\theta = \frac{1.22\lambda f}{D}$$

Typically, the focal length of a microscope lens is half its diameter, which shows that it is not possible to resolve details smaller than the wavelength being used.

$$\mathrm{RP} \approx \frac{\lambda}{2}$$

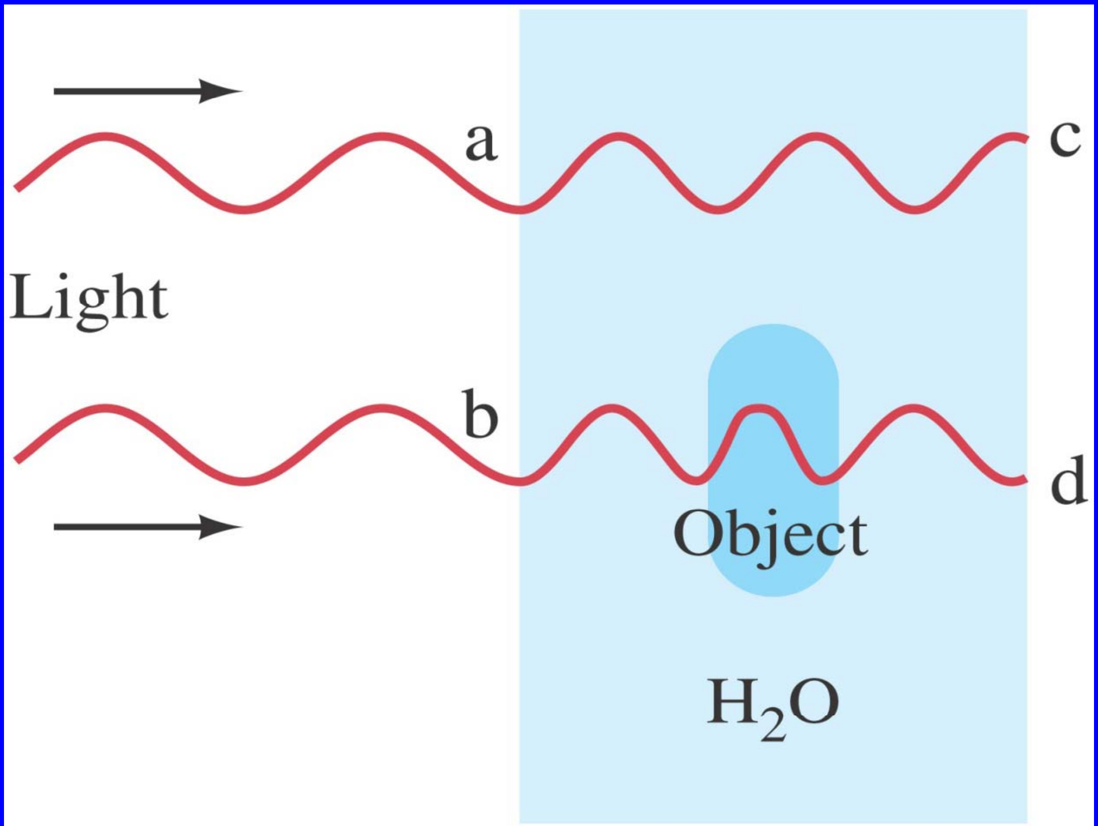
Resolution of the Human Eye and Useful Magnification

The human eye can resolve objects that are about 1 cm apart at a distance of 20 m, or 0.1 mm apart at the near point.

This limits the useful magnification of a light microscope to about 500x – 1000x.

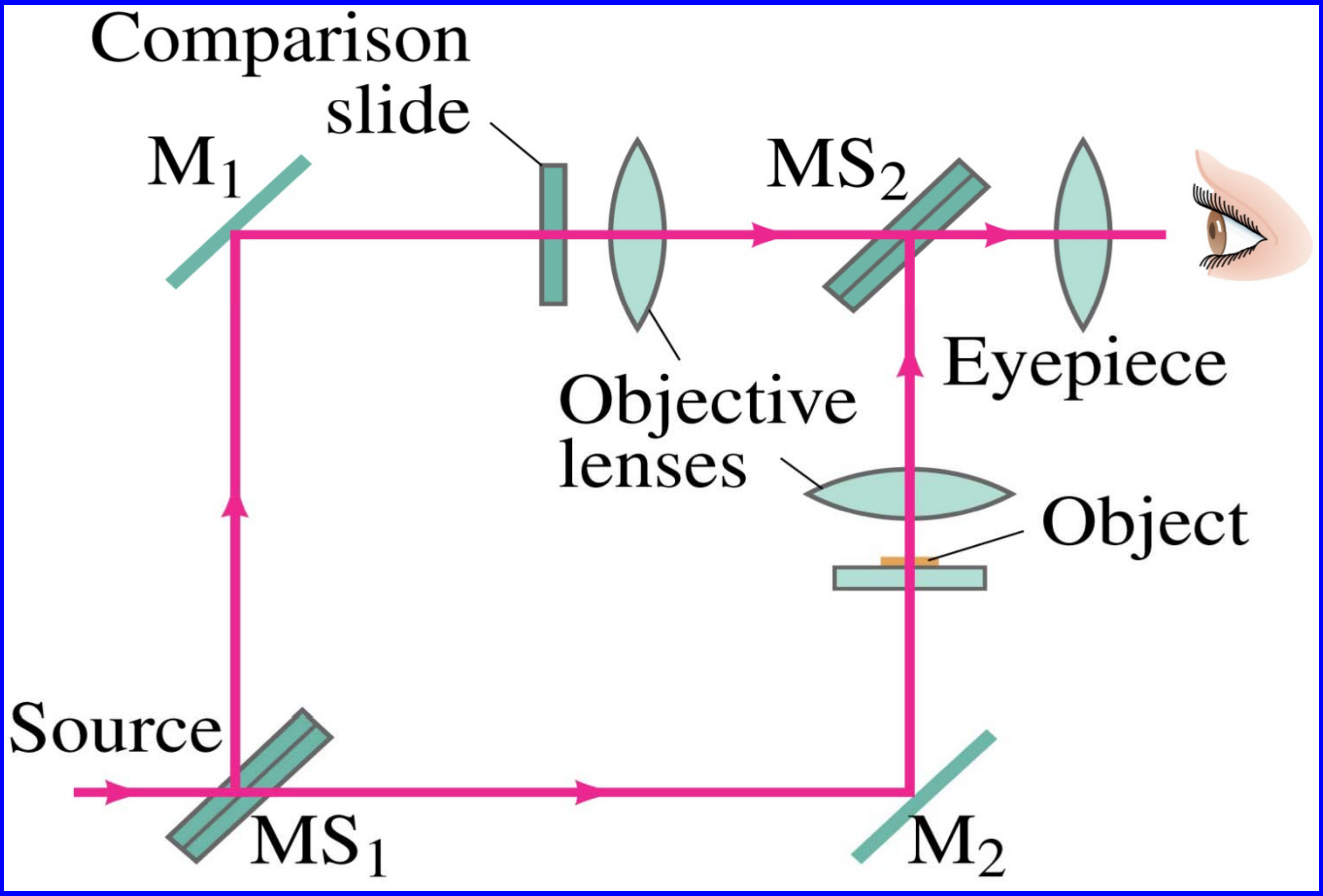
Specialty Microscopes and Contrast

In addition to sufficient resolving power, a microscope must be able to distinguish the object from its background.



Specialty Microscopes and Contrast

One way to do this is by using an interference microscope, which can detect objects by the change in wavelength as the light passes through them.

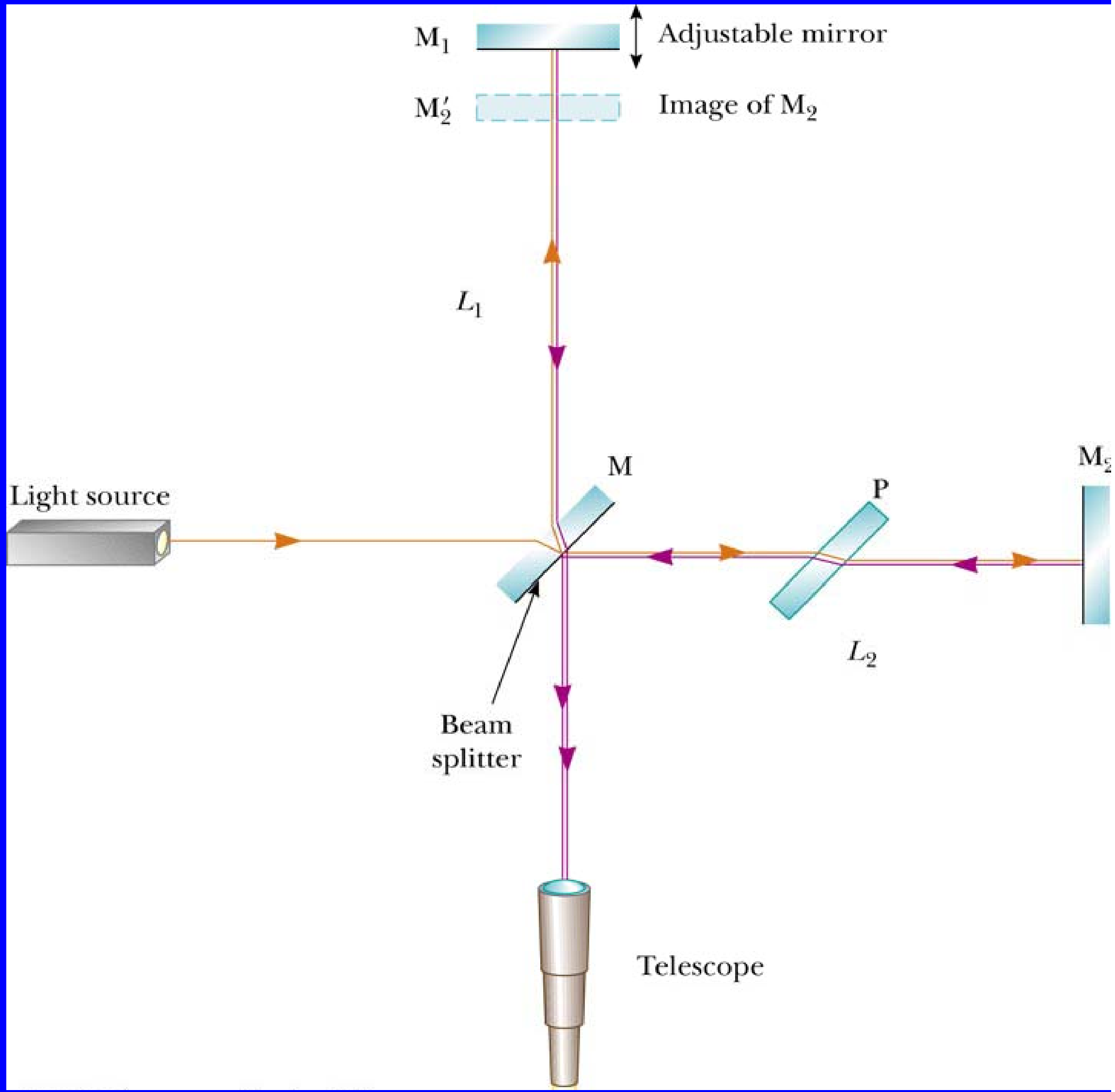


Michelson Interferometer

- The Michelson Interferometer is an optical instrument that has great scientific importance, but is unfamiliar to most people
- It splits a beam of light into two parts and then recombines them to form an interference pattern
  - It is used to make accurate length measurements

Michelson Interferometer, schematic

- A beam of light provided by a monochromatic source is split into two rays by mirror M
- One ray is reflected to M_1 and the other transmitted to M_2
- After reflecting, the rays combine to form an interference pattern
- The glass plate ensures both rays travel the same distance through glass



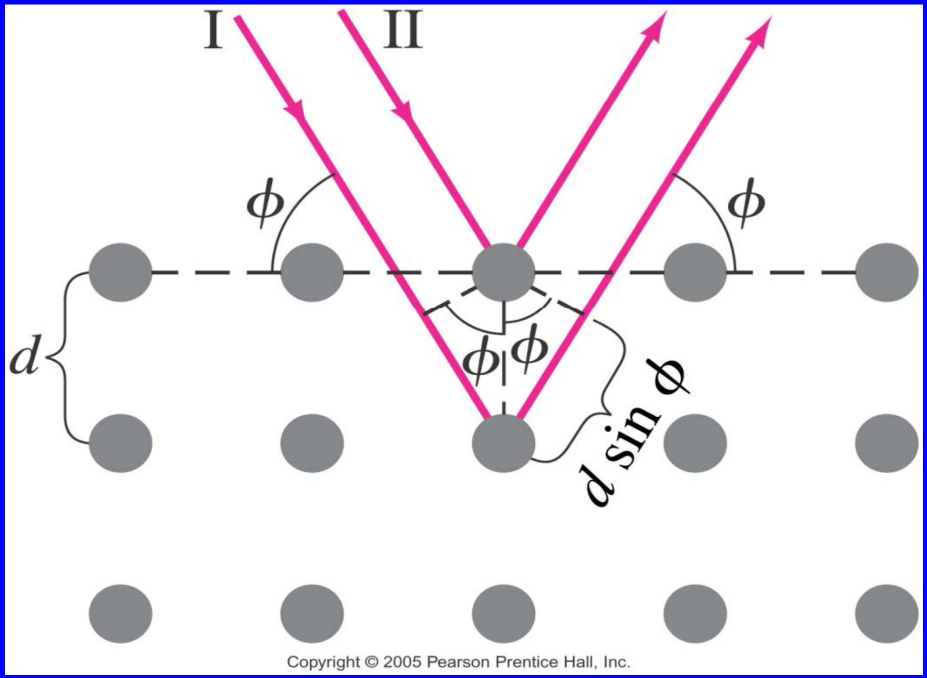
Measurements with a Michelson Interferometer

- The interference pattern for the two rays is determined by the difference in their path lengths
- When M_1 is moved a distance of $\lambda/4$, successive light and dark fringes are formed
  - This change in a fringe from light to dark is called *fringe shift*
- The wavelength can be measured by counting the number of fringe shifts for a measured displacement of M
- If the wavelength is accurately known, the mirror displacement can be determined to within a fraction of the wavelength

X-Rays and X-Ray Diffraction

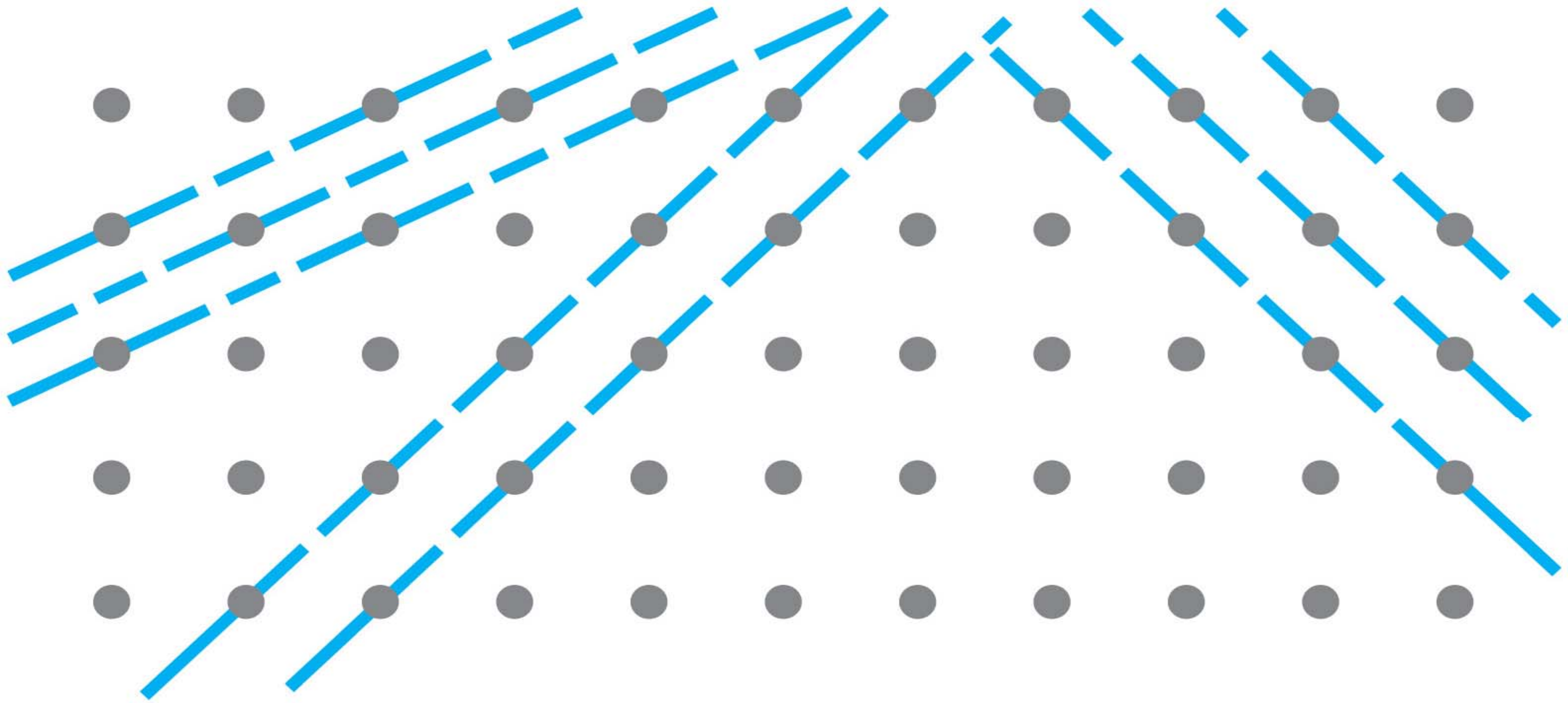
The wavelengths of X-rays are very short. Diffraction experiments are impossible to do with conventional diffraction gratings.

Crystals have spacing between their layers that is ideal for diffracting X-rays:



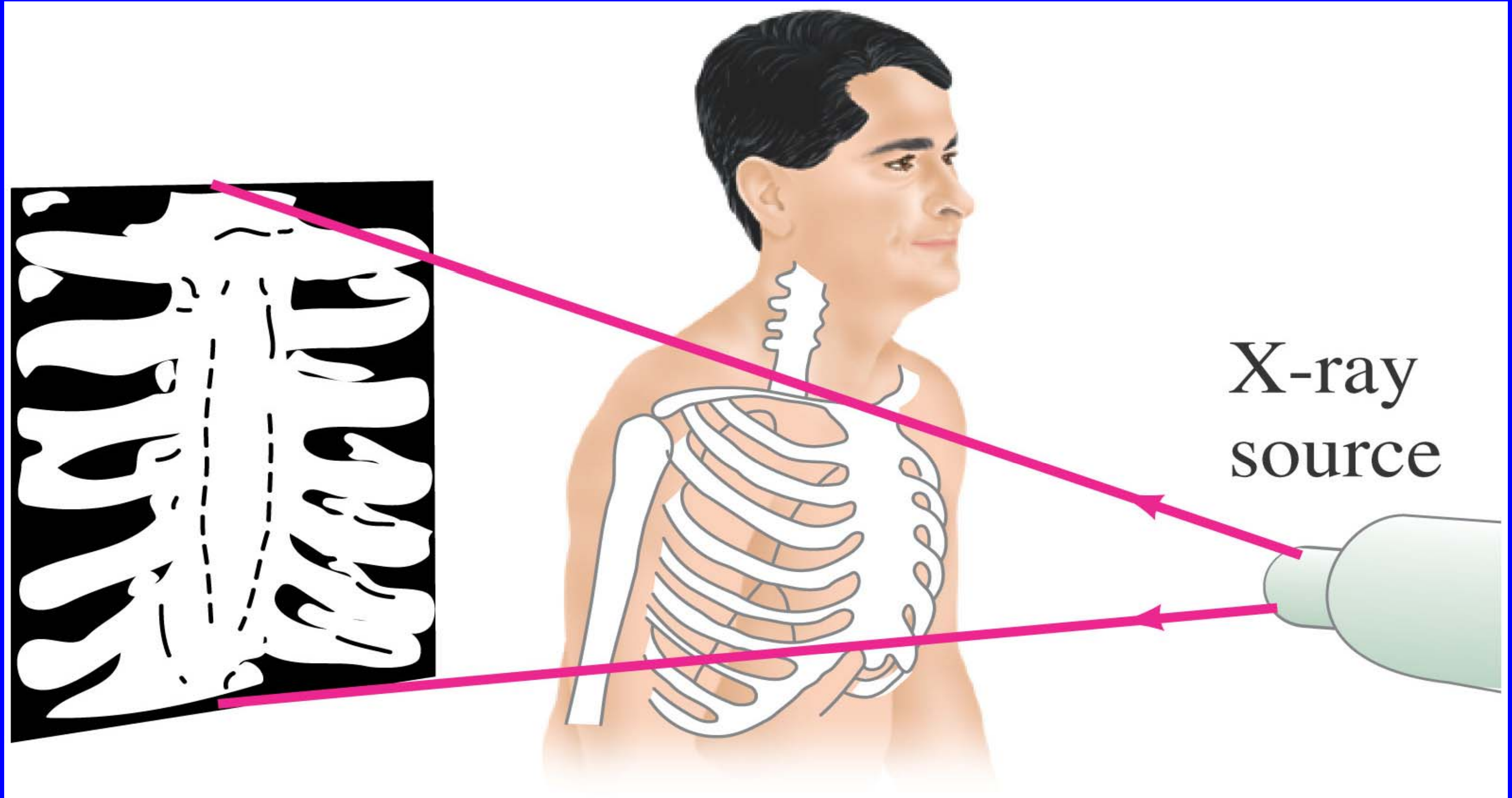
X-Rays and X-Ray Diffraction

X-ray diffraction is now used to study the internal structure of crystals; this is how the helical structure of DNA was determined.



X-Ray Imaging and Computed Tomography (CT Scan)

A conventional X-ray is essentially a shadow; there are no lenses involved.



X-Ray Imaging and Computed Tomography (CT Scan)

Computed tomography uses a narrow beam of X-rays, and takes measurements at many different angles. The measurements are sent to a computer, which combines them into a detailed image.