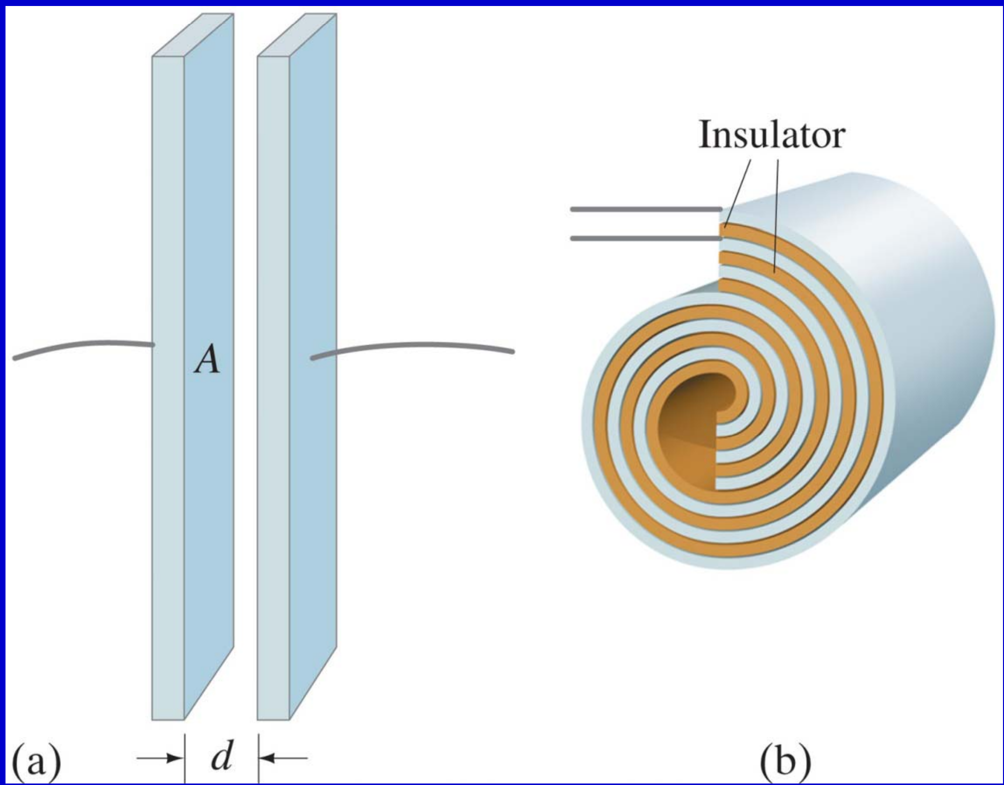
PHYSICS 1442-003
Fall 20101

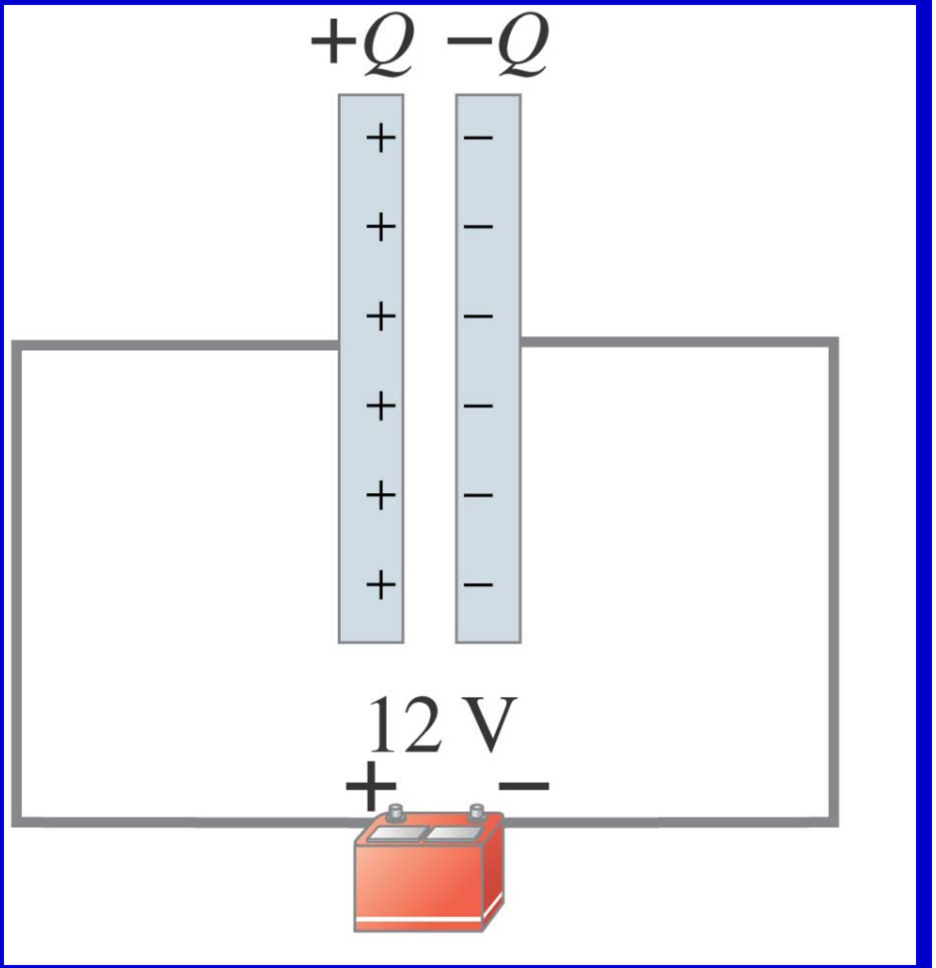
Lecture 5
Capacitance

Capacitor

A capacitor consists of two conductors that are close but not touching. A capacitor has the ability to store electric charge.



- When a capacitor is connected to a battery, electrons are transferred from the left plate through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged
- The flow of charges ceases when the voltage across the capacitor equals that of the battery
- The capacitor reaches its maximum charge when the flow of charge ceases



Capacitance

When a capacitor is connected to a battery, the charge on its plates of is proportional to the voltage:

$$Q = CV$$

The quantity C is called the capacitance.

The capacitance, C, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor (plate) to the magnitude of the potential difference between the conductors (plates)

Capacitance

- $Q = CV \quad \Rightarrow \quad C \equiv \frac{Q}{\Delta V}$

Unit of capacitance: the Farad (F)

- 1 F = 1 C / V
- A Farad is very large
  - Often will see μF or pF

Parallel-Plate Capacitor

- The capacitance of a device does not depend on the voltage; it is a function of the geometry and materials of the capacitor.
- For a parallel-plate capacitor whose plates are separated by air:

$$C = \varepsilon_o \frac{A}{d}$$

To get an idea how big a farad is, suppose you want to make a 1-F air-filled parallel-plate capacitor for a circuit you are building. To make it a reasonable size, suppose you limit the plate area to 1.0 cm^2.

What would the gap have to be between the plates? Is this practically achievable?

Capacitance C = 1F

Plate area is 1.0 cm^2.

What would the gap have to be between the plates?

$$C = \frac{\varepsilon_0 A}{d} \quad \rightarrow$$

$$d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\ C^2/N\cdot m^2\right)\left(1.0\times10^{-4}\ m^2\right)}{(1F)} = \boxed{9\times10^{-16}\ m}$$

QUICK QUIZ 1

A capacitor is designed so that one plate is large and the other is small. If the plates are connected to a battery, (a) the large plate has a greater charge than the small plate, (b) the large plate has less charge than the small plate, or (c) the plates have charges equal in magnitude but opposite in sign.

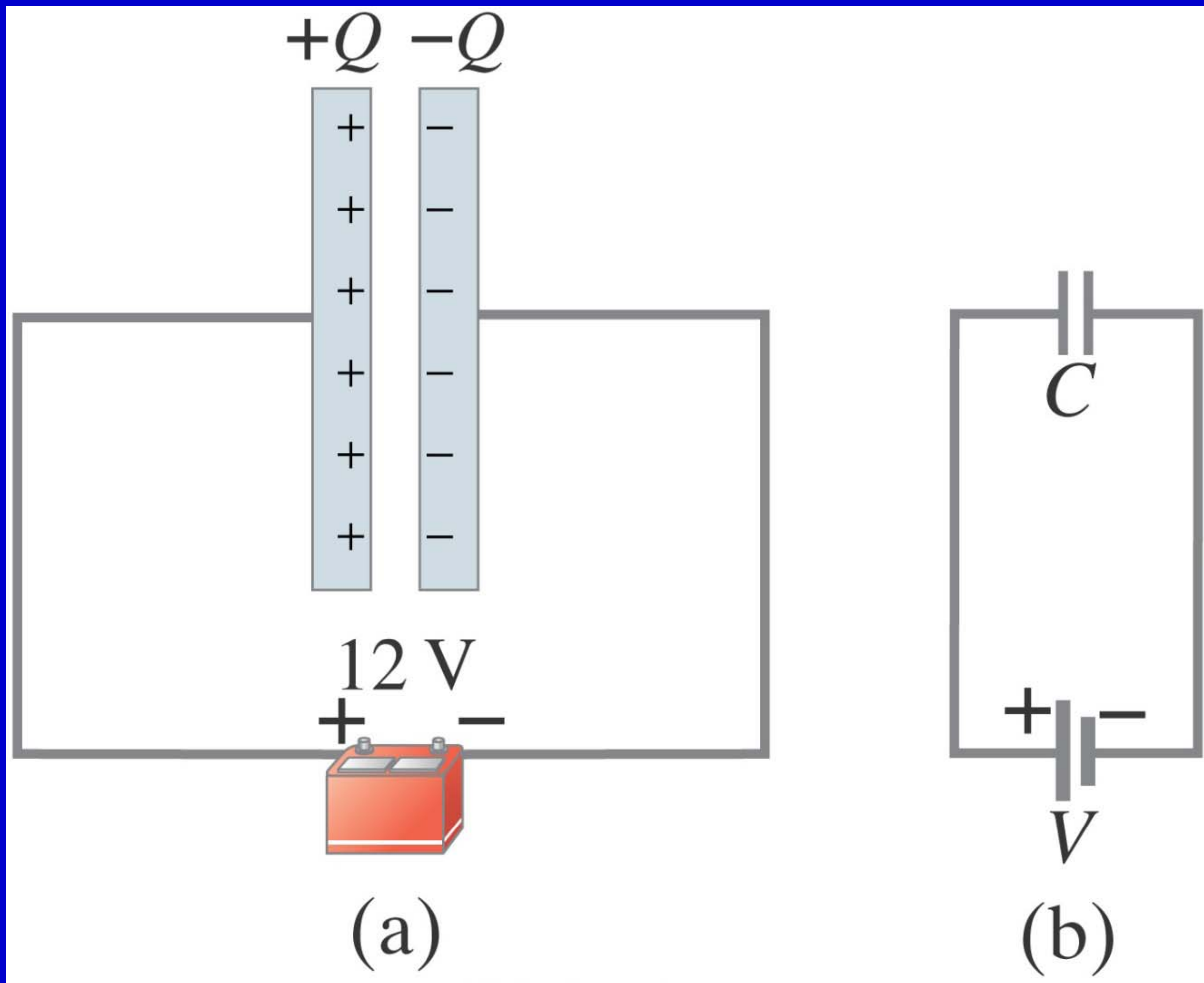
QUICK QUIZ 1 ANSWER

(c). The battery moves negative charge from one plate and puts it on the other. The first plate is left with excess positive charge whose magnitude equals that of the negative charge moved to the other plate.

Capacitors in Circuits

- A *circuit* is a collection of objects usually containing a source of electrical energy (such as a battery) connected to elements that convert electrical energy to other forms
- A *circuit diagram* can be used to show the path of the real circuit

(a) Parallel-plate capacitor connected to battery. (b) is a circuit diagram.



- When a capacitor is connected to a battery, electrons are transferred from the left plate through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged
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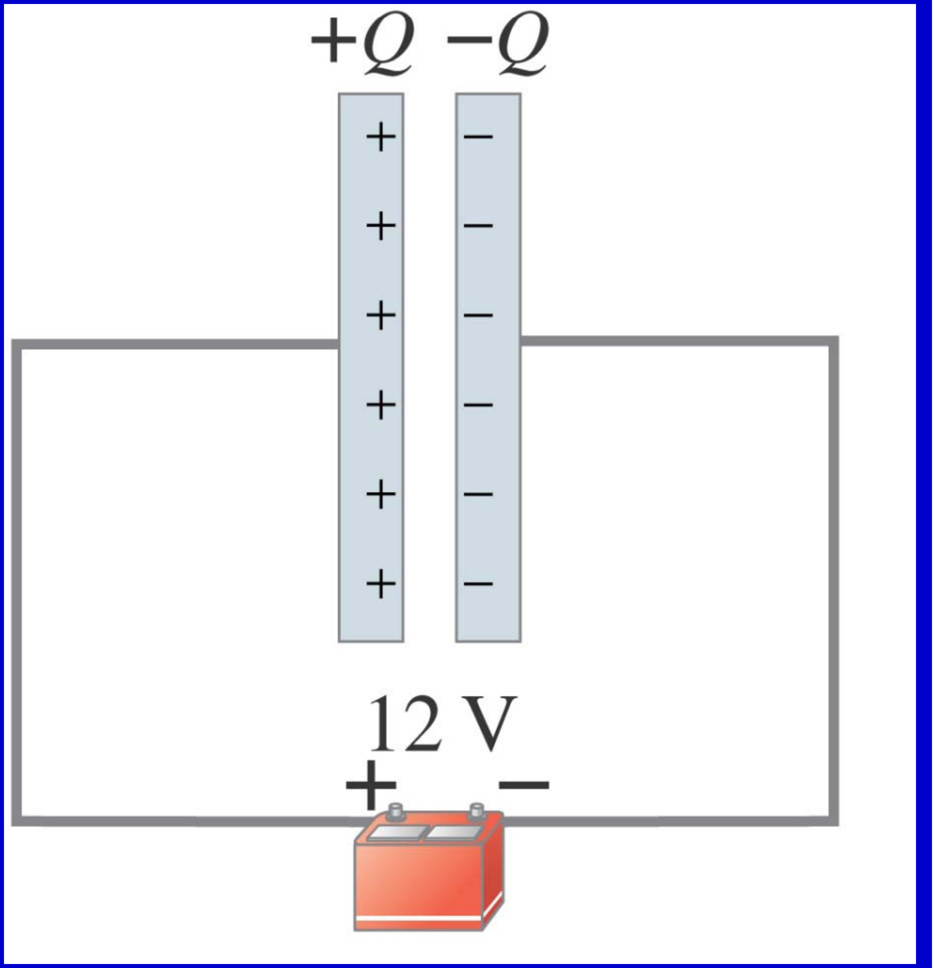
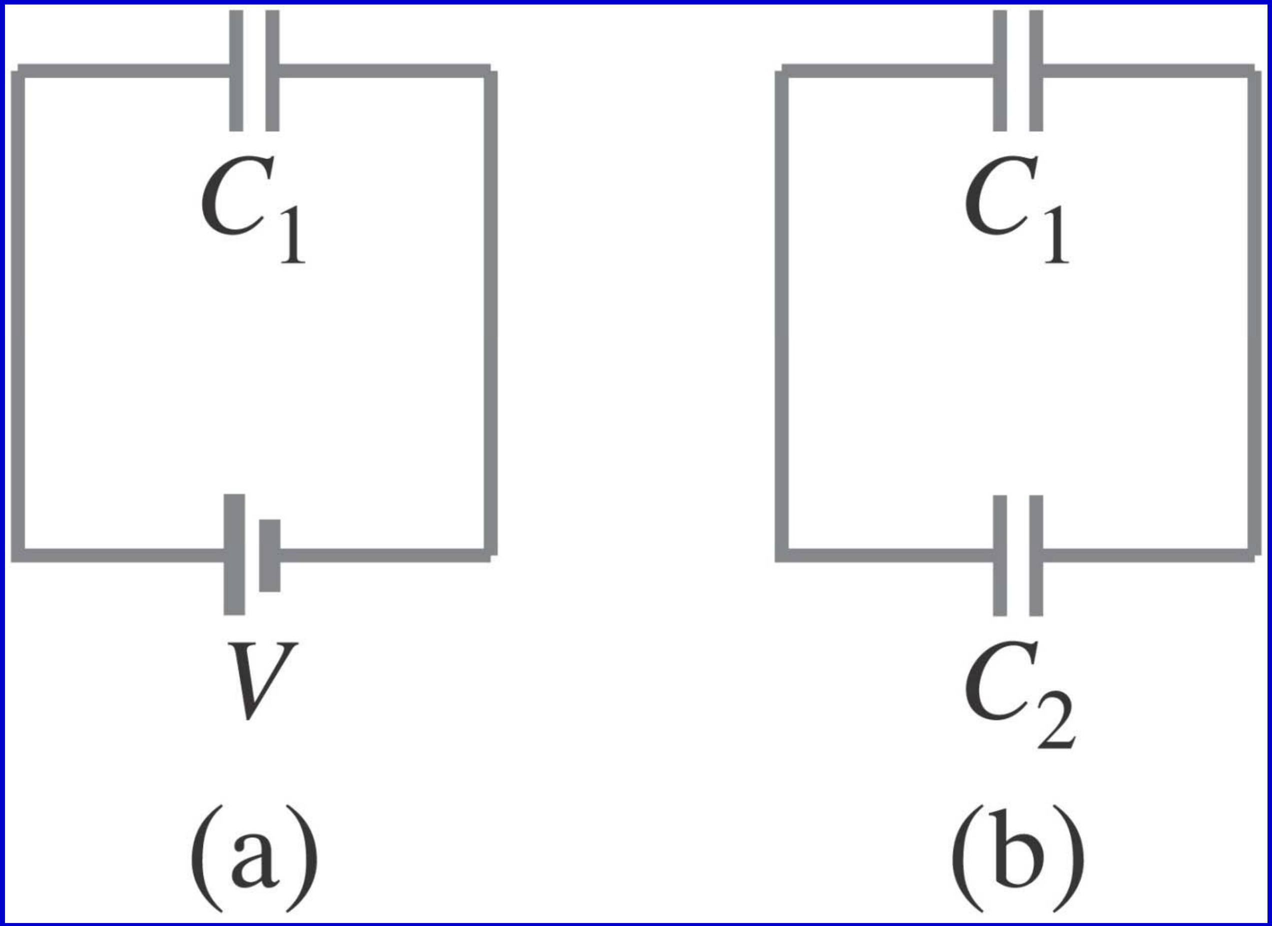
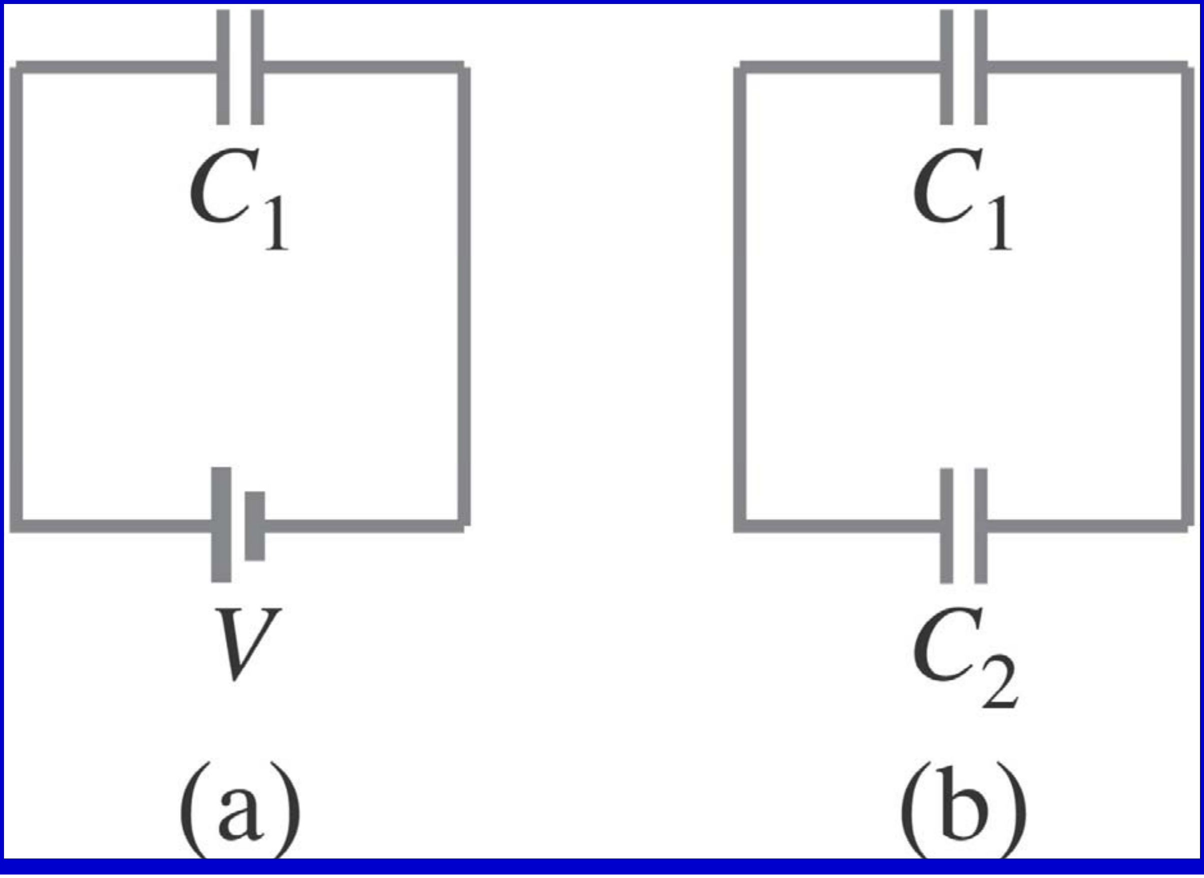


A 7.70-mF capacitor is charged by a 125-V battery (Fig. (a)) and then is disconnected from the battery. When this capacitor is then connected (Fig. (b)) to a second (initially uncharged) capacitor, the final voltage on each capacitor is 15 V. What is the value of C_2 ?



After C_1 is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected together, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire.



We use the total charge and the final potential difference to find the value of the second capacitor.

$$Q_{\text{Total}} = C_1 V_{1_{\text{initial}}} \qquad Q_{1_{\text{final}}} = C_1 V_{\text{final}} \qquad Q_{2_{\text{final}}} = C_2 V_{\text{final}}$$

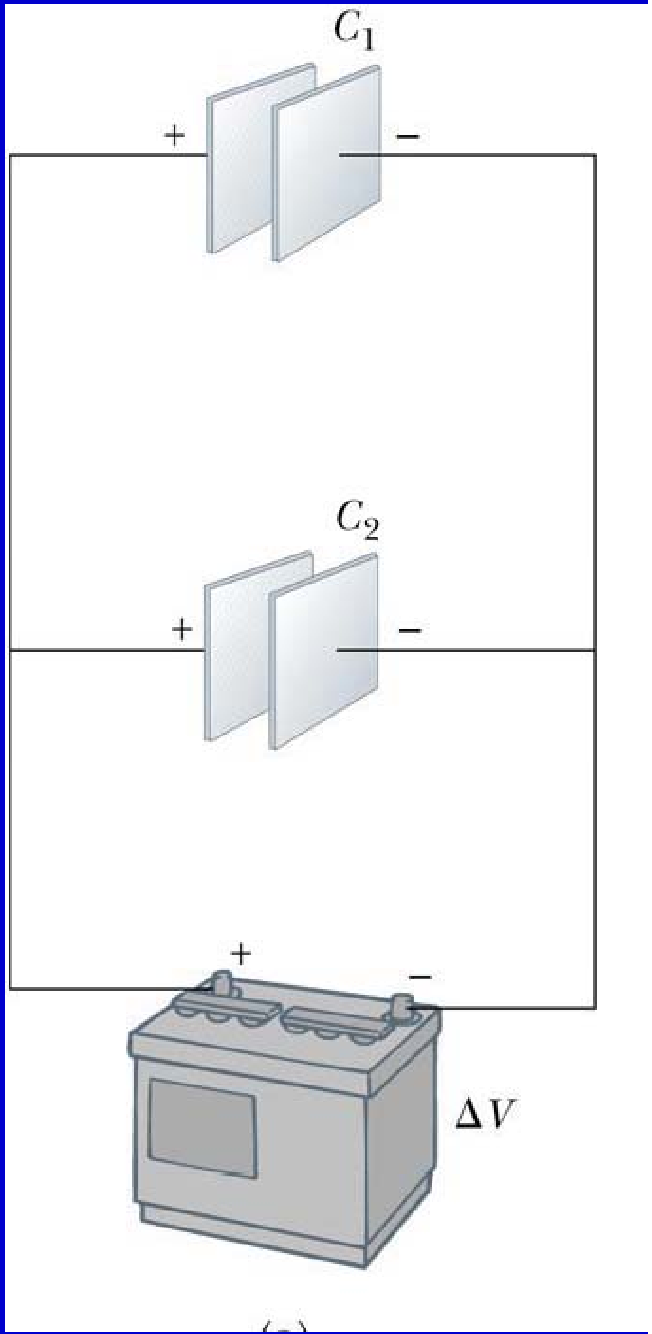
$$Q_{\text{Total}} = Q_{1_{\text{final}}} + Q_{2_{\text{final}}} = \left(C_1 + C_2\right) V_{\text{final}} \rightarrow$$

$$C_1 V_{1_{\text{initial}}} = \left(C_1 + C_2\right) V_{\text{final}} \rightarrow$$

$$C_2 = C_1 \left(\frac{V_{1_{\text{initial}}}}{V_{\text{final}}} - 1 \right) = \left(7.7 \times 10^{-6}\,\text{F}\right)\left(\frac{125\,\text{V}}{15\,\text{V}} - 1 \right) = \boxed{5.6 \times 10^{-5}\,\text{F}}$$

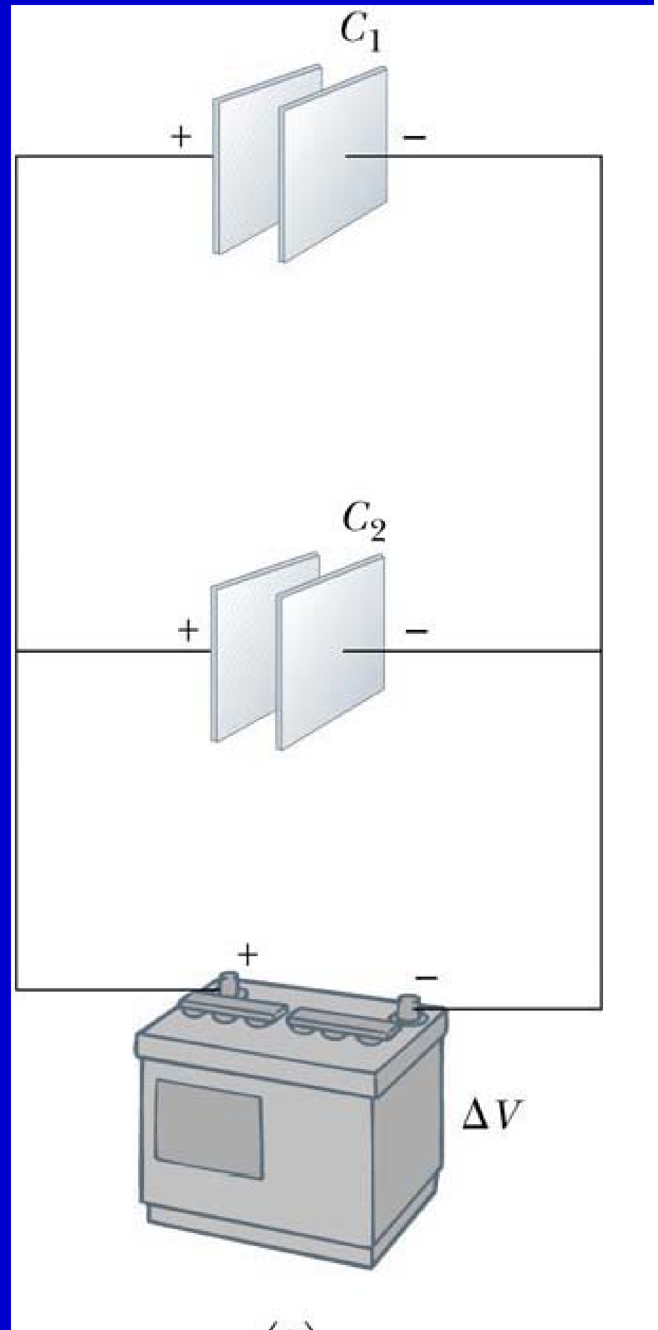
Capacitors in Parallel

- When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged
- The flow of charges ceases when the voltage across the capacitors equals that of the battery
- The capacitors reach their maximum charge when the flow of charge ceases



Capacitors in Parallel

- The total charge is equal to the sum of the charges on the capacitors
  - $Q_{total} = Q_1 + Q_2$
- The potential difference across the capacitors is the same
  - And each is equal to the voltage of the battery

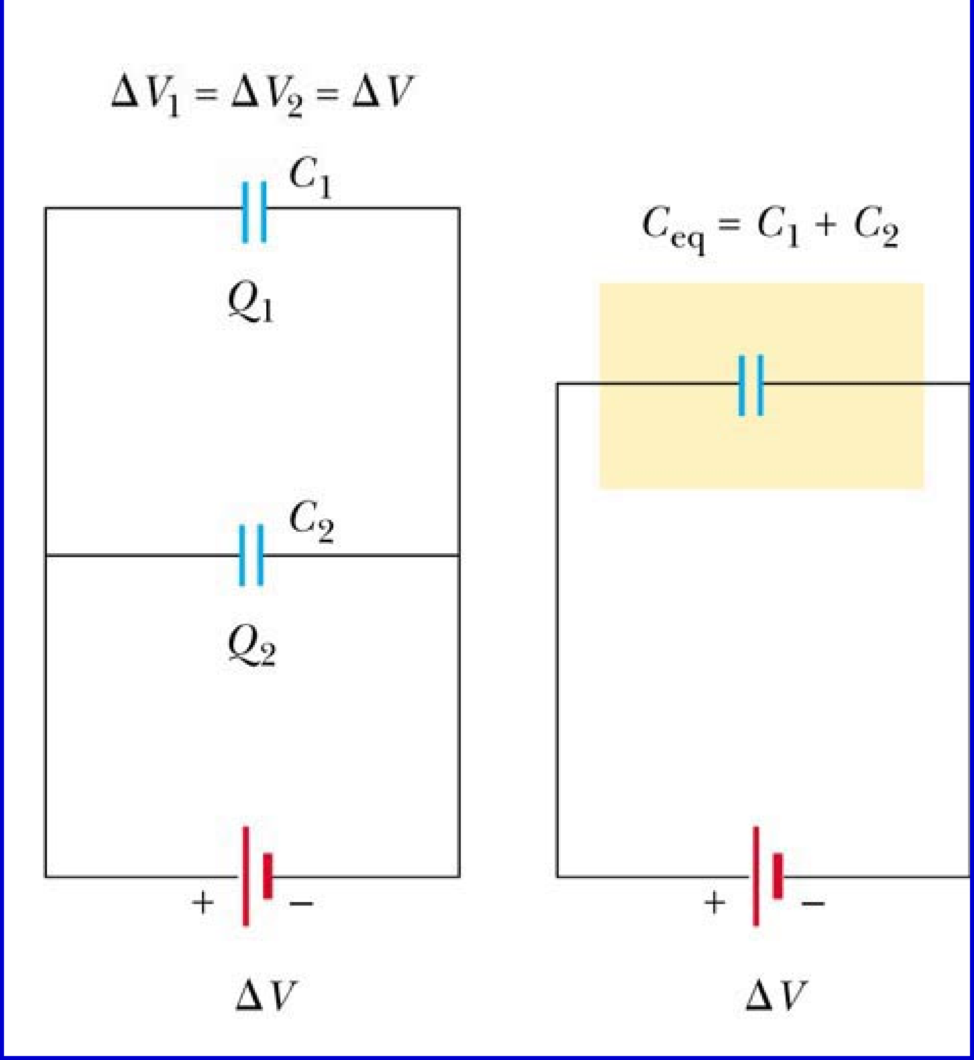


Capacitors in Parallel

The capacitors can be replaced with one capacitor with a capacitance of C_{eq}

- $C_{eq} = C_1 + C_2$

The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors

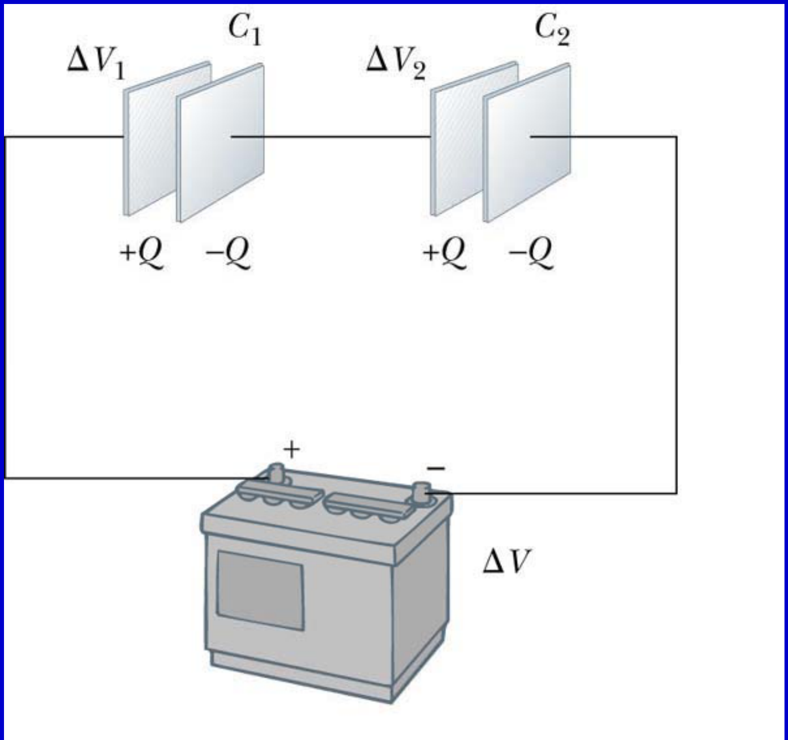


Capacitors in Parallel

- $C_{eq} = C_1 + C_2$
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors

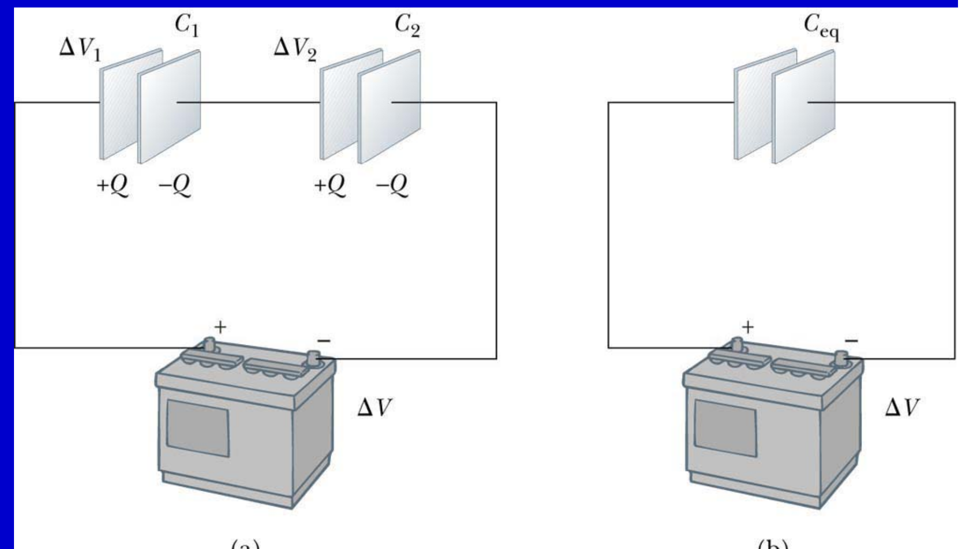
Capacitors in Series

- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery
- As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge
- All of the right plates gain charges of –Q and all the left plates have charges of +Q



Capacitors in Series

- An equivalent capacitor can be found that performs the same function as the series combination
- The potential differences add up to the battery voltage
- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination



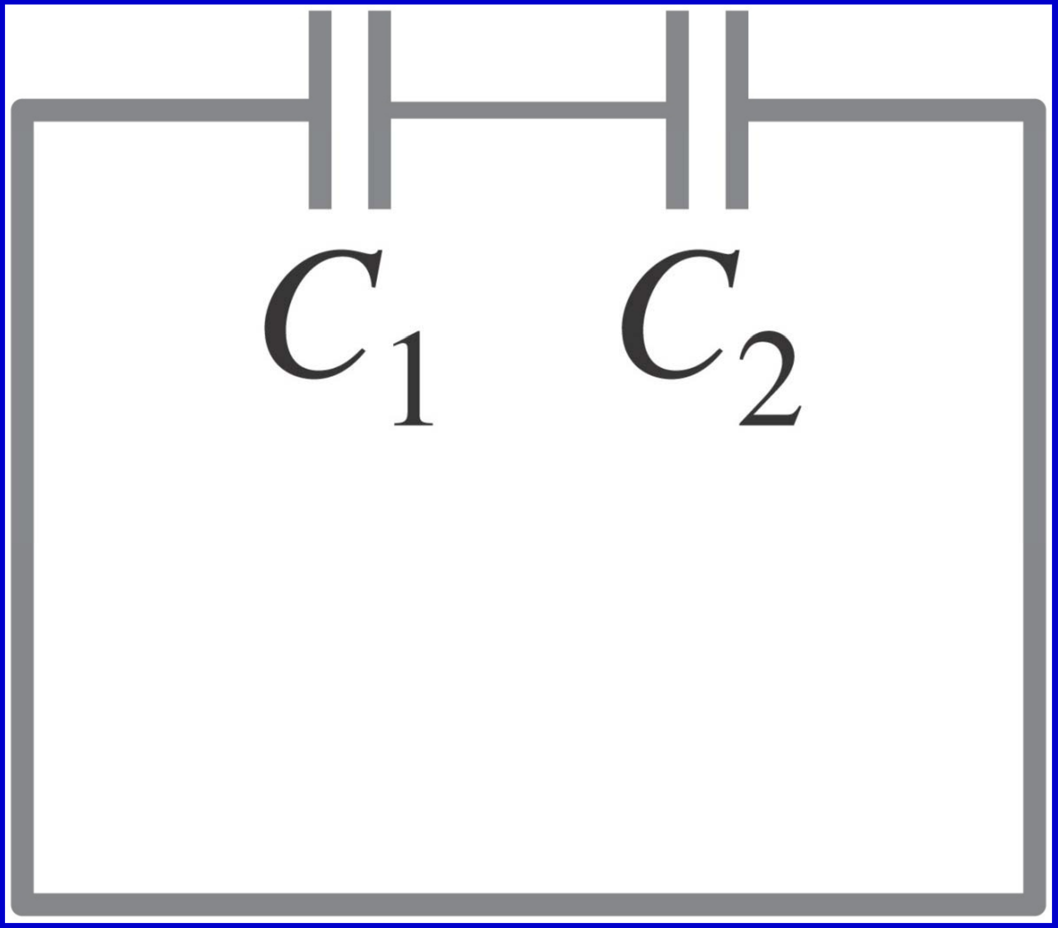
$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Problem-Solving Strategy

- A complicated circuit can often be reduced to one equivalent capacitor
  - Replace capacitors in series or parallel with their equivalent
  - Redraw the circuit and continue
- To find the charge on, or the potential difference across, one of the capacitors, start with your final equivalent capacitor and work back through the circuit reductions

A capacitor of capacitance C_1 carries a charge Q_0. It is then connected directly to a second, uncharged, capacitor of capacitance C_2 as shown . What charge will each carry now? What will be the potential difference across each?



There is no other source of charge except for the original capacitor. Thus the total charge must remain at Q_0.

Also since the plates of the capacitor C_1 are connected via equipotential wires to the plate of the capacitor C_2, the two capacitors must have the same voltage V across their plates.

$$Q_0 = Q_1 + Q_2 \qquad Q_1 = C_1 V \qquad Q_2 = C_2 V$$

$$Q_0 = C_1 V + C_2 V = \left(C_1 + C_2\right) V \rightarrow V = \boxed{\frac{Q_0}{C_1 + C_2}}$$

$$Q_1 = C_1 V = \boxed{Q_0 \frac{C_1}{C_1 + C_2}} \qquad Q_2 = C_2 V = \boxed{Q_0 \frac{C_2}{C_1 + C_2}}$$

Problem-Solving Strategy

- A complicated circuit can often be reduced to one equivalent capacitor
  - Replace capacitors in series or parallel with their equivalent
  - Redraw the circuit and continue
- To find the charge on, or the potential difference across, one of the capacitors, start with your final equivalent capacitor and work back through the circuit reductions

Energy Stored in a Capacitor

- Energy stored = ½Q ΔV
- From the definition of capacitance: $C \equiv \frac{Q}{\Delta V}$
- this can be rewritten in different forms

$$Energy = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

Applications

- In general, capacitors act as energy reservoirs that can slowly charged and then discharged quickly to provide large amounts of energy in a short pulse

QUICK QUIZ 2

You charge a parallel-plate capacitor, remove it from the battery, and prevent the wires connected to the plates from touching each other. When you pull the plates farther apart, do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV; (e) energy stored in the capacitor.

QUICK QUIZ 5 ANSWER

(a) *C* decreases

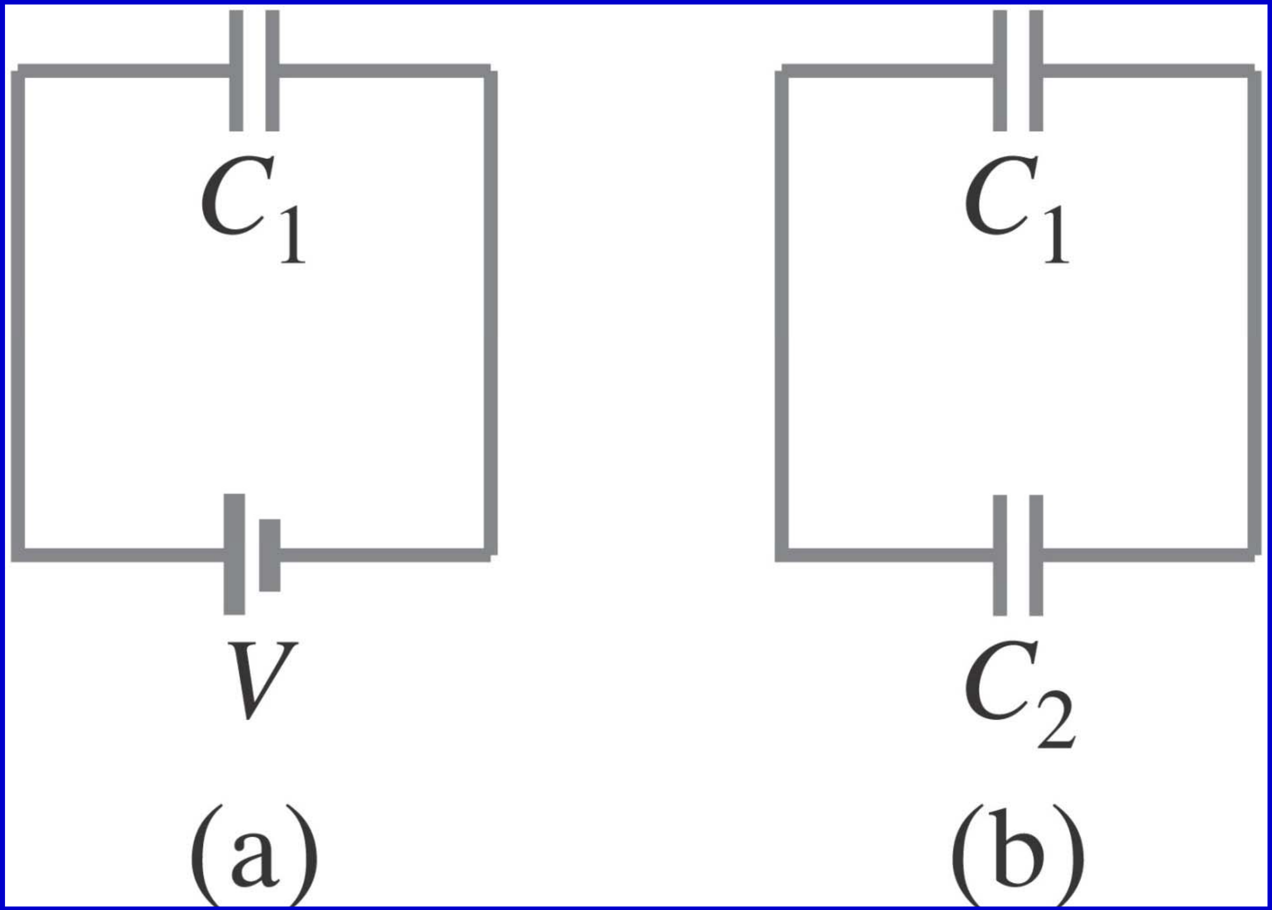
(b) *Q* stays the same

(c) *E* stays the same

(d) D*V* increases

(e) The energy stored increases.

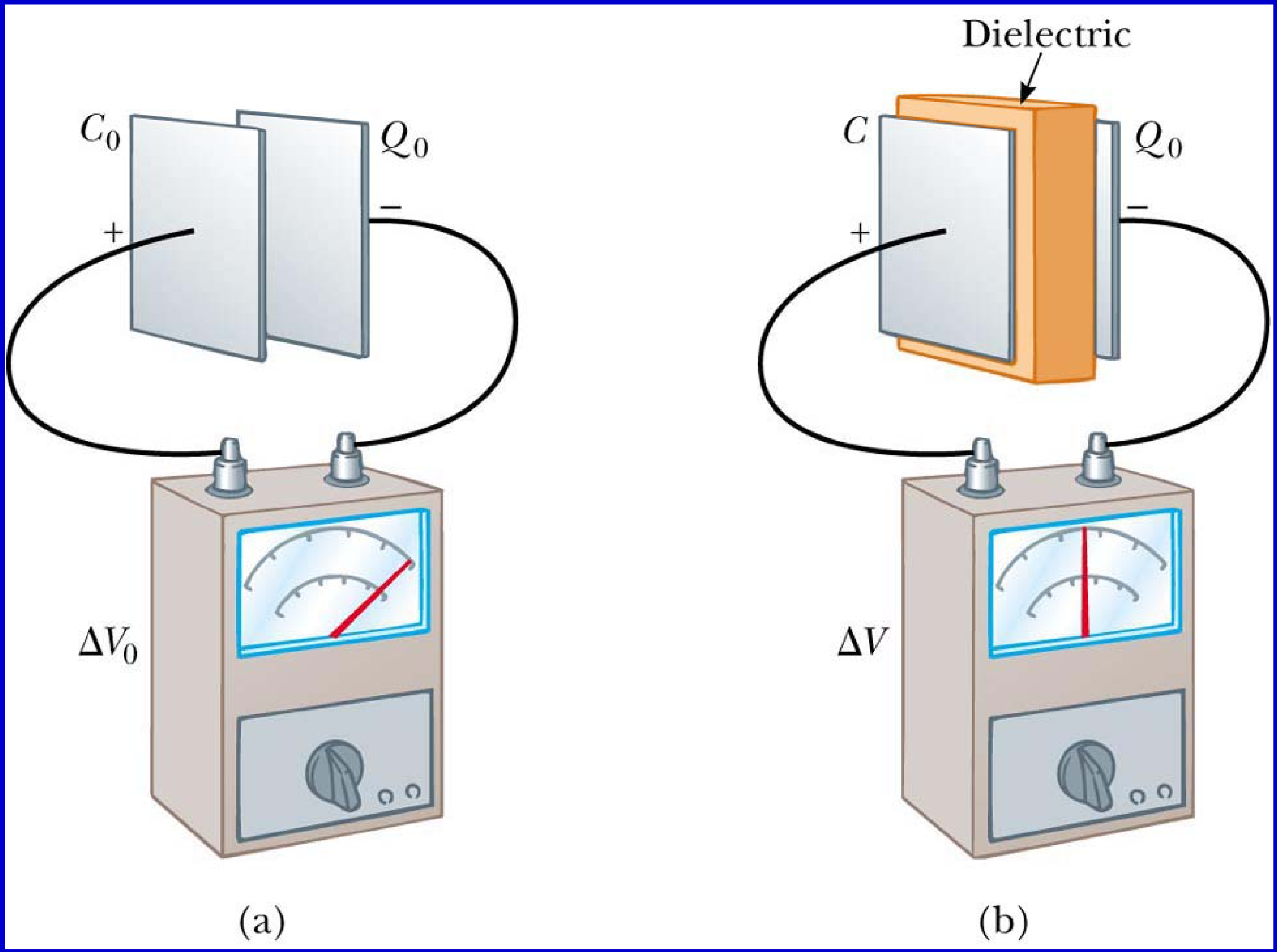
A 2.70-μF capacitor is charged by a 12.0-V battery. It is disconnected from the battery and then connected to an uncharged 4.00-μF capacitor . Determine the total stored energy (a) before the two capacitors are connected, and (b) after they are connected. (c) What is the change in energy



Capacitors with Dielectrics

- A *dielectric* is an insulating material that, when placed between the plates of a capacitor, increases the capacitance
  - Dielectrics include rubber, plastic, or waxed paper
- $C = \kappa C_o = \kappa \varepsilon_o (A/d)$
  - The capacitance is multiplied by the factor κ when the dielectric completely fills the region between the plates

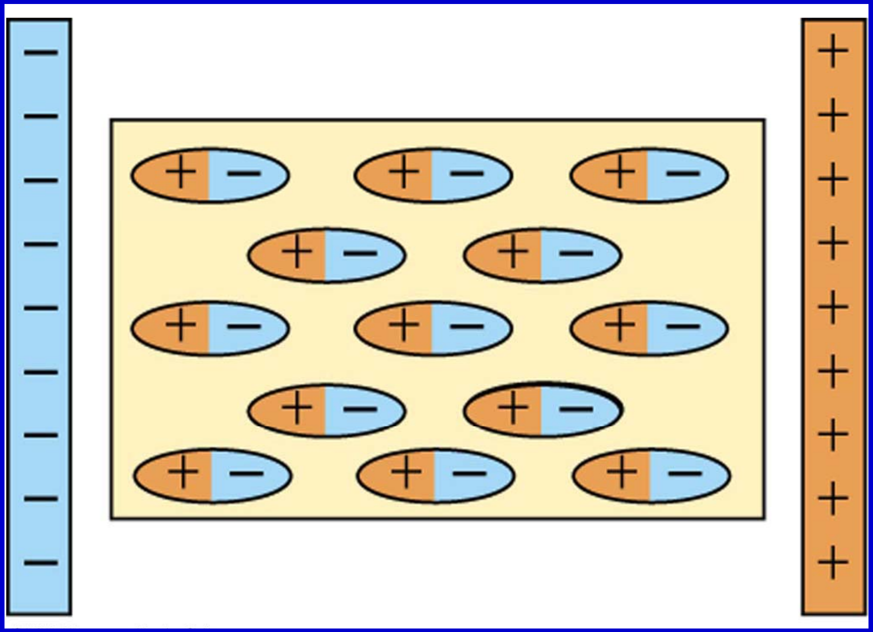
Capacitors with Dielectrics



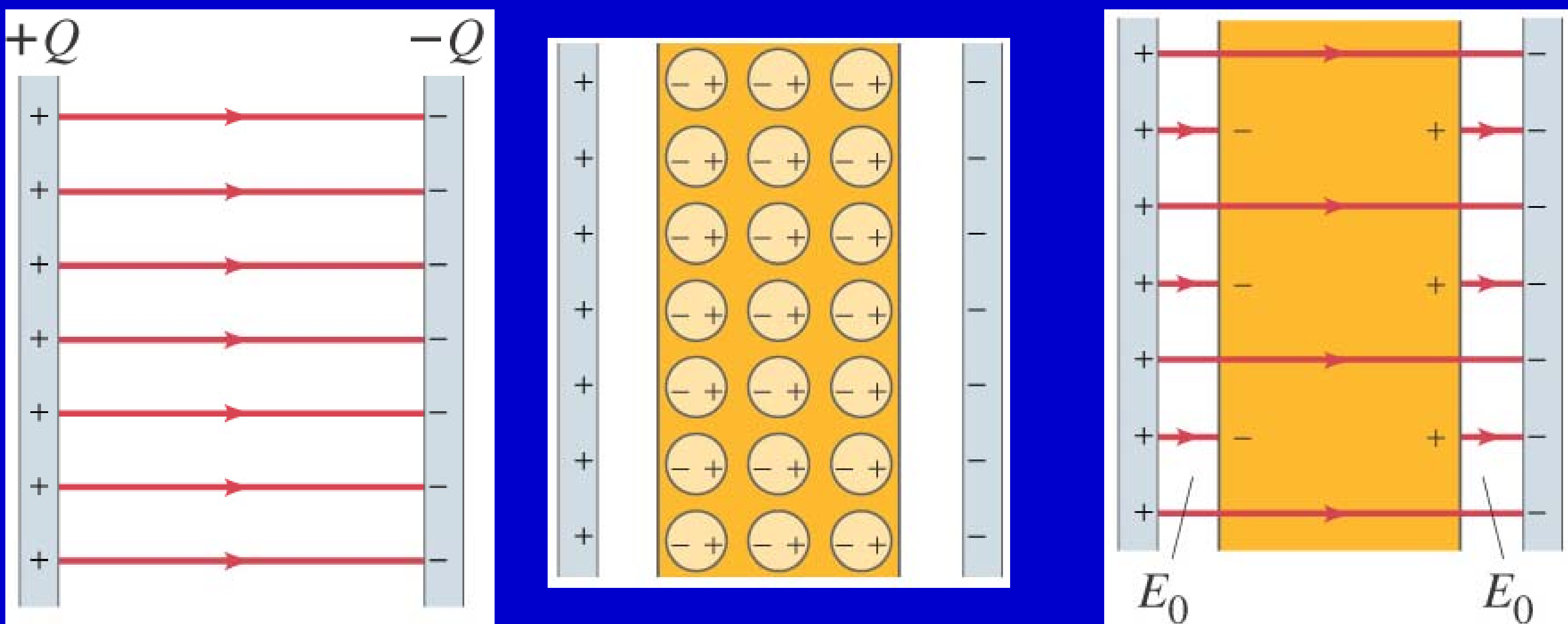
An Atomic Description of Dielectrics

- Polarization occurs when there is a separation between the “centers of gravity” of its negative charge and its positive charge
- In a capacitor, the dielectric becomes polarized because it is in an electric field that exists between the plates

- The presence of the positive charge on the dielectric effectively reduces some of the negative charge on the metal
- This allows more negative charge on the plates for a given applied voltage
- The capacitance increases



The molecules in a dielectric tend to become oriented in a way that reduces the external field.



Dielectric Strength

- For any given plate separation, there is a maximum electric field that can be produced in the dielectric before it breaks down and begins to conduct
- This maximum electric field is called the *dielectric strength*

TABLE 17–3 Dielectric constants (at 20°C)

Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Vinyl (plastic)	2–4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Rubber, neoprene	6.7	12×10^6
Porcelain	6–8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

Dielectric strength is the maximum field a dielectric can experience without breaking down.

QUICK QUIZ 6

A fully charged parallel-plate capacitor remains connected to a battery while you slide a dielectric between the plates. Do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV; (e) energy stored in the capacitor.

QUICK QUIZ 6 ANSWER

(a) C increases

(b) Q increases

(c) E stays the same

(d) DV remains the same

(e) The energy stored increases