PHYS 1444 – Section 002
Lecture #5

Dr. Koymen

- Gauss' law with many charges
- What is Gauss' law good for?
- Electric Potential Energy
- Electric Potential

Gauss' Law w/ more than one charge

- Let's consider several charges inside a closed surface.
- For each charge, Q_i, inside the chosen closed surface,

$$\oint \vec{E}_i \cdot d\vec{A} = \frac{Q_i}{\varepsilon_0}$$

What is $\vec{E}_i$?

The electric field produced by Q_i alone!

- Since electric fields can be added vectorially, following the superposition principle, the total field E is equal to the sum of the fields due to each charge $\vec{E} = \sum \vec{E}_i$ and any external field. So

$$\oint \vec{E} \cdot d\vec{A} = \oint \left(\vec{E}_{ext} + \sum \vec{E}_i \right) \cdot d\vec{A} = \frac{\sum Q_i}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0}$$

What is Q_{encl}?

The total enclosed charge!

- The value of the flux depends on the charge enclosed in the surface!! ➔ Gauss' law.

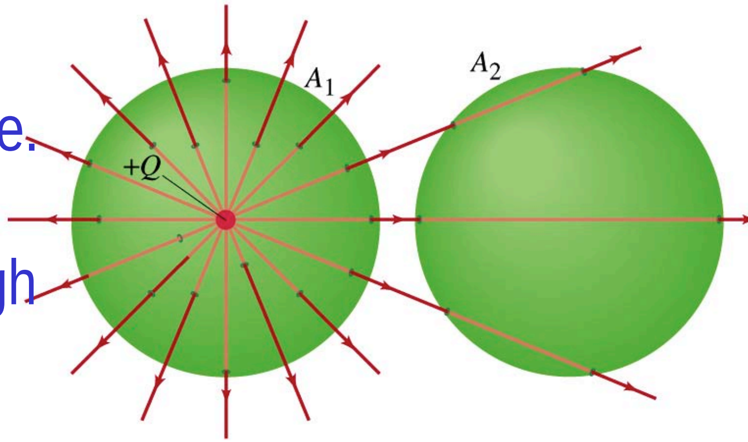
So what is Gauss' Law good for?

- Derivation of Gauss' law from Coulomb's law is only valid for static electric charge.
- Electric field can also be produced by changing magnetic fields.
  - Coulomb's law cannot describe this field while Gauss' law is still valid
- Gauss' law is more general than Coulomb's law.
  - Can be used to obtain electric field, forces or obtain charges

Gauss' Law: Any **differences** between the input and output flux of the electric field over any enclosed surface is due to the charge within that surface!!!

Example 22 – 2

Flux from Gauss' Law: Consider the two gaussian surfaces, A_1 and A_2, shown in the figure. The only charge present is the charge +Q at the center of surface A_1. What is the net flux through each surface A_1 and A_2?



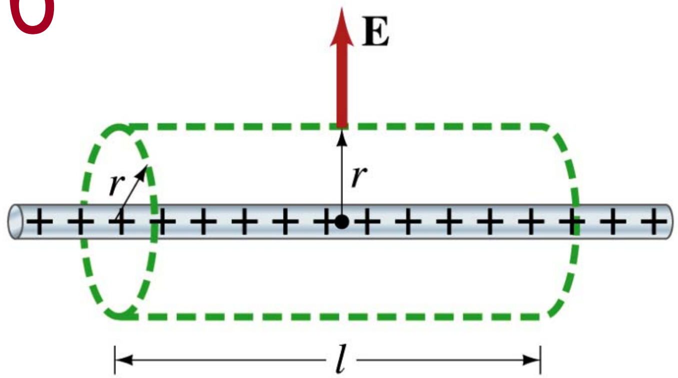
- The surface A_1 encloses the charge +Q, so from Gauss' law we obtain the total net flux

$$\oint \vec{E} \cdot d\vec{A} = \frac{+Q}{\varepsilon_0}$$

- The surface A_2 the charge, +Q, is outside the surface, so the total net flux is 0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0} = 0$$

Example 22 – 6



Long uniform line of charge: A very long straight wire possesses a uniform positive charge per unit length, λ. Calculate the electric field at points near but outside the wire, far from the ends.

- Which direction do you think the field due to the charge on the wire is?
  – Radially outward from the wire, the direction of radial vector r.
- Due to cylindrical symmetry, the field is the same on the gaussian surface of a cylinder surrounding the wire.
  – The end surfaces do not contribute to the flux at all. Why?
    - Because the field vector E is perpendicular to the surface vector dA.
- From Gauss' law

$$\oint \vec{E}\cdot d\vec{A} = E\oint dA = E\left(2\pi r l\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

Solving for E

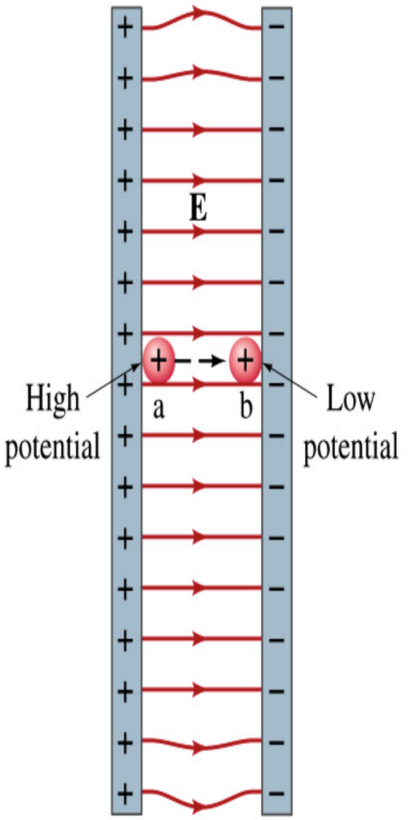
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Ch 23-Electric Potential Energy

- Concept of energy is very useful solving mechanical problems
- Conservation of energy makes solving complex problems easier.
- When can the potential energy be defined?
  - Only for a conservative force.
  - The work done by a conservative force is independent of the path. What does it only depend on??
    - The difference between the initial and final positions
  - Can you give me an example of a conservative force?
    - Gravitational force
- Is the electrostatic force between two charges a conservative force?
  - Yes. Why?
  - The dependence of the force to the distance is identical to that of the gravitational force.
    - The only thing matters is the direct linear distance between the object not the path.

Electric Potential Energy

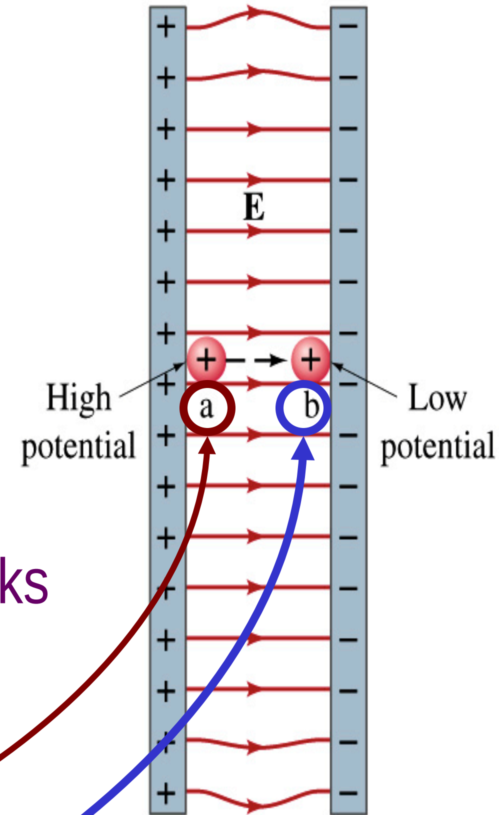
- How would you define the change in electric potential energy $U_b - U_a$?
  - The potential gained by the charge as it moves from point *a* to point *b*.
  - The negative work done on the charge by the electric force to move it from *a* to *b*.



- Let's consider an electric field between two parallel plates w/ equal but opposite charges
  - The field between the plates is uniform since the gap is small and the plates are infinitely long…
- What happens when we place a small charge, +q, on a point at the positive plate and let go?
  - The electric force will accelerate the charge toward negative plate.
  - What kind of energy does the charged particle gain?
    - Kinetic energy

Electric Potential Energy

- What does this mean in terms of energies?
  - The electric force is a conservative force.
  - Thus, the mechanical energy (K+U) is conserved under this force.
  - The charged object has only the electric potential energy at the positive plate.
  - The electric potential energy decreases and
  - Turns into kinetic energy as the electric force works on the charged object and the charged object gains speed.
- Point of greatest potential energy for
  - Positively charged object
  - Negatively charged object



PE=	U	0
KE=	0	K
ME=	U	K

U+K

Electric Potential

- How is the electric field defined?
  - Electric force per unit charge: F/q
- We can define electric potential (potential) as
  - The electric potential energy per unit charge
  - This is like the voltage of a battery…
- Electric potential is written with a symbol V
  - If a positive test charge q has potential energy U_a at a point *a*, the electric potential of the charge at that point is

$$V_a = \frac{U_a}{q}$$

Electric Potential

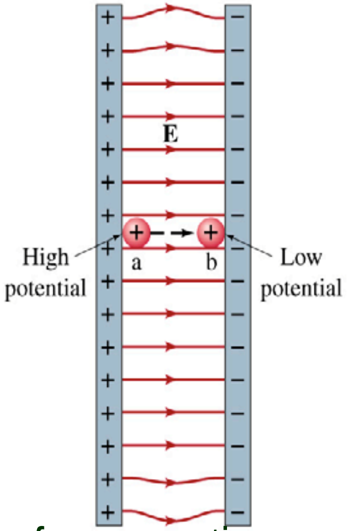
- Since only the difference in potential energy is meaningful, only the potential difference between two points is measurable
- What happens when the electric force does "positive work"?
  - The charge gains kinetic energy
  - Electric potential energy of the charge decreases
- Thus the difference in potential energy is the same as the negative of the work, W_{ba}, done on the charge by the electric field to move the charge from point a to b.
- The potential difference V_{ba} is

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = \frac{-W_{ba}}{q}$$

  - Electric potential is independent of the test charge!!

A Few Things about Electric Potential

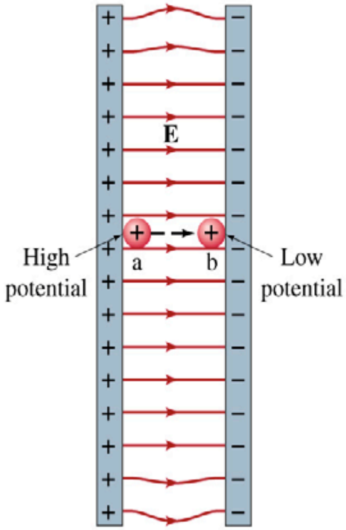
- What does the electric potential depend on?
  - Other charges that creates the field
  - What about the test charge?
    - No, the electric potential is independent of the test charge
    - Test charge gains potential energy by existing in the potential created by other charges
- Which plate is at a higher potential?
  - Positive plate. Why?
    - Since positive charge has the greatest potential energy on it.
  - What happens to the positive charge if it is let go?
    - It moves from higher potential to lower potential
  - How about a negative charge?
    - Its potential energy is higher on the negative plate. Thus, it moves from negative plate to positive. Potential difference is the same.
- The unit of the electric potential is Volt (V).
- From the definition, 1V = 1J/C.



Zero point of electric potential can be chosen arbitrarily.

Often the ground, a conductor connected to Earth is zero.

Example 23 – 1



A negative charge: Suppose a negative charge, such as an electron, is placed at point *b* in the figure. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?

- An electron placed at point *b* will move toward the positive plate since it was released at its highest potential energy point.
- It will gain kinetic energy as it moves toward left, decreasing its potential energy.
- The electron, however, moves from the point *b* at a lower potential to point *a* at a higher potential. $\Delta V=V_a-V_b>0$.
- This is because the potential is generated by the charges on the plates not by the electron.