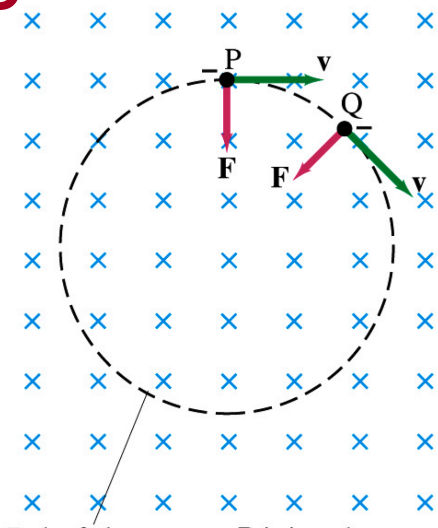
PHYS 1444 – Section 001
Lecture #15

Dr. Koymen

- Charged Particle Path in a Magnetic Field
- Cyclotron Frequency
- Torque on a Current Loop
- Magnetic Dipole Moment
- Magnetic Dipole Potential Energy
- Magnetic Field Due to Straight Wire
- Forces Between Two Parallel Wires

Charged Particle's Path in Magnetic Field



- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?
  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward
  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves on a circular path with a centripetal force F.

Example 27 – 7

Electron's path in a uniform magnetic field. An electron travels at a speed of 2.0×10^{7}m/s in a plane perpendicular to a 0.010-T magnetic field. Describe its path.

What is formula for the centripetal force? $F = ma = m\frac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is $F = evB$

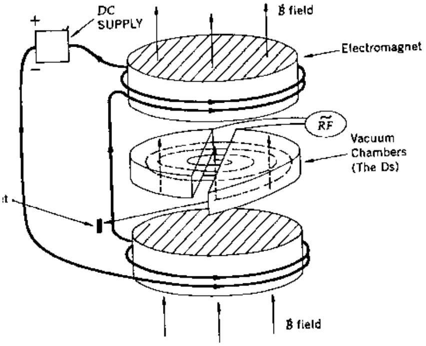
Since the magnetic force provides the centripetal force, we can establish an equation with the two forces $F = evB = m\frac{v^2}{r}$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}\,kg\right)\cdot\left(2.0\times10^{7}\,m/s\right)}{\left(1.6\times10^{-19}\,C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}\,m$$

Cyclotron Frequency

- The time required for a particle of charge q moving w/ constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



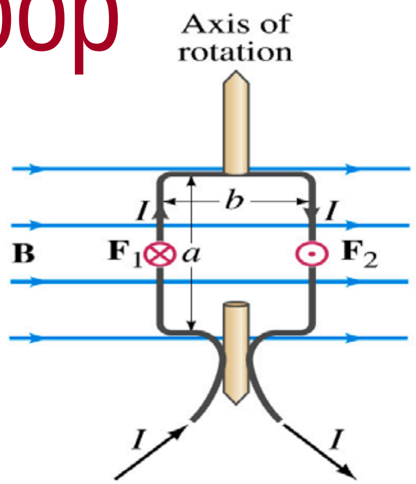
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

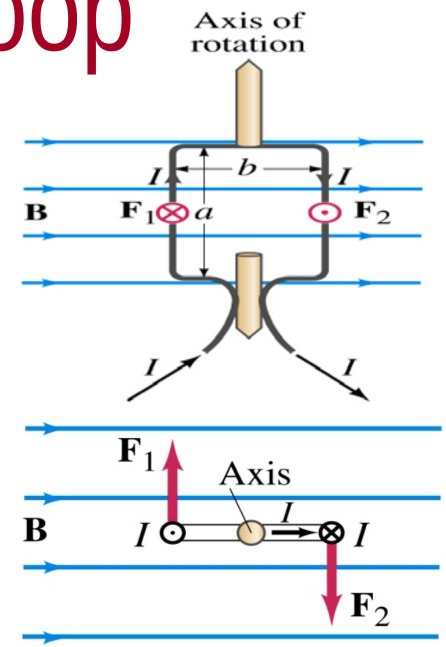
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque to the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not turn unless the direction of the current changes

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length a?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces

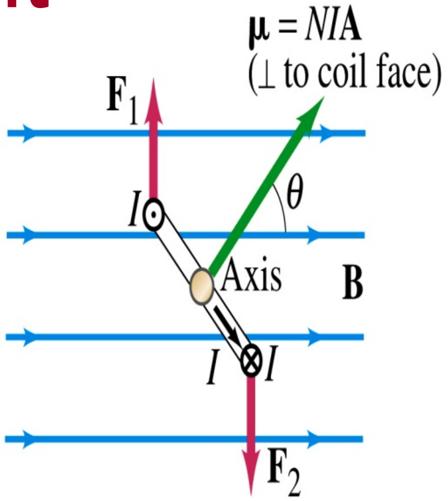
$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

    - Where $A = ab$ is the area of the coil
  - What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

  - If the coil makes an angle θ w/ the field $\tau = NIAB\sin\theta$

Magnetic Dipole Moment



- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity NIA is called the **magnetic dipole moment of the coil**
  - It is considered a vector $\vec{\mu} = NI\vec{A}$
    - Its direction is the same as that of the area vector A and is perpendicular to the plane of the coil consistent with the right-hand rule
      - Your thumb points to the direction of the magnetic moment when your finger cups around the loop in the direction of the wire
  - Using the definition of magnetic moment, the torque can be written in vector form $\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}$

Magnetic Dipole Potential Energy

- Where else did you see the same form of torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at $\theta=\pi/2$, then C=0
  - Thus the potential energy is $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$
    - Very similar to the electric dipole

Example 27 – 12

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10}m.

What provides the centripetal force? Coulomb force

So we can obtain the speed of the electron from

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

Solving for v

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \sqrt{\frac{\left(8.99\times10^{9}\,N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}} = 2.19\times10^{6}\,m/s$$

Since the electric current is the charge that passes through the given point per unit time, we can obtain the current

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

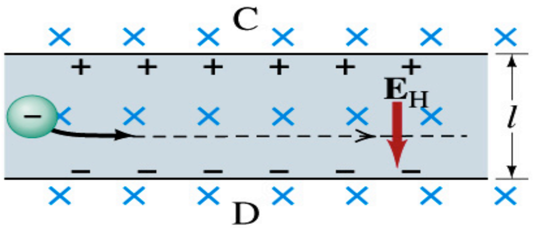
Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} = \frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m}}$$

$=9.27\times10^{-24}$A.m

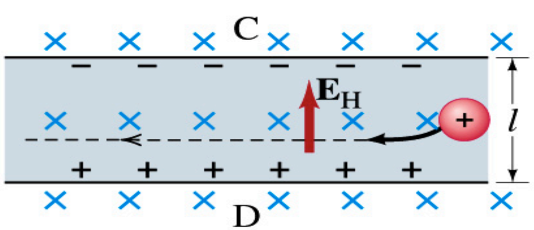
The Hall Effect

- What do you think will happen to the electrons flowing through a conductor immersed in a magnetic field?
  - Magnetic force will push the electrons toward one side of the conductor. Then what happens?
    - $\vec{F}_B = -e\vec{v}_d \times \vec{B}$
  - A potential difference will be created due to continued accumulation of electrons on one side. Till when? Forever?
  - Nope. Till the electric force inside the conductor is equal and opposite to the magnetic force



(a)

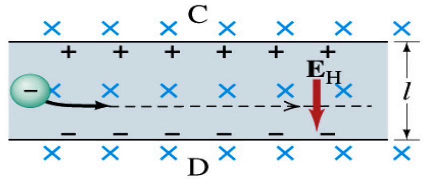
- This is called the **Hall Effect**
  - The potential difference produced is called
    - The Hall emf
  - The electric field due to the separation of charge is called the Hall field, E_H, and it points to the direction opposite to the magnetic force



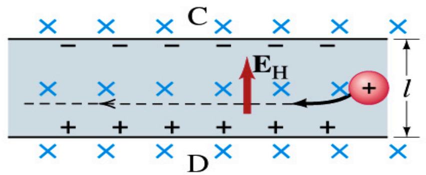
(b)

The Hall Effect

- In equilibrium, the force due to Hall field is balanced by the magnetic force ev_dB, so we obtain



(a)



(b)

- $eE_H = ev_d B$ and $E_H = v_d B$
- The Hall emf is then $\varepsilon_H = E_H l = v_d Bl$
  - Where l is the width of the conductor
- What do we use the Hall effect for?
  - The current of negative charge moving to right is equivalent to the positive charge moving to the left
  - The Hall effect can distinguish these since the direction of the Hall field or direction of the Hall emf is opposite
  - Since the magnitude of the Hall emf is proportional to the magnetic field strength ➔ can measure the B-field strength
    - Hall probe