PHYS 1444 – Section 001
Lecture #23

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- Gauss' Law of Magnetism
- Maxwell's Equations
- Production of Electromagnetic Waves
- EM Waves from Maxwell's Equations
- Speed of EM Waves
- Energy in EM Waves
- Energy Transport

Displacement Current

- Maxwell interpreted the second term in the generalized Ampere's law equivalent to an electric current
  - He called this term as the displacement current, I_D
  - While the other term is called as the conduction current, I
- Ampere's law then can be written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + I_D \right)$$

  - Where

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

  - While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself

Gauss' Law for Magnetism

- If there is symmetry between electricity and magnetism, there must be an equivalent law in magnetism as the Gauss' Law in electricity
- For a magnetic field B, the magnetic flux Φ_B through the surface is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

  - Where the integration is over the area of either an open or a closed surface
- The magnetic flux through a closed surface which completely encloses a volume is

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- What was the Gauss' law in the electric case?
  - The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ε_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

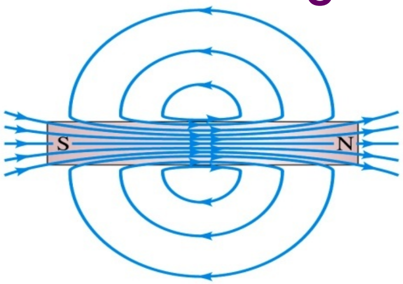
- Why is result of the integral zero?
  - There is no isolated magnetic poles, the magnetic equivalent of single electric charges

Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\oint \vec{B} \cdot d\vec{A} = 0$$

  - There are as many magnetic flux lines that enter the enclosed volume as leave it
  - If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
    - Electricity do have the source and the sink
  - Magnetic field lines must be continuous
  - Even for bar magnets, the field lines exist both insides and outside of the magnet



Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law, relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampére's Law

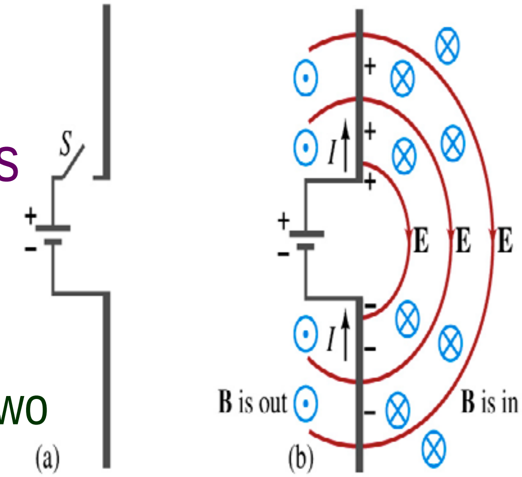
A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces the magnetic field that also changes
    - This changing magnetic field then in turn produces the electric field that changes
    - This process continues
  - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space

Production of EM Waves

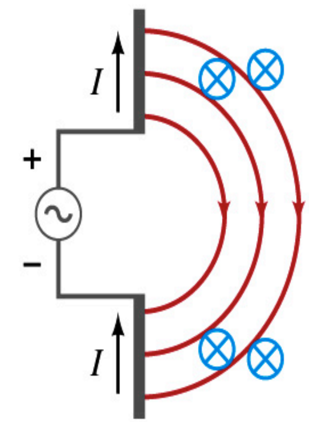
- Consider two conducting rods that will serve as an antenna are connected to a DC power source
  - What do you think will happen when the switch is closed?
    - The rod connected to the positive terminal is charged positive and the other negative
    - Then the electric field will be generated between the two rods
    - Since there is current that flows through the rods, a magnetic field around them will be generated

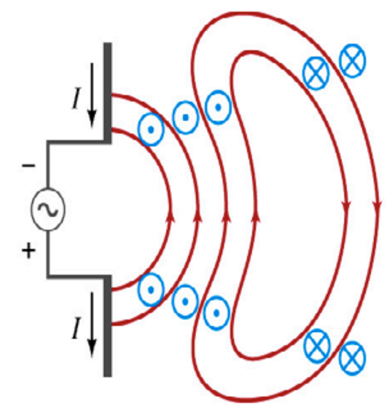


- How far would the electric and magnetic fields extend?
  - In static case, the field extends indefinitely
  - When the switch is closed, the fields are formed near the rods quickly but
  - The stored energy in the fields won't propagate w/ infinite speed

Production of EM Waves

- What happens if the antenna is connected to an ac power source?
  - When the connection was initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
    - The field lines form as in the dc case
    - The field lines propagate away from the antenna
  - Then the direction of the voltage reverses
    - New field lines in the opposite direction forms
    - While the original field lines still propagates away from the rod reaching out far
      - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
    - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge
  - The fields far from the antenna is called the <u>radiation field</u>
  - Both electric and magnetic fields form closed loops perpendicular to each other





Properties of Radiation Fields

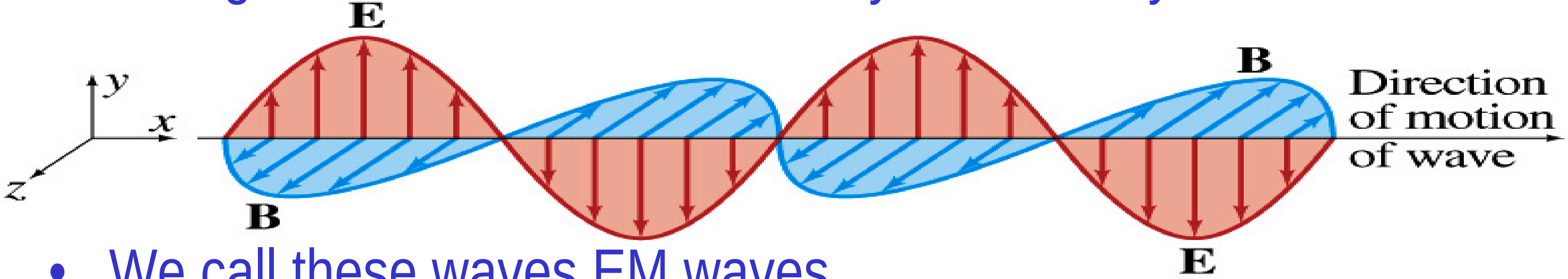
- The fields travel on the other side of the antenna as well
- The field strength are the greatest in the direction perpendicular to the oscillating charge while along the parallel direction is 0
- The magnitude of **E** and **B** in the radiation field decrease with distance as 1/r
- The energy carried by the EM wave is proportional to the square of the amplitude, E^2 or B^2
  - So the intensity of wave decreases as $1/r^2$

Properties of Radiation Fields

- The electric and magnetic fields at any point are perpendicular to each other and to the direction of motion
- The fields alternate in direction
  - The field strengths vary from maximum in one direction, to 0 and to maximum in the opposite direction
- The electric and magnetic fields are in phase
- Very far from the antenna, the field lines are pretty flat over a reasonably large area
  - Called plane waves

EM Waves

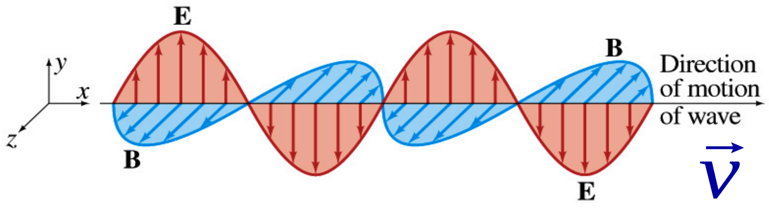
- If the voltage of the source varies sinusoidally, the field strengths of the radiation field vary sinusoidally



- We call these waves EM waves
- They are transverse waves
- EM waves are always waves of fields
  - Since these are fields, they can propagate through an empty space
- In general accelerating electric charges give rise to electromagnetic waves
- This prediction from Maxwell's equations was experimentally proven by Heinrich Hertz through the discovery of radio waves

EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
  - An area of space where there is no charges or conduction currents
  - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
  - What are these flat waves called?
    - Plane waves
    - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
  - So we can also assume that the wave is traveling in the x-direction w/ velocity, **v**=v**i**, and that **E** is parallel to y axis and **B** is parallel to z axis



Maxwell's Equations w/ Q=I=0

- In this region of free space, Q=0 and I=0, thus the four Maxwell's equations become

$$\oint \vec{E}\cdot d\vec{A}=\frac{Q_{encl}}{\varepsilon_0}$$ → $Q_{encl}=0$ → $$\oint \vec{E}\cdot d\vec{A}=0$$

$$\oint \vec{B}\cdot d\vec{A}=0$$ → No Changes → $$\oint \vec{B}\cdot d\vec{A}=0$$

$$\oint \vec{E}\cdot d\vec{l}=-\frac{d\Phi_B}{dt}$$ → No Changes → $$\oint \vec{E}\cdot d\vec{l}=-\frac{d\Phi_B}{dt}$$

$$\oint \vec{B}\cdot d\vec{l}=\mu_0 I_{encl}+\mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$ → $I_{encl}=0$ → $$\oint \vec{B}\cdot d\vec{l}=\mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!

EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength λ and frequency *f*, such traveling wave can be written as

$$E = E_y = E_0 \sin(kx - \omega t)$$

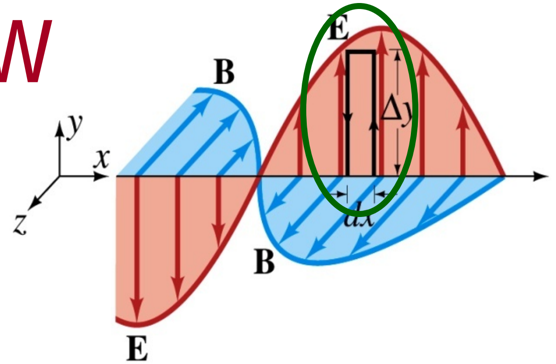
$$B = B_z = B_0 \sin(kx - \omega t)$$

  – Where

$$k = \frac{2\pi}{\lambda} \qquad \varpi = 2\pi f \quad \text{Thus} \Rightarrow \quad f\lambda = \frac{\omega}{k} = v$$

  – What is v?
    - It is the speed of the traveling wave
  – What are E_0 and B_0?
    - The amplitudes of the EM wave. Maximum values of E and B field strengths.

From Faraday's Law



- Let's apply Faraday's law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

  – to the rectangular loop of height Δy and width dx

- $\vec{E}\cdot d\vec{l}$ along the top and bottom of the loop is 0. Why?
  – Since E is perpendicular to dℓ.
  – So the result of the integral through the loop counterclockwise becomes

$$\oint \vec{E}\cdot d\vec{l} = \vec{E}\cdot d\vec{x} + \left(\vec{E}+d\vec{E}\right)\cdot\Delta\vec{y} + \vec{E}\cdot d\vec{x}' + \vec{E}\cdot\Delta\vec{y}' =$$

$$= 0 + \left(E+dE\right)\Delta y - 0 - E\Delta y = dE\Delta y$$

  – For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$-\frac{d\Phi_B}{dt} = \frac{dB}{dt}dx\Delta y$$

Thus

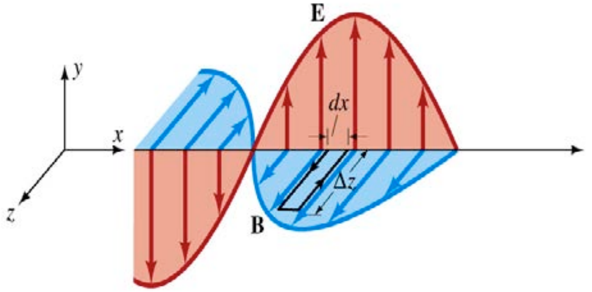
$$dE\Delta y = -\frac{dB}{dt}dx\Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Since E and B depend on x and t

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

From Modified Ampére's Law



- Let's apply Maxwell's 4th equation

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

  – to the rectangular loop of length Δz and width dx

- $\vec{B}\cdot d\vec{l}$ along the x-axis of the loop is 0
  – Since B is perpendicular to d*l*.
  – So the result of the integral through the loop counterclockwise becomes

$$\oint \vec{B}\cdot d\vec{l} = B\Delta Z - \left(B + dB\right)\Delta Z = -dB\Delta Z$$

  – For the right-hand side of the equation is

$$\mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 \frac{dE}{dt} dx\Delta z$$

Thus

$$-dB\Delta z = \mu_0\varepsilon_0 \frac{dE}{dt} dx\Delta z$$

  – $$\frac{dB}{dx} = -\mu_0\varepsilon_0 \frac{dE}{dt}$$

Since E and B depend on x and t

$$\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0 \frac{\partial E}{\partial t}$$

Relationship between E, B and v

- Let's now use the relationship from Faraday's law $\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial E}{\partial x}=\frac{\partial}{\partial x}\left(E_0\sin\left(kx-\omega t\right)\right)=kE_0\cos\left(kx-\omega t\right)$$

$$\frac{\partial B}{\partial t}=\frac{\partial}{\partial t}\left(B_0\sin\left(kx-\omega t\right)\right)=-\omega B_0\cos\left(kx-\omega t\right)$$

Since $\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$ We obtain $kE_0\cos\left(kx-\omega t\right)=\omega B_0\cos\left(kx-\omega t\right)$

Thus $\frac{E_0}{B_0}=\frac{\omega}{k}=v$

- Since E and B are in phase, we can write $E/B=v$
  - This is valid at any point and time in space. What is v?
    - The velocity of the wave

Speed of EM Waves

- Let's now use the relationship from Ampere's law $\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial B}{\partial x} = \frac{\partial}{\partial x}\left(B_0 \sin\left(kx - \omega t\right)\right) = kB_0 \cos\left(kx - \omega t\right)$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t}\left(E_0 \sin\left(kx - \omega t\right)\right) = -\omega E_0 \cos\left(kx - \omega t\right)$$

Since $\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$ We obtain $kB_0 \cos\left(kx - \omega t\right) = \varepsilon_0 \mu_0 \omega E_0 \cos\left(kx - \omega t\right)$

Thus $\frac{B_0}{E_0} = \frac{\varepsilon_0 \mu_0 \omega}{k} = \varepsilon_0 \mu_0 v$

- However, from the previous page we obtain $E_0 / B_0 = v = \frac{1}{\varepsilon_0 \mu_0 v}$
- Thus $v^2 = \frac{1}{\varepsilon_0 \mu_0}$

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{\left(8.85 \times 10^{-12}\ C^2/N \cdot m^2\right) \cdot \left(4\pi \times 10^{-7}\ T \cdot m/A\right)}} = 3.00 \times 10^8\ m/s$$

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.

Speed of Light w/o Sinusoidal Wave Forms

- Taking the time derivative on the relationship from Ampere's laws, we obtain $\frac{\partial^2 B}{\partial x \partial t} = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$
- By the same token, we take position derivative on the relationship from Faraday's law $\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$
- From these, we obtain

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2} \quad \text{and} \quad \frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2}$$

- Since the equation for traveling wave is $\frac{\partial^2 x}{\partial t^2} = v^2 \frac{\partial^2 x}{\partial x^2}$
- By correspondence, we obtain $v^2 = \frac{1}{\varepsilon_0 \mu_0}$
- A natural outcome of Maxwell's equations is that E and B obey the wave equation for waves traveling w/ speed $v = 1/\sqrt{\varepsilon_0 \mu_0}$
  - Maxwell predicted the existence of EM waves based on this

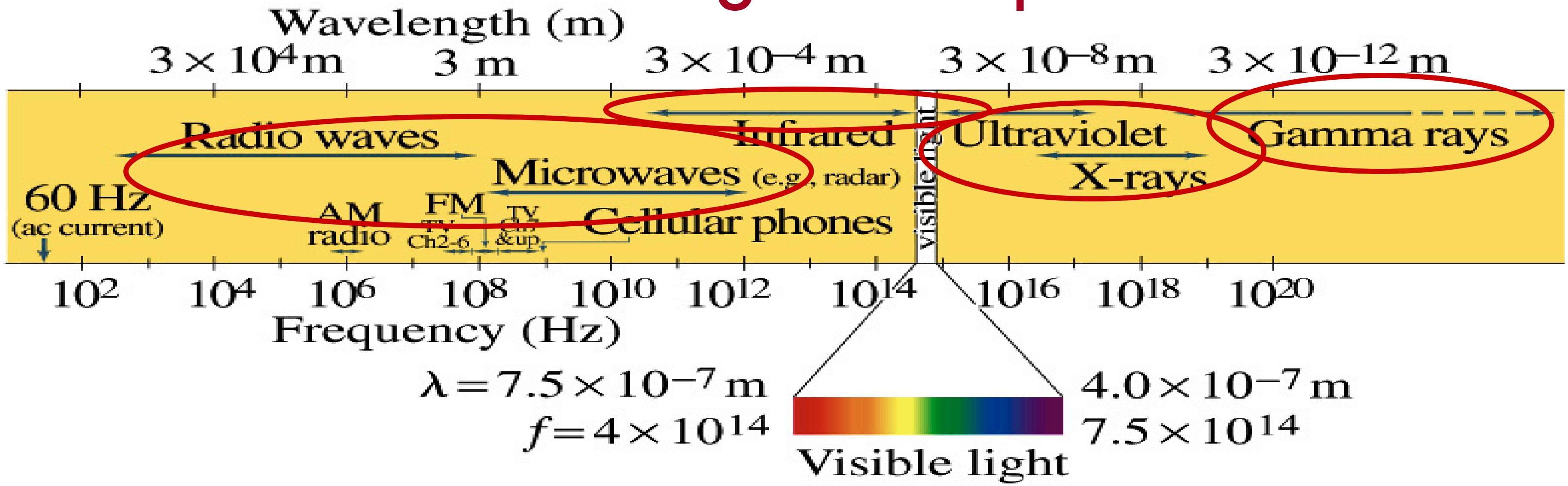
Light as EM Wave

- People knew some 60 years before Maxwell that light behaves like a wave, but …
  - They did not know what kind of waves they are.
    - Most importantly what is it that oscillates in light?
- Heinrich Hertz first generated and detected EM waves experimentally in 1887 using a spark gap apparatus
  - Charge was rushed back and forth in a short period of time, generating waves with frequency about 10^9Hz (these are called radio waves)
  - He detected using a loop of wire in which an emf was produced when a changing magnetic field passed through
  - These waves were later shown to travel at the speed of light

Light as EM Wave

- The wavelengths of visible light were measured in the first decade of the 19th century
  - The visible light wave length were found to be between 4.0×10^{-7}m (400nm) and 7.5×10^{-7}m (750nm)
  - The frequency of visible light is $f\lambda = c$
    - Where f and λ are the frequency and the wavelength of the wave
      - What is the range of visible light frequency?
      - 4.0×10^{14}Hz to 7.5×10^{14}Hz
    - c is 3×10^{8}m/s, the speed of light
- EM Waves, or EM radiation, are categorized using EM spectrum

Electromagnetic Spectrum



- Low frequency waves, such as radio waves or microwaves can be easily produced using electronic devices
- Higher frequency waves are produced natural processes, such as emission from atoms, molecules or nuclei
- Or they can be produced from acceleration of charged particles
- Infrared radiation (IR) is mainly responsible for the heating effect of the Sun
  - The Sun emits visible lights, IR and UV
    - The molecules of our skin resonate at infrared frequencies so IR is preferentially absorbed and thus warm up

Example 31 – 3

Wavelength of EM waves. Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave and (c) of a beam of visible red light from a laser at frequency $4.74\text{x}10^{14}$Hz.

What is the relationship between speed of light, frequency and the wavelength? $c = f\lambda$

Thus, we obtain $\lambda = \frac{c}{f}$

For f=60Hz
$$\lambda = \frac{3\times10^8\ m/s}{60s^{-1}} = 5\times10^6\,m$$

For f=93.3MHz
$$\lambda = \frac{3\times10^8\ m/s}{93.3\times10^6\,s^{-1}} = 3.22m$$

For f=$4.74\text{x}10^{14}$Hz
$$\lambda = \frac{3\times10^8\ m/s}{4.74\times10^{14}\,s^{-1}} = 6.33\times10^{-7}\,m$$

EM Wave in the Transmission Lines

- Can EM waves travel through a wire?
  - Can it not just travel through the empty space?
  - Nope. It sure can travel through a wire.
- When a source of emf is connected to a transmission line, the electric field within the wire does not set up immediately at all points along the line
  - When two wires are separated via air, the EM wave travel through the air at the speed of light, c.
  - However, through medium w/ permittivity e and permeability m, the speed of the EM wave is given $v = 1/\sqrt{\varepsilon\mu} < c$
    - Is this faster than c? Nope! It is slower.

Energy in EM Waves

- Since B=E/c and $c = 1/\sqrt{\varepsilon_0 \mu_0}$, we can rewrite the energy density

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{\varepsilon_0 \mu_0 E^2}{\mu_0} = \varepsilon_0 E^2$$

$$u = \varepsilon_0 E^2$$

  - Note that the energy density associate with B field is the same as that associate with E
  - So each field contribute half to the total energy
- By rewriting in B field only, we obtain

$$u = \frac{1}{2}\varepsilon_0 \frac{B^2}{\varepsilon_0 \mu_0} + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{B^2}{\mu_0}$$

$$u = \frac{B^2}{\mu_0}$$

- We can also rewrite to contain both E and B

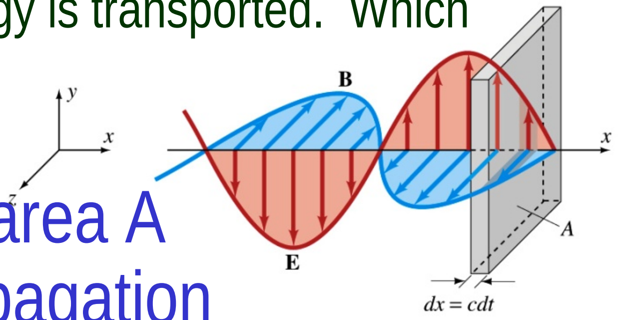
$$u = \varepsilon_0 E^2 = \varepsilon_0 EcB = \frac{\varepsilon_0 EB}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} EB$$

$$u = \sqrt{\frac{\varepsilon_0}{\mu_0}} EB$$

-

Energy Transport

- What is the energy the wave transport per unit time per unit area?
  - This is given by the vector **S**, the Poynting vector
    - The unit of S is W/m^2.
    - The direction of **S** is the direction in which the energy is transported. Which direction is this?
      - The direction the wave is moving



- Let's consider a wave passing through an area A perpendicular to the x-axis, the axis of propagation
  - How much does the wave move in time dt?
    - dx=cdt
  - The energy that passes through A in time dt is the energy that occupies the volume dV, $dV = Adx = Acdt$
  - Since the energy density is $u=\varepsilon_0 E^2$, the total energy, dU, contained in the volume V is $dU = udV = \varepsilon_0 E^2 Acdt$

Energy Transport

- Thus, the energy crossing the area A per time dt is

$$S = \frac{1}{A}\frac{dU}{dt} = \varepsilon_0 c E^2$$

- Since E=cB and $c = 1/\sqrt{\varepsilon_0 \mu_0}$, we can also rewrite

$$S = \varepsilon_0 c E^2 = \frac{cB^2}{\mu_0} = \frac{EB}{\mu_0}$$

- Since the direction of S is along v, perpendicular to E and B, the Poynting vector S can be written

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$$

  – This gives the energy transported per unit area per unit time at any instant

Average Energy Transport

- The average energy transport in an extended period of time since the frequency is so high we do not detect the rapid variation with respect to time.
- If E and B are sinusoidal, $\overline{E^2} = E_0^2 / 2$
- Thus we can write the magnitude of the average Poynting vector as

$$\overline{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0}$$

  - This time averaged value of S is the intensity, defined as the average power transferred across unit area. E_0 and B_0 are maximum values.
- We can also write

$$\overline{S} = \frac{E_{rms} B_{rms}}{\mu_0}$$

  - Where Erms and Brms are the rms values ($E_{rms} = \sqrt{\overline{E^2}}$, $B_{rms} = \sqrt{\overline{B^2}}$)

Example 31 – 6

E and B from the Sun. Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about 1350W/m^2. Assume that this is a single EM wave and calculate the maximum values of E and B.

What is given in the problem? The average S!!

$$\bar{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0}$$

For E_0, $E_0 = \sqrt{\frac{2\bar{S}}{\varepsilon_0 c}} = \sqrt{\frac{2 \cdot 1350 W/m^2}{\left(8.85\times10^{-12} C^2/N\cdot m^2\right)\cdot\left(3.00\times10^8 m/s\right)}} = 1.01\times10^3 V/m$

For B_0 $\quad B_0 = \frac{E_0}{c} = \frac{1.01\times10^3 V/m}{3\times10^8 m/s} = 3.37\times10^{-6} T$