PHYS 1444 – Section 001
Lecture #21

Dr. Koymen

- AC Circuit w/ Resistance only
- AC Circuit w/ Inductance only
- AC Circuit w/ Capacitance only
- AC Circuit w/ LRC

Why do we care about circuits on AC?

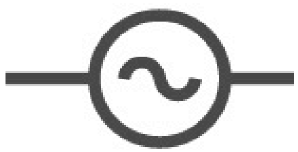
- The circuits we've learned so far contain resistors, capacitors and inductors and have been connected to a DC source or a fully charged capacitor
  - What? This does not make sense.
  - The inductor does not work as an impedance unless the current is changing. So an inductor in a circuit with DC source does not make sense.
  - Well, actually it does. When does it impede the current flow?
    - Immediately after the circuit is connected to the source so the current is still changing.
    - So what?
      - It causes the change of magnetic flux.
  - Now does it make sense?
- Anyhow, learning the responses of resistors, capacitors and inductors in a circuit connected to an AC emf source is important. Why is this?
  - Since most the generators produce sinusoidal current
  - Any voltage that varies over time can be expressed in the superposition of sine and cosine functions

AC Circuits – the preamble

- Do you remember how the rms and peak values for current and voltage are related?

$$V_{rms} = \frac{V_0}{\sqrt{2}} \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

- The symbol for an AC power source is

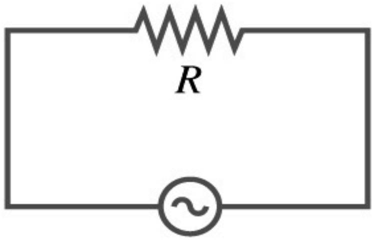


- We assume that the voltage gives rise to current

$$I = I_0 \sin 2\pi ft = I_0 \sin \varpi t$$

  – where $\varpi = 2\pi f$

AC Circuit w/ Resistance only



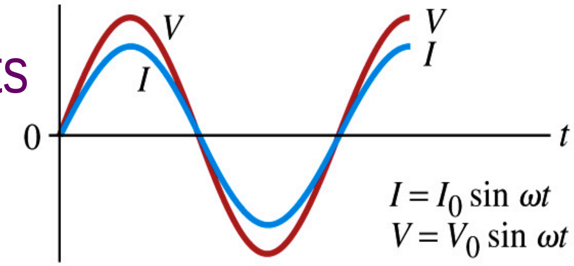
- What do you think will happen when an ac source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain

$$V - IR = 0$$

- Thus

$$V = I_0 R \sin \varpi t = V_0 \sin \varpi t$$

  – where $V_0 = I_0 R$
- What does this mean?
  – Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
  – Current and voltage are "in phase"



- Energy is lost via the transformation into heat at an average rate $\overline{P} = \overline{I}\,\overline{V} = I_{rms}^2 R = V_{rms}^2 / R$
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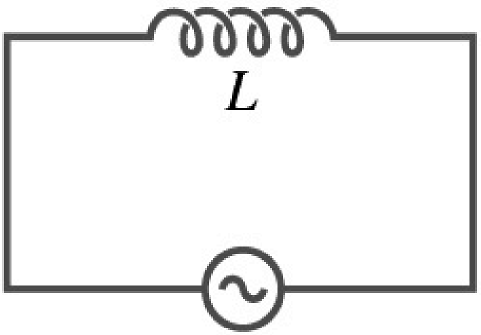
AC Circuit w/ Inductance only

- From Kirchhoff's loop rule, we obtain

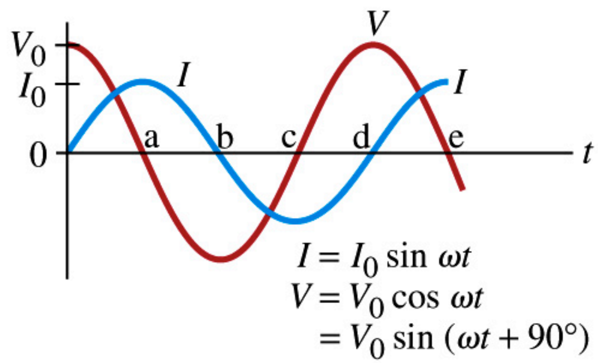
$$V - L\frac{dI}{dt} = 0$$



- Thus

$$V = L\frac{dI}{dt} = L\frac{d\left(I_0 \sin \varpi t\right)}{dt} = \varpi L I_0 \cos \varpi t$$

  - Using the identity $\cos\theta = \sin\left(\theta + 90^\circ\right)$
- $$V = \varpi L I_0 \sin\left(\varpi t + 90^\circ\right) = V_0 \sin\left(\varpi t + 90^\circ\right)$$
  - where $V_0 = \varpi L I_0$
- What does this mean?
  - Current and voltage are "out of phase by $\pi/2$ or 90°".
  - In other words the current reaches its peak ¼ cycle after the voltage
- What happens to the energy?
  - No energy is dissipated
  - The average power is 0 at all times
  - The energy is stored temporarily in the magnetic field
  - Then released back to the source



AC Circuit w/ Inductance only

- How are the resistor and inductor different in terms of energy?
  - Inductor — Stores the energy temporarily in the magnetic field and then releases it back to the emf source
  - Resistor — Does not store energy but transforms it to thermal energy, getting it lost to the environment
- How are they the same?
  - They both impede the flow of charge
  - For a resistance R, the peak voltage and current are related to $V_0 = I_0 R$
  - Similarly, for an inductor we can write $V_0 = I_0 X_L$
    - Where X_L is the inductive reactance of the inductor $X_L = \varpi L$ 0 when ω=0.
    - What do you think is the unit of the reactance? Ω
    - The relationship $V_0 = I_0 X_L$ is not valid at a particular instance. Why not?
      - Since V_0 and I_0 do not occur at the same time

$V_{rms} = I_{rms} X_L$ is valid!

Example 30 – 9

Reactance of a coil. A coil has a resistance R=1.00Ω and an inductance of 0.300H. Determine the current in the coil if (a) 120 V dc is applied to it; (b) 120 V ac (rms) at 60.0Hz is applied.

Is there a reactance for dc? Nope. Why not? Since ω=0, $X_L = \varpi L = 0$

So for dc power, the current is from Kirchhoff's rule $V - IR = 0$

$$I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120A$$

For an ac power with f=60Hz, the reactance is

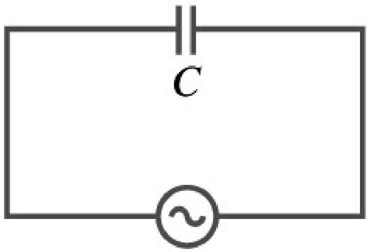
$$X_L = \varpi L = 2\pi fL = 2\pi \cdot \left(60.0s^{-1}\right) \cdot 0.300H = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06A$$

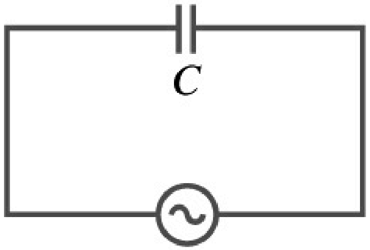
AC Circuit w/ Capacitance only

- What happens when a capacitor is connected to a dc power source?
  - The capacitor quickly charges up.
  - There is no steady current flow in the circuit
    - Since a capacitor prevents the flow of a dc current



- What do you think will happen if it is connected to an ac power source?
  - The current flows continuously. Why?
  - When the ac power turns on, charge begins to flow one direction, charging up the plates
  - When the direction of the power reverses, the charge flows in the opposite direction

AC Circuit w/ Capacitance only



- From Kirchhoff's loop rule, we obtain

$$V = \frac{Q}{C}$$

- Current at any instance is $I = \frac{dQ}{dt} = I_0 \sin \varpi t$

- Thus, the charge Q on the plate at any instance is

$$Q = \int_{Q=0}^{Q} dQ = \int_{t=0}^{t} I_0 \sin \varpi t dt = -\frac{I_0}{\varpi} \cos \varpi t$$

- The voltage across the capacitor is

$$V = \frac{Q}{C} = -I_0 \frac{1}{\varpi C} \cos \varpi t$$

  – Using the identity $\cos\theta = -\sin\left(\theta - 90^\circ\right)$

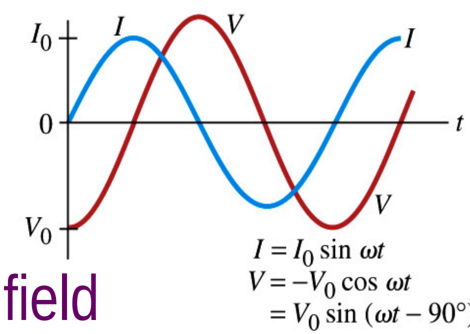
$$V = I_0 \frac{1}{\varpi C} \sin\left(\varpi t - 90^\circ\right) = V_0 \sin\left(\varpi t - 90^\circ\right)$$

  – Where

  – $V_0 = \frac{I_0}{\varpi C}$

AC Circuit w/ Capacitance only

- So the voltage is $V = V_0 \sin\left(\varpi t - 90^\circ\right)$
- What does this mean?
  - Current and voltage are "out of phase by π/2 or 90º" but in this case, the voltage reaches its peak ¼ cycle after the current
- What happens to the energy?
  - No energy is dissipated
  - The average power is 0 at all times
  - The energy is stored temporarily in the electric field
  - Then released back to the source

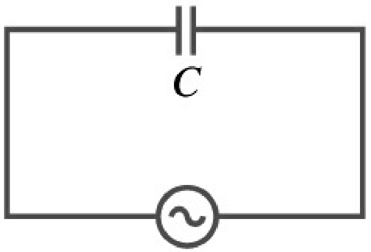


- Relationship between the peak voltage and the peak current in the capacitor can be written as $V_0 = I_0 X_C$
  - Where the capacitance reactance X_C is defined as $X_C = \frac{1}{\varpi C}$
  - Again, this relationship is only valid for rms quantities $V_{rms} = I_{rms} X_C$

Infinite when ω=0.

Example 30 – 10

Capacitor reactance. What are the peak and rms current in the circuit in the figure if C=1.0μF and V_{rms}=120V? Calculate for (a) f=60Hz, and then for (b) f=6.0x10^{5}Hz.



The peak voltage is $V_0 = \sqrt{2}V_{rms} = 120V \cdot \sqrt{2} = 170V$

The capacitance reactance is

$$X_C = \frac{1}{\varpi C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \cdot \left(60s^{-1}\right) \cdot 1.0 \times 10^{-6} F} = 2.7k\Omega$$

Thus the peak current is

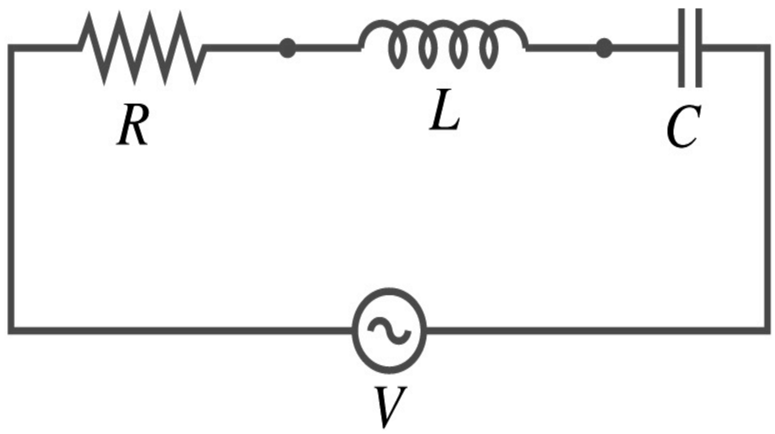
$$I_0 = \frac{V_0}{X_C} = \frac{170V}{2.7k\Omega} = 63mA$$

The rms current is

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{120V}{2.7k\Omega} = 44mA$$

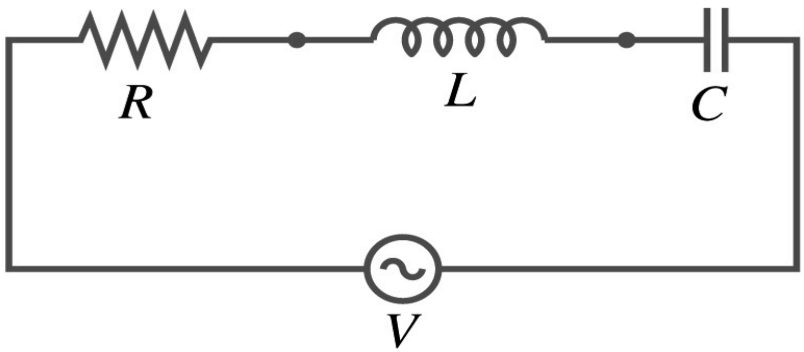
AC Circuit w/ LRC

- The voltage across each element is
  - V_R is in phase with the current
  - V_L leads the current by 90°
  - V_C lags the current by 90°
- From Kirchhoff's loop rule
- $V=V_R+V_L+V_C$
  - However since they do not reach the peak voltage at the same time, the peak voltage of the source V_0 will not equal $V_{R0}+V_{L0}+V_{C0}$
  - The rms voltage also will not be the simple sum of the three
- Let's try to find the total impedance, peak current I_0 and the phase difference between I_0 and V_0.



AC Circuit w/ LRC

- The current at any instance is the same at all point in the circuit
  - The currents in each elements are in phase
  - Why?
    - Since the elements are in series
  - How about the voltage?
    - They are not in phase.



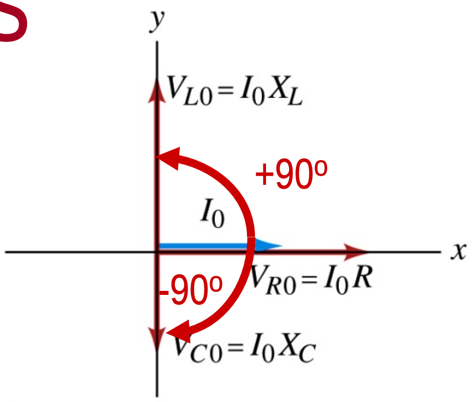
- The current at any given time is

$$I = I_0 \sin \varpi t$$

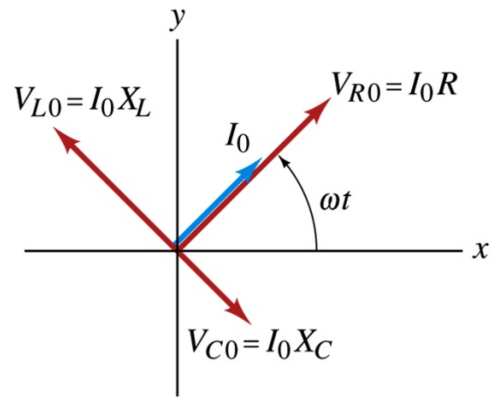
- The analysis of LRC circuit is done using the "phasor" diagram in which arrows are drawn in an xy plane to represent the amplitude of each voltage, just like vectors
  - The lengths of the arrows represent the magnitudes of the peak voltages across each element; $V_{R0}=I_0R$, $V_{L0}=I_0X_L$ and $V_{C0}=I_0X_C$
  - The angle of each arrow represents the phase of each voltage relative to the current, and the arrows rotate at the angular frequency ω to take into account the time dependence.
    - The projection of each arrow on y axis represents voltage across each element at any given time

Phasor Diagrams

- At t=0, I=0.
  - Thus $V_{R0}=0$, $V_{L0}=I_0X_L$, $V_{C0}=I_0X_C$
- At t=t, $I = I_0 \sin \varpi t$



(a)



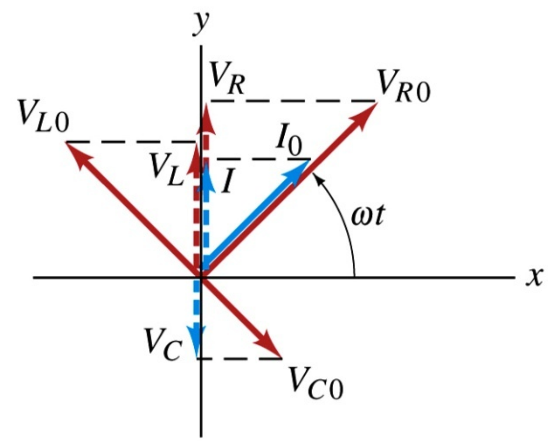
(b)

- Thus, the voltages (y-projections) are

$$V_R = V_{R0} \sin \varpi t$$

$$V_L = V_{L0} \sin\left(\varpi t + 90^\circ\right)$$

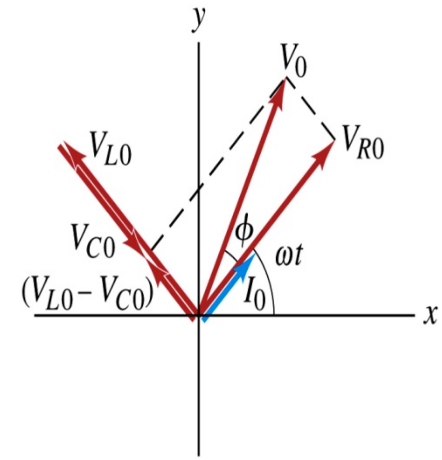
$$V_C = V_{C0} \sin\left(\varpi t - 90^\circ\right)$$



(c)

AC Circuit w/ LRC

- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum,
  - The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage
  - So we can use the sum of all vectors as the representation of the peak source voltage V_0.



- V_0 forms an angle ϕ to V_{R0} and rotates together with the other vectors as a function of time, $V = V_0 \sin\left(\varpi t + \phi\right)$
- We determine the total impedance Z of the circuit defined by the relationship $V_{rms} = I_{rms} Z$ or $V_0 = I_0 Z$
- From Pythagorean theorem, we obtain

$$V_0 = \sqrt{V_{R0}^2 + \left(V_{L0} - V_{C0}\right)^2} = \sqrt{I_0^2 R^2 + I_0^2\left(X_L - X_C\right)^2} = I_0\sqrt{R^2 + \left(X_L - X_C\right)^2} = I_0 Z$$

- Thus the total impedance is $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(\varpi L - \frac{1}{\varpi C}\right)^2}$

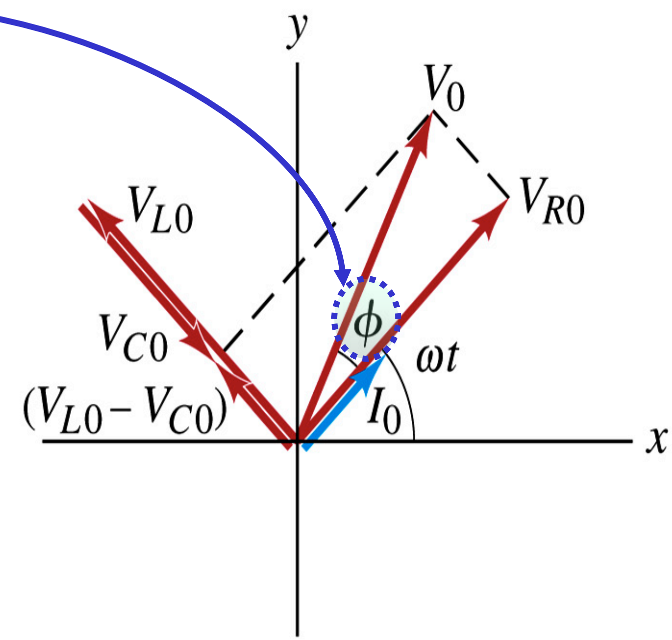
AC Circuit w/ LRC

- The phase angle ϕ is

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0\left(X_L - X_C\right)}{I_0 R} = \frac{\left(X_L - X_C\right)}{R}$$

- or

$$\cos\phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$



- What is the power dissipated in the circuit?
  - Which element dissipates the power?
  - Only the resistor
- The average power is $\overline{P} = I_{rms}^2 R$
  - Since $R = Z\cos\phi$
  - We obtain

$$\overline{P} = I_{rms}^2 Z\cos\phi = I_{rms} V_{rms}\cos\phi$$

  - The factor $\cos\phi$ is referred as the power factor of the circuit
  - For a pure resistor, $\cos\phi = 1$ and $\overline{P} = I_{rms} V_{rms}$
  - For a capacitor or inductor alone $\phi = -90^o$ or $+90^o$, so $\cos\phi = 0$ and $\overline{P} = 0.$