Microtorus: a high-finesse microcavity with whispering-gallery modes

Vladimir S. Ilchenko
Jet Propulsion Laboratory, California Institute of Technology, MS 298-100, 4800 Oak Grove Drive, Pasadena, California 91109

Michael L. Gorodetsky
Department of Physics, Moscow State University, Moscow 119899, Russia

X. Steve Yao and Lute Maleki
Jet Propulsion Laboratory, California Institute of Technology, MS 298-100, 4800 Oak Grove Drive, Pasadena, California 91109

Received September 7, 2000

We demonstrate a 165-μm oblate spheroidal microcavity with a free spectral range of 383.7 GHz (3.06 nm), a resonance bandwidth of 23 MHz (quality factor \(Q\approx10^7\)) at 1550 nm, and finesse \(F\approx10^4\). The highly oblate spheroidal dielectric microcavity combines a very high \(Q\) factor, typical of microspheres, with a vastly reduced number of excited whispering-gallery modes (by 2 orders of magnitude). The very large free spectral range in this novel microcavity—a few hundred gigahertz instead of a few gigahertz as in typical microspheres—is desirable for applications in spectral analysis, narrow-linewidth optical and rf oscillators, and cavity QED.

\(\text{OCIS codes: 060.2340, 140.4780, 120.2440, 230.3990.}\)

Microspherical resonators supporting optical whispering-gallery (WG) modes have attracted considerable attention in various fields of research and technology. The combination of a very high \(Q\) factor (\(10^8\)–\(10^9\) or greater) and submillimeter dimensions (typical diameters ranging from a few tens of to several hundred micrometers) makes these resonators attractive new components for a number of applications, including basic physics research, molecular spectroscopy, narrow-linewidth lasers, optoelectronic oscillators, and sensors.\(^1\)\(^-\)\(^5\) Effective methods of coupling light in and out of WG modes in microspheres, including single-mode fiber couplers and integrated waveguides, are currently being developed.\(^6\)\(^,\)\(^7\)

WG modes are essentially closed circular waves trapped by total internal reflection inside an axially symmetric dielectric body. The very high \(Q\) of microspheres results from ultralow optical loss in the material (typically, fiber-grade fused silica), a fire-polished surface with subnanometer-scale inhomogeneities, high-index contrast for steep reduction of radiative and scattering losses with increasing radius, and two-dimensional curvature providing for grazing reflection of all wave-vector components. Grazing incidence is essential for minimizing surface scattering that would otherwise limit \(Q\) to far less than the value imposed by attenuation in the material.\(^8\) For example, in integrated optical microring and microdisk cavities\(^8\) based on planar waveguide technology (the light in planar devices is effectively bouncing from flat surfaces at a finite angle), the typical \(Q\) factor is only \(10^4\) to \(10^5\). The substantially higher \(Q\) in the spheres than in microdisks and microrings comes at the price of a relatively dense spectrum of modes. In ideal spheres the spectrum consists of \(\text{TE}(\text{TM})_{\ell m q}\) modes separated by a large free spectral range (FSR) defined by the circumference of the sphere and related to consecutive values of index \(\ell\). In silica spheres of diameter 150–400 μm the large FSR should be in the range 437–165 GHz or, on the wavelength scale, 3.5 to 1.3 nm near the center wavelength of 1550 nm. Each \(\text{TE}(\text{TM})_{\ell m q}\) mode is \((2\ell + 1)\)-fold degenerate with respect to the index \(m\). Residual nonsphericity removes this degeneracy and leads to a series of observable \(\text{TE}(\text{TM})_{\ell m q}\) modes separated by a small FSR in the range 6.8–2.5 GHz, for the same sphere dimensions, center wavelength, and eccentricity \(\epsilon^2 = 3 \times 10^{-2}\) that is typical of current fabrication methods.

A relatively dense spectrum complicates important applications of microsphere resonators, such as spectral analysis and laser stabilization, and necessitates using intermediate filtering. In this Letter we demonstrate a microcavity with a novel geometry that retains two-dimensional curvature confinement, low scattering loss, and the very high \(Q\) that is typical of microspheres and yet approaches the spectral properties of the single-mode waveguide ring, i.e., a highly oblate spheroidal microcavity, or microtorus.

Calculation of the spectrum of the dielectric spheroid (i.e., the ellipsoid of revolution) is not a trivial task, even numerically. In contrast to the case of small eccentricity (see, for example, Ref. 10), analysis in highly eccentric spheroids cannot be based on simple approximations. Unlike in the case of cylindrical or spherical coordinates, orthogonal spheroidal vector harmonics that satisfy boundary conditions may not be derived from a single scalar wave function.\(^11\) Furthermore, the calculation of spheroidal angular and radial functions is also a nontrivial task.

To analyze the properties of eigenfrequencies we shall rely on physical considerations; however, the same approximations can be derived from the asymptotic forms of the spheroidal functions. A very good approximation of WG-mode eigenfrequencies in a dielectric sphere with radius \(a\) much larger than the wavelength can be found as follows:

\[ \lambda = 2a \left( \frac{\pi}{\lambda} \right)^2 \]

\(\lambda\) is the free spectral range in the WG spectrum, \(a\) is the radius of the sphere, and \(\lambda\) is the free spectral range in the WG spectrum.
where \( t_{iq} \) is the \( q \)th zero of the spherical Bessel function of order \( l \) and \( \chi = n \) for the TE mode and \( \chi = 1/n \) for the TM mode. For large \( l \), \( t_{iq} \sim l + O(l^{1/3}) \), and it may either be calculated directly or approximated by the zeros of the Airy function.\(^{12}\) The meaning of the small second term on the right-hand side of Eq. (1) can be understood if we remember that a WG mode is quasi-classically a closed circular beam supported by total internal reflection, the optical field tunnels outside at the depth \( 1/k\sqrt{n^2 - 1} \), and the tangential component of \( E \) (TE mode), or normal of \( D \) (TM mode), is continuous at the boundary of the dielectric. Eigenfrequencies of high-order WG modes \((l \gg 1; l = m)\) in dielectric spheres can be approximated from solutions of the scalar wave equation with zero boundary conditions, because most of the energy is concentrated in one component of the field \((E_\theta \text{ for the TE mode and } E_r \text{ for the TM mode})\).

Based on the considerations above, let us estimate the WG-mode eigenfrequencies in oblate spheroids of large semiaxis \( a \), small semiaxis \( b \), and eccentricity \( \varepsilon = \sqrt{(1 - b^2)/a^2} \). Since WG modes are localized in the equatorial plane, we shall approximate the radial distribution by the cylindrical Bessel function \( J_m(nk_{mq}r) \), with \( nk_{mq}a = na\sqrt{k^2_{mq} - k_{l1}^2} = T_{mq} \), where \( J_m(T_{mq}) = 0 \) and \( k_{l1} \) is the wave number of the quasi-classical solution for angular spheroidal functions. For our purposes a rough approximation of the wave number is enough:

\[
k_{l1}^2 \approx \frac{2(l - m) + 1}{a^2 \sqrt{1 - \varepsilon^2}} m;
\]

more-rigorous considerations can follow the approach given in Ref. 13. Taking into account that \( T_{mq} \approx t_{iq} - (l - m + 1/2) \), we finally obtain the following approximation:

\[
nk_{mq}a - \frac{\chi}{\sqrt{n^2 - 1}} = na\sqrt{k^2_{mq} + k_{l1}^2} \approx T_{mq} + \frac{k^2 a^2}{2T_{mq}} \Rightarrow nk_{mq}a = t_{iq} + \frac{2(l - m) + 1}{2} \left( \frac{1}{\sqrt{1 - \varepsilon^2}} - 1 \right). \tag{2}
\]

For very small \( \varepsilon \), Eq. (2) yields the same value for a small FSR (frequency splitting between modes with successive \( m = l \)) as obtained with the more-rigorous perturbation methods used in the study reported in Ref. 10. In addition, we compared the prediction of approximation (2), \( l = 100 \), with the numerically calculated zeros of the radial spheroidal functions. Even with \( \varepsilon = 0.8 \), the error in the estimate of \( m, m + 1 \) mode splitting is no more than 5\%, and that in the absolute mode frequencies is no more than 0.1\%. For larger \( l \) and smaller \( \varepsilon \), the error will evidently be smaller. As follows from approximation (2), with increasing eccentricity a small FSR (frequency interval between modes with successive \( m \)) becomes compatible with the large FSR: \( (k_{l1mq} - k_{l1mq}) \sim (k_{lm+1q} - k_{l1mq}) \sim k_{l1mq}/l \). In addition, excitation conditions for modes with different \( m \) become more selective; e.g., optimal angles for prism coupling vary with \( \varepsilon \) as \( k_\perp/k_{l1mq} \propto (1 - \varepsilon^2)^{-1/4} \). In our experiment we prepared a spheroidal (microtorus) cavity by compressing a small sphere of low-melting silica glass between cleaved fiber tips. The combined action of surface tension and axial compression resulted in the desired geometry (see a typical microcavity in Fig. 1). One of the fiber stems was then cut, and the whole structure was installed next to a standard prism coupler. The WG-mode spectrum was observed by use of a distributed-feedback laser near the wavelength of 1550 nm. The laser was continuously frequency scannable within a range of \( \sim 80 \text{ GHz} \) (by modulation of the current) and temperature tunable from 1545.1 to 1552.4 nm. A high-resolution spectrum over 900 GHz was compiled from 15 individual scans in 60-GHz increments that we obtained by changing the laser temperature. We obtained the frequency reference for stitching the spectral fragments by recording the fringes of a high-finesse \((F \approx 120)\) Fabry–Perot etalon (FSR, 30 GHz) and additional frequency marks provided by 3.75-GHz amplitude modulation. The total drift of the etalon was less than 400 MHz over the full 15-min measurement time. The compiled spectrum is presented in Fig. 2. The spectrum is reduced to only two WG modes of selected polarization that are excited within the large FSR of the cavity, 383 GHz, or 3.06 nm in the wavelength domain. The transmission of parasitic modes is at least 6 \text{ dB} smaller than that of the principal ones. With an individual mode bandwidth of 23 MHz, finesse \( F = 1.7 \times 10^4 \) is therefore demonstrated with this microresonator. We can compare this result with the

![Fig. 1. Photograph and schematic of the microcavity geometry.](Image)
predictions of approximation (2). For the dimensions of our cavity, \( a = 82.5 \, \mu m, \quad b = 42.5 \, \mu m, \quad c = 0.86 \), and a refractive index of the material of \( n = 1.453 \), the principal mode number of the TE modes at the wavelength of 1550 nm should be \( l \approx 473 \), and the large FSR \( \nu_{lmq} - \nu_{l-1,mq} = \frac{c}{2\pi na}(t_{ll} - r_{l-1}) = \frac{c}{2\pi na}[1 + 0.617(1/3) + O(l^{-5/3})] \approx 402 \, GHz \). The small FSR should be

\[
\nu_{lmq} - \nu_{l,m-1,q} = \frac{c}{2\pi na} \left( \frac{1}{\sqrt{1 - e^2}} - 1 \right) = 383 \, GHz.
\]

In the experimental spectrum the frequency separation between the largest peaks, 1 and 2, in Fig. 2 is equal to 383.7 ± 0.5 GHz. This separation can therefore be attributed as corresponding to the small FSR, in good agreement with the estimate above, if we take into account an approximately 2% uncertainty in the measurement (limited mainly by the precision of the geometrical evaluation of the cavity dimensions). The separation between peaks 3 and 4 in Fig. 2 is equal to 400.3 ± 0.5 GHz and is in turn close to the estimate of the large FSR.

It may be argued that, despite the large splitting between the modes with adjacent values of index \( m \) (of the order of the large FSR), one may still expect a dense spectrum resulting from the overlap of many mode families with a different principal number \( l \). In practice, however, it is exactly the coincidence of the frequency domain of the WG modes with different main indices \( l \), and the rapidly increasing difference \( l - m \), that should be responsible for effective dephasing of the idle modes from the evanescent coupler, resulting in the reduction of modes in the observed spectrum. In addition to increasing phase mismatch for excitation of WG modes with complex transverse structure \( (l - m > 1) \), reduced amplitude of these modes may also be the result of their lower intrinsic quality factor. (With a smaller unloaded \( Q \), critical coupling of these modes would require closer positioning of the cavity to the excitation prism of the waveguide.\(^{15} \)) A complete interpretation of the obtained spectra goes beyond the format of this Letter. Such an interpretation will be based on a wider-range spectral mapping, study of microcavities of different eccentricities, calculations of field distribution, detailed analysis of phase-matched excitation in the evanescent coupler, and measurements of unloaded quality factors for different WG-mode families.

In conclusion, experimental results indicate a substantial reduction (up to 2 orders of magnitude) in the number of excited WG modes in highly oblate spheroids compared with those of typical microspheres. This reduction in mode density is obtained without sacrificing the high \( Q \) associated with these structures. This novel type of optical microcavity demonstrates a true finesse of the order of \( 10^4 \), a free spectral range of the order of few nanometers, and a quality factor \( Q \sim 1 \times 10^7 \). Based on these results, we conclude that a complete elimination of transverse WG modes may be expected in spheroidal cavities of higher eccentricity. Further increases of \( Q \) and of the finesse may also be expected with refinement of the fabrication technique. The decrease in the density of the mode spectrum of ultrahigh-\( Q \) microcavities makes possible new applications in laser stabilization, microlasers, various passive photonics devices, and fundamental physics experiments.

The original concept and the experimental research described in this Letter were conceived and performed at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with NASA. V. S. Ilchenko's e-mail address is ilchenko@jpl.nasa.gov.

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