Diode Models

Solution by
Professor R. L. Carter
EE 2303/601 - Electronics I
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ronc@uta.edu

Purpose and Theoretical Background

This project is an exercise in the basic Shockley model for the ideal diode, the semiconductor diode and the piecewise linear (PWL) approximation for the diode.

The ideal diode has the properties that

$$i_D = 0,\ v_D < 0, \text{ and}$$
$$v_D = 0,\ i_D > 0,$$

where i_D is the diode current and v_D is the voltage of the anode relative to the cathode. The Shockley diode model is used in SPICE[1,2] to represent the semiconductor diode with the two parameters IS, the saturation current density, and N, the ideality factor. The model equation is

$$i_D = IS*[\exp(v_D/(N*V_t)) - 1],$$

where $Vt \equiv kT/q$ (= 25.8642 mV at 27 C)[3] is the thermal voltage. SPICE also inserts a conducting path with value GMIN between each terminal of the diode and the circuit common. Consequently, the SPICE model has the net effect that

$$i_{D,SPICE} = IS*[\exp(v_D/(N*V_t)) - 1] + v_D*GMIN,$$

when the anode has an applied voltage v_D and the cathode is at the circuit common potential. The PWL model represents a diode with an offset or turn-on voltage V_f and a series resistance, R_f in series with an ideal diode. The model equations are thus

$$i_D = 0,\ v_D < V_f, \text{ and}$$
$$i_D = (v_D - V_f)/ R_f,\ v_D \geq V_f.$$

A special case of the PWL model occurs when a specific diode operating condition, i_{DQ}, v_{DQ} is applied, and in that case, the offset voltage is

$$V_f = v_{DQ} - N* V_t, \text{ and}$$

$$R_f = r_d \equiv N* V_t/ i_{DQ}.$$

Procedure

All numerical values were obtained from PSpice[4] simulations or direct evaluation of the Shockley equation or PWL model using Microsoft Excel[5].

Diode Models

A. Ideal Diode The purpose of this part of the Project is to find values of IS and N in order to best approximate the PWL ideal diode.

A1. PSpice was used to simulate diodes with IS values of 1E-12, 1E-14, and 1E-16, and with N values of 0.2, 1, and 5 for each IS value (a total of 9 model instances). [Note: The D-Break model was used with IS and N defined as specified. No value was specified for any other parameter, so the SPICE simulation used only the Shockley model. According to Carter[3] GMIN was set to 1E-21, so as to give $|v_D*GMIN| \leq$

15E-21 A throughout the simulation range. In order to obtain sufficient numerical precision, the parameter NUMDGT was set to 6 or more.]

(A1a) SPICE was used as described above to solve for, iR, the diode current when the applied voltage is -15.0 volts, and solve for (A1b) the diode voltage, vF, when the forward diode current is 100 mA. (A1c) The numerical results are reported in Table 1. The PSpice schematic is shown in Figure 1. The output including netlist is given in Appendix 1. The simulations for later parts are similar to this simulation, so additional schematics and outputs will not always be cited.

Table 1. Diode model reverse current iR for vD = -15.0V, and forward voltage drop vF for iD = 100mA. Specific IS and N values used in PSpice are as shown.

N=	IS =	1.00E-12	1.00E-14	1.00E-16
0.2	iR= ,(A)	-1.0E-12	-1.0E-14	-1.0E-16
0.2	vF= ,(V)	0.131	0.155	0.179
1.0	iR= ,(A)	-1.0E-12	-1.0E-14	-1.0E-16
1.0	vF= ,(V)	0.655	0.774	0.893
5.0	iR= ,(A)	-1.0E-12	-1.0E-14	-1.0E-16
5.0	vF= ,(V)	3.275	3.871	4.467

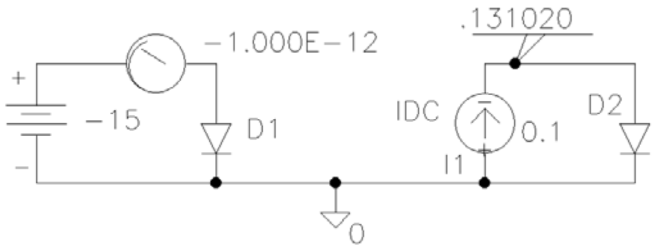

Figure 1. The Pspice schematic for simulating the Shockley diode (D1 and D2) with IS = 1E-12 A and N = 0.20, showing the values iR = -1.000E-12 and vF = 0.131020.

(A1d) Which of these 9 instances best approximates the ideal diode? The best reverse current characteristics, iR are given for the smallest value of IS (i.e., 1E-16 A). The best forward voltage drop characteristics are given by the lowest N value (i.e., N=0.2). Overall, IS=1E-16A and N=0.2 is best.

A2. The purpose of this part is to design a model instance for which an ideal diode has iR < -1 pA (when vD = -20 volts) and vF < 1 nV (when iD = 100 mA). The results are (A2a) IS < 1E-12 A and N < 1.53E-9. (A2b) The schematic and resulting values of iR and vF for this case are shown in Figure 2.

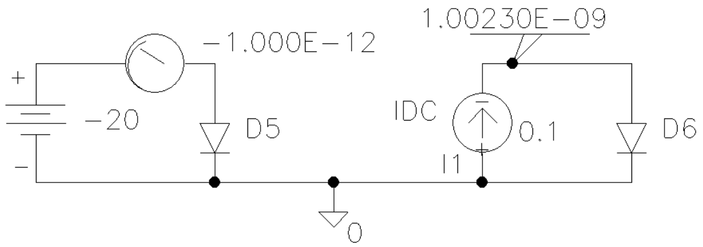

Figure 2. An ideal diode model using PSpice, showing iR = 1 pA for vR = -20 V, and vF = 1.0023 nV for iF = 100 mA. D5 and D6 have IS = 1E-12 A and N = 1.53e-9.

(A2b) The SPICE circuit is shown is Figure 2. The system parameters GMIN = 1E-21 and NUMDGT = 6.

(A2c) The values of iR and vF are identified by the markers in the schematic. The output is similar that reported in Appendix 1.

(A2d) Justify the values of IS and N chosen in (A2a) by using the Shockley equation model to develop a set of equations for deriving values of IS and N which will satisfy the criterion of iR < -1 pA (for a reverse bias of vD = -20 volts) and vF < 1 nV (for a forward current of iD = 100 mA).

The reverse current will be given by IS. The forward voltage drop is obtained from the Shockley equation by solving for N when v_D = 1 nV and i_D = 100 mA in the equation

$$N = v_D/\{V_t * \ln[(i_D/IS) + 1]\}.$$

Thus, for the values cited, N = 1.53E-9

B. Small Signal Model The purpose of this part of the Project is to examine the accuracy of the small signal diode model relative to the Shockley equation. The small-signal piece-wise linear model (ssPWL) can be modeled in PSpice as a ideal diode in series with a turn-on voltage, Vf, and a diode resistance, rd, defined by the operating conditions at the diode bias point iDQ and vDQ, such that

$$Vf = vDQ - N*Vt, \text{ and}$$

$$rd = N*Vt/iDQ.$$

Where Vt = kT/q (= 25.8642 mV at T = 300K, see reference 3). The schematic with the ssPWL model and current flow definitions (see Table 2) are shown in Figure 3. This is the schematic used throughout the rest of this section.

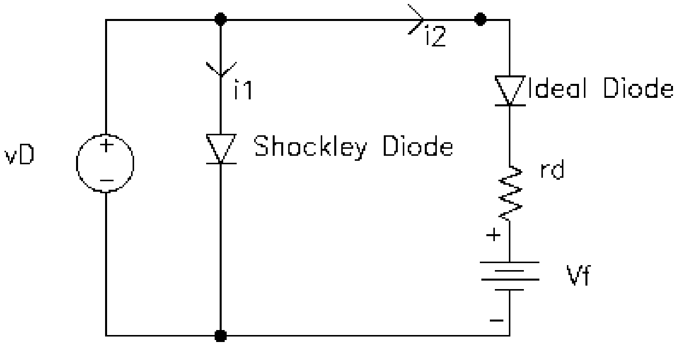

Figure 3. The circuit showing the definition of the diode currents and the effective small-signal piece-wise linear diode model.

(B1) IS = 3.0E-14A and N = 1.05, a ssPWL model (for v_{DQ} = 0.60 volts) was developed and compared to the Shockley model by generating data shown in Table 2. The values used for the ssPWL are

$$i_{DQ} \equiv IS*[\exp(v_{DQ}/(N*V_t)) - 1] = 1.180784E-4 \text{ A},$$

$$V_f = v_{DQ} - N*V_t = 0.5728426 \text{ V, and}$$

$$R_f = r_d \equiv N*V_t/\ i_{DQ} = 229.995 \text{ Ohms}.$$

The parameters used for the "ideal diode" were IS(ideal) = 3E-14 A, and N(ideal) = 1.341E-12. This can be verified to give a reverse current of 3E-14 A (like D1, which models `i1`) and a forward voltage drop of less than 1 pV in the range of forward currents (≤ 100 mA) used.

The absolute value of the error relative to iDQ used in the last column of Table 2. is defined by

Relative error = |I2 – I1|/iDQ

The figure of merit, M ≡ SUM{|I2 – I1|/iDQ} = 1.85E+03, shown in Table 2 is the sum of the absolute values of the relative errors, and gives a numerical measure of the degree to which the ssPWL model represents the Shockley equation. The current iDQ is the Shockley diode current for vDQ=0.60V.

Table 2. Comparison of the Shockley diode equation (IS = 3.0E–14A, N=1.05) to the small signal piece-wise linear model (vDQ = 0.60 V). The total relative error, M = SUM{|I2 – I1|/iDQ} is a figure of merit of the degree to which the model represents the Shockley equation.

B1	Shockley model	PWL,ss model	relative error
vD(V)	I1(A)	I2(A)	|I2-I1|/iD
-20.00	-3.00E-14	-3.00E-14	0.00E+00
-15.00	-3.00E-14	-3.00E-14	0.00E+00
-10.00	-3.00E-14	-3.00E-14	0.00E+00
-5.00	-3.00E-14	-3.00E-14	0.00E+00
0.00	0.00E+00	-3.00E-14	2.54E-10
0.05	1.59E-13	-3.00E-14	1.60E-09
0.10	1.16E-12	-3.00E-14	1.01E-08
0.15	7.48E-12	-3.00E-14	6.36E-08
0.20	4.73E-11	-3.00E-14	4.01E-07
0.25	2.99E-10	-3.00E-14	2.53E-06
0.30	1.88E-09	-3.00E-14	1.59E-05
0.35	1.19E-08	-3.00E-14	1.00E-04
0.40	7.48E-08	-3.00E-14	6.33E-04
0.45	4.71E-07	-3.00E-14	3.99E-03
0.50	2.97E-06	-3.00E-14	2.52E-02
0.55	1.87E-05	-3.00E-14	1.59E-01
0.60	1.18E-04	1.18E-04	3.68E-11
0.65	7.44E-04	3.35E-04	3.46E+00
0.70	4.69E-03	5.53E-04	3.51E+01
0.75	2.96E-02	7.70E-04	2.44E+02
0.80	1.86E-01	9.88E-04	1.57E+03
total relative error = M =			1.85E+03

C. Generalized PWL Model The purpose of this part of the Project is to develop a generalized PWL model for which rd is replaced by an a general diode resistance, Rf ≠ rd ≡ N∗Vt/iDQ and Vfi takes on an arbitrary value Vfi ≠ Vf ≡ [vDQ − N∗Vt]. The approach will be identical to part B1.

(C1a) Table C1a reports the generalized PWL model with `i1` defined as in Part B, and `i2a` given by Rfa = 1.1∗rd = 1.1∗N∗Vt/iDQ = 252.994 Ohms and Vfa = [vDQ - 10∗n∗Vt] = 0.3284259 Volts. The figure of merit Ma = 1.86E+03.

(C1b) Table C1b reports the generalized PWL model with `i1` defined as in Part B, and `i2a` given by Rfa = 1.1∗rd = 1.1∗N∗Vt/iDQ = 252.994 Ohms and Vfa = [vDQ - 20∗n∗Vt] = 0.0568518 Volts. The figure of merit Mb = 1.92E+03.

(C1c) Table C1c reports the generalized PWL model with `i1` defined as in Part B, and `i2a` given by Rfa = 1.2∗rd = 1.2∗N∗Vt/iDQ = 275.994 Ohms and Vfa = [vDQ - 20∗n∗Vt] = 0.0568518 Volts. The figure of merit Mc = 1.91E+03.

Table C1a. Comparison of the Shockley diode equation (IS = 3.0E−14A, N=1.05) to a generalized piece-wise linear model (Rfa = 252.994 Ohms, Vfa = 0.3284259 V). The sum relative error, Ma = 1.86E+03 = SUM{|I2 − I1|/iDQ} is a figure of merit of the degree of fit to the Shockley equation.

C1a	Shockley model	PWL,ss model	relative error
vD(V)	I1(A)	I2(A)	|I2-I1|/iD
-20.00	-3.00E-14	-3.00E-14	0.00E+00
-15.00	-3.00E-14	-3.00E-14	0.00E+00
-10.00	-3.00E-14	-3.00E-14	0.00E+00
-5.00	-3.00E-14	-3.00E-14	0.00E+00
0.00	0.00E+00	-3.00E-14	2.54E-10
0.05	1.59E-13	-3.00E-14	1.60E-09
0.10	1.16E-12	-3.00E-14	1.01E-08
0.15	7.48E-12	-3.00E-14	6.36E-08
0.20	4.73E-11	-3.00E-14	4.01E-07
0.25	2.99E-10	-3.00E-14	2.53E-06
0.30	1.88E-09	-3.00E-14	1.59E-05
0.35	1.19E-08	8.53E-05	7.22E-01
0.40	7.48E-08	2.83E-04	2.40E+00
0.45	4.71E-07	4.81E-04	4.07E+00
0.50	2.97E-06	6.78E-04	5.72E+00
0.55	1.87E-05	8.76E-04	7.26E+00
0.60	1.18E-04	1.07E-03	8.09E+00
0.65	7.44E-04	1.27E-03	4.46E+00
0.70	4.69E-03	1.47E-03	2.73E+01
0.75	2.96E-02	1.67E-03	2.36E+02
0.80	1.86E-01	1.86E-03	1.56E+03
Sum relative error = Ma =			1.86E+03

Table C1b. Comparison of the Shockley diode equation (IS = 3.0E−14A, N=1.05) to a generalized piece-wise linear model (Rfa = 252.994 Ohms, Vfa = 0.0568518 V). The sum relative error, Mb = 1.92E+03 = SUM{|I2 − I1|/iDQ} is a figure of merit of the degree of fit to the Shockley equation.

C1b	Shockley model	PWL,ss model	relative error
vD(V)	I1(A)	I2(A)	|I2-I1|/iD
-20.00	-3.00E-14	-3.00E-14	0.00E+00
-15.00	-3.00E-14	-3.00E-14	0.00E+00
-10.00	-3.00E-14	-3.00E-14	0.00E+00
-5.00	-3.00E-14	-3.00E-14	0.00E+00
0.00	0.00E+00	-3.00E-14	2.54E-10
0.05	1.59E-13	-3.00E-14	1.60E-09
0.10	1.16E-12	1.71E-04	1.44E+00
0.15	7.48E-12	3.68E-04	3.12E+00
0.20	4.73E-11	5.66E-04	4.79E+00
0.25	2.99E-10	7.63E-04	6.47E+00
0.30	1.88E-09	9.61E-04	8.14E+00
0.35	1.19E-08	1.16E-03	9.81E+00
0.40	7.48E-08	1.36E-03	1.15E+01
0.45	4.71E-07	1.55E-03	1.32E+01
0.50	2.97E-06	1.75E-03	1.48E+01
0.55	1.87E-05	1.95E-03	1.63E+01
0.60	1.18E-04	2.15E-03	1.72E+01
0.65	7.44E-04	2.34E-03	1.36E+01
0.70	4.69E-03	2.54E-03	1.82E+01
0.75	2.96E-02	2.74E-03	2.27E+02
0.80	1.86E-01	2.94E-03	1.55E+03
Sum relative error = Mb =			1.92E+03

Table C1c. Comparison of the Shockley diode equation (IS = 3.0E−14A, N=1.05) to a generalized piece-wise linear model (Rfa = 275.994 Ohms, Vfa = 0.0568518 V). The sum relative error, Mc = 1.91E+03= SUM{|I2 − I1|/iDQ} is a figure of merit of the degree of fit to the Shockley equation.

C1c	Shockley model	PWL,ss model	relative error
vD(V)	I1(A)	I2(A)	|I2-I1|/iD
-20.00	-3.00E-14	-3.00E-14	0.00E+00
-15.00	-3.00E-14	-3.00E-14	0.00E+00
-10.00	-3.00E-14	-3.00E-14	0.00E+00
-5.00	-3.00E-14	-3.00E-14	0.00E+00
0.00	0.00E+00	-3.00E-14	2.54E-10
0.05	1.59E-13	-3.00E-14	1.60E-09
0.10	1.16E-12	1.56E-04	1.32E+00
0.15	7.48E-12	3.38E-04	2.86E+00
0.20	4.73E-11	5.19E-04	4.39E+00
0.25	2.99E-10	7.00E-04	5.93E+00
0.30	1.88E-09	8.81E-04	7.46E+00
0.35	1.19E-08	1.06E-03	9.00E+00
0.40	7.48E-08	1.24E-03	1.05E+01
0.45	4.71E-07	1.42E-03	1.21E+01
0.50	2.97E-06	1.61E-03	1.36E+01
0.55	1.87E-05	1.79E-03	1.50E+01
0.60	1.18E-04	1.97E-03	1.57E+01
0.65	7.44E-04	2.15E-03	1.19E+01
0.70	4.69E-03	2.33E-03	2.00E+01
0.75	2.96E-02	2.51E-03	2.29E+02
0.80	1.86E-01	2.69E-03	1.56E+03
Sum relative error = Mc =			1.91E+03

(C1d) Which PWL model, [rd,Vf], [Rfa,Vfa], [Rfb,Vfb], or [Rfc,Vfc] fits the Shockley model better according to the figure of merit defined by M, Ma, Mb, or Mc, respectively? The smallest figure of merit is for the original ssPWL model, M = 1.85E+03. Consequently, the smaller value of Rf and the larger value of Vf seems to give the best fit.

(C2) Design and report the parameter values and figure of merit for a generalized PWL model [Rfg,Vfg] which gives the smallest Mg value you can determine. Describe how you developed your final design.

Note that the largest contribution to the relative error is for the 0.75 V and 0.80 V data points with 13% and 85% respectively of the total contribution to M in the ssPWL model of Part B1. A generalized PWL model can be constructed which fits these two data points exactly. The model parameters can be calculated from

$$Rfg = (0.8\text{ V} - 0.75\text{ V})/[iD(0.8) - iD(0.75V)]$$
$$= (0.05\text{ V})/(186.432mA-29.576mA),\text{ so}$$

$$Rfg = 0.318763\text{ Ohm, and}$$

$$Vfg = 0.8V - iD(0.8)*Rfg = 0.740572\text{ V}$$

Table C2 reports the generalized fit of the PWL model with `i1` defined as in Part B, and `i2a` given by Rfg = 318.763 mOhms and Vfg = 740.572 mVolts. The figure of merit Mg = 47.2 which is much less than M = 1853 which was the best previously obtained value (for part B1). Additional attempts at improvements using optimization methods achieved only minor improvements.

Table C2. Comparison of the Shockley diode equation (IS = 3.0E−14A, N=1.05) to a generalized piece-wise linear model (Rfg = 318.763 mOhms, Vfg = 740.572 mVolts). The sum relative error, Mg = 47.2 = SUM{\|I2 − I1\|/iDQ} is a figure of merit of the degree of fit to the Shockley equation and is the best fit achieved in this investigation.

C2	Shockley model	PWL,ss model	relative error
vD(V)	I1(A)	I2(A)	|I2-I1|/iD
-20.00	-3.00E-14	-3.00E-14	0.00E+00
-15.00	-3.00E-14	-3.00E-14	0.00E+00
-10.00	-3.00E-14	-3.00E-14	0.00E+00
-5.00	-3.00E-14	-3.00E-14	0.00E+00
0.00	0.00E+00	-3.00E-14	2.54E-10
0.05	1.59E-13	-3.00E-14	1.60E-09
0.10	1.16E-12	-3.00E-14	1.01E-08
0.15	7.48E-12	-3.00E-14	6.36E-08
0.20	4.73E-11	-3.00E-14	4.01E-07
0.25	2.99E-10	-3.00E-14	2.53E-06
0.30	1.88E-09	-3.00E-14	1.59E-05
0.35	1.19E-08	-3.00E-14	1.00E-04
0.40	7.48E-08	-3.00E-14	6.33E-04
0.45	4.71E-07	-3.00E-14	3.99E-03
0.50	2.97E-06	-3.00E-14	2.52E-02
0.55	1.87E-05	-3.00E-14	1.59E-01
0.60	1.18E-04	-3.00E-14	1.00E+00
0.65	7.44E-04	-3.00E-14	6.30E+00
0.70	4.69E-03	-3.00E-14	3.97E+01
0.75	2.96E-02	2.96E-02	2.66E-08
0.80	1.86E-01	1.86E-01	2.66E-08
Sum relative error = Mg =			4.72E+01

Summary and Conclusion

The Shockley model equation for the semiconductor diode has been compared to several piecewise linear (PWL) models. It has been observed that the best overall fit of a PWL model to the Shockley model equation is obtained when the parameter values are extracted in the range of the maximum diode current values to be used in the model.

Appendix 1

```
* Schematics Version 6.2 - April 1995
* Mon Jul 24 22:00:10 2000

** Analysis setup **
.OPTIONS GMIN=1e-21
.OPTIONS NUMDGT=6
.OP
.LIB project.lib
.OP

* From [SCHEMATICS NETLIST] section of msim.ini:
.lib nom.lib

.INC "ProjTable1.net"

**** INCLUDING ProjTable1.net ****
* Schematics Netlist *

D_D1         $N_0001 0 Dbreak-X1
D_D2         $N_0002 0 Dbreak-X
I_I1         0 $N_0002 DC 0.1
V_V1         $N_0003 0 -15
V_V2         $N_0003 $N_0001 0

**** RESUMING PROJTABLE1.CIR ****
.INC "ProjTable1.als"

**** INCLUDING ProjTable1.als ****
* Schematics Aliases *

.ALIASES
D_D1            D1(1=$N_0001 2=0 )
D_D2            D2(1=$N_0002 2=0 )
I_I1            I1(+=0 -=$N_0002 )
V_V1            V1(+=$N_0003 -=0 )
V_V2            V2(+=$N_0003 -=$N_0001 )
.ENDALIASES


**** RESUMING PROJTABLE1.CIR ****

.probe

.END

**** 07/24/100 22:00:12 ******** NT Evaluation PSpice (April 1995) ***********

 * C:\DATA\AMIPRO\EE2303\Project\ProjTable1.sch

 ****     Diode MODEL PARAMETERS

*******************************************************************************

               Dbreak-X1       Dbreak-X
          IS    1.000000E-12    1.000000E-12
           N     .2              .2

**** 07/24/100 22:00:12 ******** NT Evaluation PSpice (April 1995) ***********

 * C:\DATA\AMIPRO\EE2303\Project\ProjTable1.sch

 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C

*******************************************************************************

 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE

($N_0001)-15.000000                   ($N_0002)   .131020

($N_0003)-15.000000

    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT
```

```
    V_V1          1.000E-12
    V_V2         -1.000E-12

    TOTAL POWER DISSIPATION   1.50E-11  WATTS


**** 07/24/100 22:00:12 ******** NT Evaluation PSpice (April 1995) ***********

 * C:\DATA\AMIPRO\EE2303\Project\ProjTable1.sch


 ****     OPERATING POINT INFORMATION      TEMPERATURE =   27.000 DEG C

******************************************************************************

**** DIODES

NAME         D_D1         D_D2
MODEL        Dbreak-X1    Dbreak-X
ID          -1.00E-12     1.00E-01
VD          -1.50E+01     1.31E-01
REQ          1.00E+21     5.17E-02
CAP          0.00E+00     0.00E+00

          JOB CONCLUDED

          TOTAL JOB TIME            .05
```

References

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[1] *SPICE: A Guide to Circuit Simulation and Analysis Using PSpice*, 3rd ed., by Paul W. Tuinenga, Prentice Hall, Englewood Cliffs, NJ, ©1995.

[2] *MicroSim PSpice for Windows, 2nd ed*, by Goody, Prentice-Hall, Upper Saddle River, N.J., ©1998.

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[4] *PSpice*$^{\text{TM MicroSim}}$ is available by download from http://www.orcad.com/Product/Analog/Analog.asp

[5] Microsoft® Excel 97 SR-2, Copyright© 1985-97 Microsoft Corporation.