Semiconductor Resistor Characteristics

Solution for Project 1 - EE 5342

Ronald L. Carter, Professor

ronc@uta.edu

A. Purpose of the Project

The purpose of this project is to use the model equations given by Fjeldly, *et al.*[1], in order to determine the quadratic temperature and field coefficients for resistors modeled in SPICE™.

B. Narrative of Project Procedure

Model equations related to resistance values are given by Fjeldly, *et al.*[1]. The electric field (F[V/cm]), impurity concentration (N_T[cm^{-3}] = $\Sigma N_d + \Sigma N_a$) and temperature (T[K]) dependencies of electron drift velocities and mobilities (v_n and μ_n) are given by:

Equation 1. $v_n = \mu_n F/(1 + (\mu_n F/v_s)^2)^{1/2}$, where

Equation 2. $\mu_n(N_T, T) = \mu_{mn} + \mu_{on}/(1 + (N_T/N_{cn})^{\nu})$,

Equation 3. $v_s = (2.4E7\ \text{cm/s})/(1 + 0.8 \exp(T/600))$,

Equation 4. $\mu_{mn} = (88\ \text{cm}^2/\text{V-s})(T/300)^{-0.57}$,

Equation 5. $\mu_{on} = (1250\ \text{cm}^2/\text{V-s})(T/300)^{-2.33}$,

Equation 6. $N_{cn} = (1.26E17\ \text{cm}^{-3})(T/300)^{2.4}$, and

Equation 7. $\nu = 0.88(T/300)^{-0.146}$.

In the same source, the electric field (F[V/cm]), impurity concentration (N_T[cm^{-3}] = $\Sigma N_d + \Sigma N_a$) and temperature (T[K]) dependencies of hole drift velocities and mobilities (v_p and μ_p) are given by:

Equation 8. $v_p = \mu_p F/(1 + \mu_p F/v_s)$, where

Equation 9. $\mu_p(N_T, T) = \mu_{mp} + \mu_{op}/(1 + (N_T/N_{cp})^{\nu})$,

Equation 10. $v_s = (2.4E7\ \text{cm/s})/(1 + 0.8 \exp(T/600))$,

Equation 11. $\mu_{mp} = (54\ \text{cm}^2/\text{V-s})(T/300)^{-0.57}$,

Equation 12. $\mu_{op} = (407\ \text{cm}^2/\text{V-s})(T/300)^{-2.23}$,

Equation 13. $N_{cp} = (2.35E17\ \text{cm}^{-3})(T/300)^{2.4}$, and

Equation 14. $\nu = 0.88(T/300)^{-0.146}$.

For this assignment, we will be interested in the parameters for the voltage nonlinearity and the temperature dependence. The Cadence Manual[2] gives the following form to be used for the and voltage nonlinearity (Equation 15a) and temperature dependence (Equation 15b) of a resistor is calculated by:

Equation 15a. R(V) = R(inst) / (1 + c1 * V + c2 * V ^ 2 + ...), where ck is the kth entry in the coefficient vector. R(inst) is the value of the resistance at V=0 for the particular instance chosen. (Consider only the two lowest order non-zero ck values.

Equation 15b. R(T) = R(tnom) * [1 + tc1 * (T - tnom) + tc2 * (T - tnom)^2]

To do Assignment 1a and 1b, the electron (a) and hole (b) Fjeldly mobilities were computed for T = 300K using Equations 2, 4, 5, 6, and 7 (electron) and Equations 9, 11, 12, 13, and 14 (hole).

Equation 16. $\mu_n(N_T, 300)$ = 88 + 1250/(1 + (N_T/1.26e17)^.88), and

Equation 17. $\mu_p(N_T, 300)$ = 54 + 407/(1 + (N_T/2.35e17)^.88).

The models taken from the Lecture (which quoted Muller and Kamins[3]) at 300K for electrons (a, phosphorous doped) and holes (b, boron doped) are

Equation 18. $\mu_n(N_T, 300)$ = 68.5 + (1414-68.5)/(1 + (N_T/9.2e16)^.711), and

Equation 19. $\mu_p(N_T, 300)$ = 44.9 + (470.5-44.9)/(1 + (N_T/2.23e17)^.719).

To complete **Assignment 1a**, the results of the Fjeldly and Lecture electron mobility models (Fjeldly and Lecture) for 1E15, 1E16, 1E17 and 1E18 N_T values have been computed using Equations 16 and 17 and tabulated in Table 1. The error computed using Equation 20 is also tabulated.

Table 1. The Fjeldly and Lecture model mobilities for electrons (**Assignment 1a**) for various concentrations of phosphorous are shown. The relative error is also shown.

N_T	μ_n		error
	(Fjeldly)	(Lecture)	
1.E+15	1321	1362	3.1%
1.E+16	1217	1184	2.7%
1.E+17	776	721	7.1%
1.E+18	262	277	5.8%

.

Equation 20. Relative error ≡ |Fjeldly-Lecture|/(Fjeldly)*100%

Comment on Assignment 1a (per Assignment 1e): The difference is small (<10%) between these two models. The difference probably arises from use of different data (perhaps Fjeldly used all n-type dopants) and different sources of the silicon material.

To complete **Assignment 1b**, the results of the Fjeldly and Lecture electron mobility models (Fjeldly and Lecture) for 1E15, 1E16, 1E17 and 1E18 N_T values have been computed using Equations 18 and 19 and tabulated in Table 2. The error computed using Equation 20 is also tabulated.

Table 2. The Fjeldly and Lecture model mobilities (**Assignment 1b**) for holes for various boron concentrations and relative error are shown.

N_T	μ_p		error
	(Fjeldly)	(Lecture)	
1.E+15	458	462	0.9%
1.E+16	437	429	1.8%
1.E+17	331	317	4.0%
1.E+18	143	153	7.0%

Comment on Assignment 1b (per Assignment 1e): The difference is small (<10%) between these two models. As in the case of 1b, the difference probably arises from use of different data (perhaps Fjeldly used all p-type dopants) and different sources of the silicon material.

To complete **Assignment 1c**, values for electron velocity at T = 300K were calculated for the Fleldly model for N_T =0, and for F = 1E3, 1E4 and 1E5 V/cm using Equations 1 through 7. Values were read from the graph given in the Lectures (which were taken from Sze[4]). The results are shown in Table 3. The error was calculated using Equation 20.

Table 3. The Fjeldly and Lecture model electron velocity for various electric fields (**Assignment 1c**) for 300K and N_T =0. The relative error is also shown.

F	v_n (cm/s)		error
[V/cm]	Fjeldly	lecture	
1.E+03	1.32E+06	1.30E+06	1.7%
1.E+04	7.25E+06	7.20E+06	0.7%
1.E+05	8.61E+06	1.00E+07	16.2%

Comment on Assignment 1c (per Assignment 1e): The difference is small for low fields, but ~16% at high field. This is probably due to the use of different sources of the silicon material.

To complete **Assignment 1d**, values for hole velocity at T = 300K were calculated for the Fleldly model for N_T =0, and for F = 1E3, 1E4 and 1E5 V/cm using Equations 8 through 14. Values were read from the graph given in the Lectures (which were taken from Sze[4]). The results are shown in Table 4. The error was calculated using Equation 20.

Table 4. The Fjeldly and Lecture model hole velocity for various electric fields (**Assignment 1d**) for 300K and N_T =0. The relative error is also shown.

F	v_p (cm/s)		error
[V/cm]	Fjeldly	lecture	
1.E+03	4.38E+05	4.80E+05	9.7%
1.E+04	3.00E+06	3.40E+06	13.2%
1.E+05	7.27E+06	8.70E+06	19.7%

Comment on Assignment 1d (per Assignment 1e): The difference is small for low field, but ~20% at intermediate and high field. This is probably due to the use of different sources of the silicon material.

Assignment 2a. First, derive the equations to use in parts b through g for the 2 lowest order non-zero ck, for tc1 and tc2. Note that

Equation 21. R(V) = (n*q*MUn + p*q*MUp)^-1*L/A, but since

Equation 22. vn = MUn*F, and

Equation 23. vp = MUp*F,

at low fields, where F = V/L, we replace MU with v/F = vL/V, and Equation 21 becomes

R = (n*q*vn(F) + p*q*vp(F))^-1*(F)*L/A, but since F = V/L

Equation 24. R = (n*q*vn(*F*, N_T, T) + p*q*vp(*F*, N_T, T))^-1*(V/L)*L/A

To complete **Assignment 2b, through 2e (voltage dependent part)**, the T=300K resistance is computed using Equation 24 (and Equations 1 through 14) as a function of applied voltage from 0 to 3V for each of the structures defined. These resistances are given in Table 5.

Table 5. Calculated values of the resistance of the structures given in Assignments 2b through 2e. The majority carrier velocities were first calculated per Equations 1 through 10 and then the effective resistance was calculated per Equation 24.

	Part b.	Part c.	Part d.	Part e.				
	Nd = 1.00E+17	Nd = 2.00E+16	Nd = 1.00E+15	Nd = 0.00E+00				
	Na = 0	Na = 2.00E+17	Na = 3.00E+18	Na = 3.00E+17				
	L = 4.00E-03	L = 4.00E-03	L = 1.00E-04	L = 2.00E-03	vs(cm/sec) 8.62E+06	vs(cm/sec) 8.62E+06	vs(cm/sec) 8.62E+06	vs(cm/sec) 8.62E+06
	A = 1.00E-07	A = 1.00E-06	A = 1.00E-08	A = 1.00E-07	MUnb cm2/V-S 776	MUpc cm2/V-S 263	MUpc cm2/V-S 93	MUpc cm2/V-S 236
V(Volts)	R(Ohm)	R(Ohm)	R(Ohm)	R(Ohm)	vnb cm/S	vpc cm/S	vpd cm/S	vpe cm/S
0.0	3215.9	526.57	223.52	1765.24	0.00E+00	0.00E+00	0.00E+00	0.00E+00
0.5	3215.9	528.58	235.59	1777.30	9.70E+04	3.28E+04	4.42E+05	5.85E+04
1.0	3215.9	530.59	247.66	1789.36	1.94E+05	6.54E+04	8.40E+05	1.16E+05
1.5	3215.9	532.60	259.72	1801.43	2.91E+05	9.77E+04	1.20E+06	1.73E+05
2.0	3215.9	534.61	271.79	1813.49	3.88E+05	1.30E+05	1.53E+06	2.29E+05
2.5	3215.9	536.62	283.85	1825.55	4.85E+05	1.62E+05	1.83E+06	2.85E+05
3.0	3215.9	538.63	295.92	1837.61	5.82E+05	1.93E+05	2.11E+06	3.40E+05

In order to fit the above data to Equation 15a to second order, the relative resistance defined as

Equation 25. r ≡ R(inst)/R(V) = R(V=0)/R(V) = 1 + c1 * V + c2 * V ^ 2

These relative resistance values computed for structures b, c, d, and e are quoted in Table 6.

Table 6. The relative resistances of structures b, c, d, and e for applied voltage from 0 to 3 Volts. The second order polynomial fit equation parameters are also given.

V(Volts)	rb	rc	rd	re
0.0	1.00000	1.00000	1.00000	1.00000
0.5	1.00000	0.99620	0.94879	0.99321
1.0	1.00000	0.99242	0.90256	0.98652
1.5	1.00000	0.98868	0.86063	0.97991
2.0	1.00000	0.98496	0.82243	0.97340
2.5	1.00000	0.98127	0.78747	0.96696
3.0	1.00000	0.97761	0.75536	0.96062
c1	0.00000	-7.60E-03	-1.05E-01	-1.37E-02
c2	0.00000	5.63E-05	7.76E-03	1.76E-04

These data were then plotted as a function of applied voltage, V, and then the second order polynomial curve-fitting option of Excel™ was used to determine the parameters c1 and c2. The data are shown plotted in Figure 1. The second order polynomial fit equations are also printed on the graphs. The pertinent values of c1b, c1c, c1d, c1e, c2b, c2c, c2d, and c2e are quoted in Table 6.

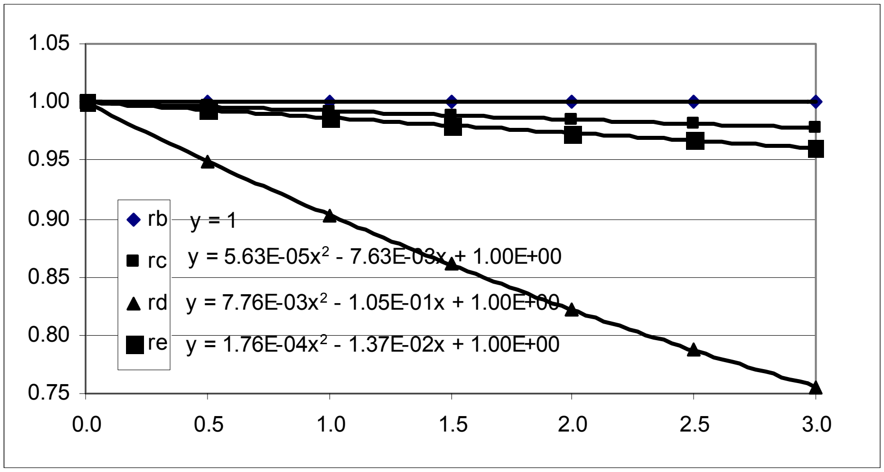

Figure 1. The relative resistance of structures b, c, d, and e as a function of voltage and the second order polynomial equations to fit this data. These equations were forced to the value of 1 at V = 0.

The temperature coefficients tc1 and tc2 defined in Equation 15b were determined in a similar manner. The parameters tc1 and tc2 were evaluated by computing the resistance over the temperature range of 260K < T < 380K, and fitting the data to a second order polynomial. The results of these calculations are given in Table 7.

Table 7. The resistance of structures b, c, d, and e for temperatures from 260K to 380K.

	Part b.	Part c.	Part d.	Part e.
	Nd = 1.00E+17	Nd = 2.00E+16	Nd = 1.00E+15	Nd = 0.00E+00
	Na = 0	Na = 2.00E+17	Na = 3.00E+18	Na = 3.00E+17
	L = 4.00E-03	L = 4.00E-03	L = 1.00E-04	L = 2.00E-03
	A = 1.00E-07	A = 1.00E-06	A = 1.00E-08	A = 1.00E-07
T (K)	R(Ohm)	R(Ohm)	R(Ohm)	R(Ohm)
260	2702.4	456.46	213.67	1562.33
280	2947.9	490.09	218.52	1659.38
300	3215.9	526.57	223.52	1765.24
320	3505.5	565.70	228.70	1879.27
340	3816.2	607.31	234.05	2000.90
360	4147.3	651.25	239.59	2129.61
380	4498.2	697.36	245.32	2264.95

In order to fit the data to Equation15b, a relative resistance, rt, was defined. In this case

Equation 27. $rt = R(T)/R(tnom) = R(T)/R(T=300) = 1 + tc1*(T - tnom) + tc2*(T - tnom)^2$

The rtb, rtc, rtd, rte values were calculated for the corresponding values of T - tnom (tnom = 300K) and fit using a second order polynomial in Excel™. The values calculated are given in Table 8, and the corresponding data are plotted in Figure 2. The second order polynomial equations are shown in Figure 2, and the coefficients tc1b, tc1c, tc1d, tc1e, tc2b, tc2c, tc2d, and tc2e are given in Table 8.

Table 8. The relative resistances rt of structures b, c, d, and e as a function of the temperature departure from tnom (=300K). The second order polynomial fit equation parameters are also given.

T-tnom	rb	rc	rd	re
-40	0.84035	0.86686	0.95593	0.88505
-20	0.91668	0.93073	0.97763	0.94003
0	1.00000	1.00000	1.00000	1.00000
20	1.09007	1.07432	1.02314	1.06460
40	1.18668	1.15334	1.04708	1.13350
60	1.28964	1.23678	1.07187	1.20641
80	1.39876	1.32435	1.09751	1.28308
tc1	4.33E-03	3.58E-03	1.14E-03	3.10E-03
tc2	8.26E-06	6.01E-06	9.82E-07	5.51E-06

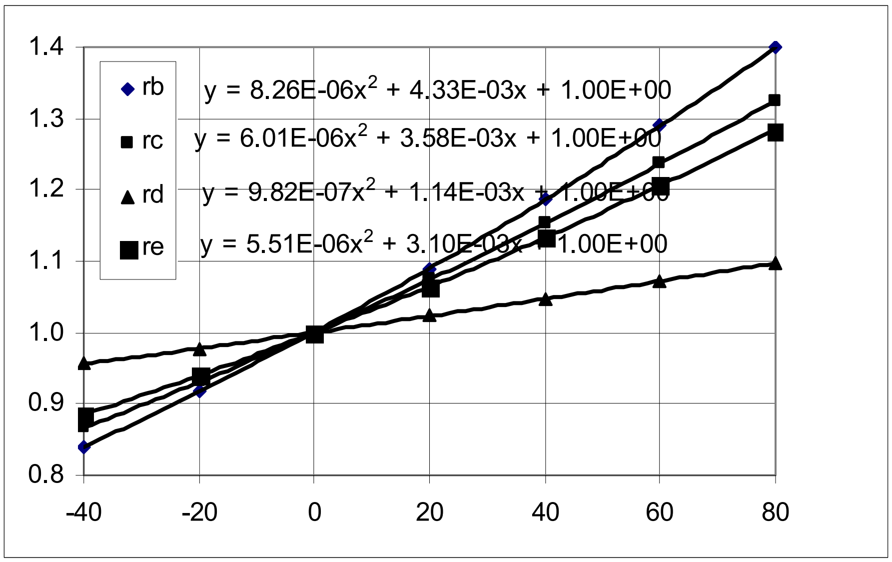

Figure 2. The relative resistance, rt, of structures b, c, d, and e as a function of T-tnom (tnom=300K) and the second order polynomial equations to fit this data. These equations were forced to the value of 1 at T = tnom.

C. Answers to Questions

Assignment 1e. Comments were included in the text relevant to parts 1a through 1d.

Assignment 2 requested comment in each case on the values and the practicality of the values obtained: In this case, the concentrations are all at levels that are appropriate for integrated circuit fabrication. The resistor lengths are all reasonable values, except for structure d where L ~ $A^{1/2}$ means that the structure is very short relative to the cross-sectional dimensions. The areas in all cases are realistically obtainable, except for structure d where the $A^{1/2}$ ~ 1 micron implies an extremely small structure for a resistor.

Assignment 2f. What values of N_d and N_a would you choose to minimize the voltage coefficients? According to Equations 1 and 8, the velocity-field relationship is characterized by

$$v = \mu_{lf}F/(1 + \mu_{lf}F/v_s)^n,$$

where μ_{lf} is the low-field mobility and n is 1/2 for electrons and 1 for holes. The parameters c1 and c2 will be minimized if the saturation velocity is reached at higher fields. Consequently, the ratio, $\mu_{lf}F/v_s$, should be minimized, requiring the smallest possible value of $\mu_{lf}F$. This is achieved by using high doping concentrations. This is substantiated by comparing structures b and c to structure e. Furthermore, the lower values of c1 and c2 will be achieved by choosing longer structures so F is lower. This is substantiated by comparing structures b, c and e with structure d which has the shortest length.

Assignment 2g. What values of N_d and N_a would you choose to minimize the thermal coefficients? With regard to temperature coefficients, the only difference in the temperature dependencies for electrons and holes are in the N_{cn} and N_{cp} values, where $N_{cn} \sim N_{cp}/2$. Consequently, there will be a slight difference between the tc1 and tc2 values for n-type and p-type material. The larger effect seems to be net impurity concentration as noting the net change upon temperature variation as shown in Figure 2. Consequently, the smaller values in tc1 and tc2 are achieved by using higher concentrations.

D. References

1. *Introduction to Device Modeling and Circuit Simulation*, by Tor A. Fjeldly, Trond Ytterdal, and Michael Shur, Wiley Interscience, New York, 1998.
2. *Affirma Spectre Circuit Simulator Device Model Equations,* Product Version 4.4.6, February 2001, 2001 Cadence Design Systems, Inc., 2000.
3. *Device Electronics for Integrated Circuits*, 2nd ed., by Richard S. Muller and Theodore I. Kamins, John Wiley and Sons, New York, 1986.
4. *Physics of Semiconductor Devices*, by S. M. Sze, Wiley-Interscience, New York, 1981.