

Instructions:

Seat Number _____

1. Do your own work. **DO NOT REMOVE THE STAPLE ON THIS EXAM.**
2. You may use a legal copy of the text by Massobrio and Antognetti. You may write notes in your text. You may NOT pass a book or note sheet to another student, or class notes or previously solved problems. **You may use your Project 1 solution and must submit it with this test in your exam packet.**
3. Calculator allowed. You may NOT share a calculator with another student.
4. Where values or equations are given on this cover sheet, use them in lieu of any other source. If a value is not given, explicitly state definitions and assumptions that you use.
5. Where possible, calculate parameters rather than read them from a graph.
6. Do all work in the spaces provided on this exam paper. If you write on the back of a sheet, make the notation "PTO" in your solution in order to assure that material written on the back of the page is evaluated for a grade. **AN EXTRA BLANK SHEET IS ATTACHED AT THE BACK OF THE EXAM.**
7. Show all calculations, making numerical substitutions and giving numerical results where possible.
8. **The total for the test is 75. Up to 25 additional points will be given for the Project report.**
9. Unless stated otherwise,

$$T = 300\text{K},$$

$$V_t = 25.852 \text{ mV}$$

10. Unless otherwise stated, the material is silicon (300K) with

$$n_i = 1.45\text{E}10 \text{ cm}^{-3}$$

$$N_c = 2.8\text{E}19 \text{ cm}^{-3}$$

$$q\chi_{\text{Si}} = 4.05 \text{ eV}$$

$$E_{g,\text{Si}} = 1.124 \text{ eV.}$$

$$N_v = 1.04\text{E}19 \text{ cm}^{-3}$$

11. For the work function of poly silicon, use

$$\phi_{n+} = \chi_{\text{Si}} = 4.05 \text{ V}$$

$$\phi_{p+} = \chi_{\text{Si}} + E_{g,\text{Si}}/q = 5.174 \text{ V.}$$

12. For minority carrier (either electrons or holes) lifetime in silicon, use the relationship

$$\tau_{\text{min}} = (4.5\text{E}-5 \text{ sec}) / (1 + N_i/1\text{E}17 + (N_i/5\text{E}17)^2),$$

$$\text{where } N_i = \text{the total impurity concentration in cm}^{-3}$$

13. For holes in silicon doped primarily with boron*, assume

$$\mu_p = \{470.5 \div [1 + (N_i \div 2.23\text{E}17)^{0.719}]\} + 44.9, \text{ in cm}^2/\text{V-sec.}$$

14. For electrons in silicon doped primarily with phosphorous*, assume

$$\mu_n = \{1414 \div [1 + (N_i \div 9.2\text{E}16)^{0.711}]\} + 68.5, \text{ in cm}^2/\text{V-sec.}$$

15. For electrons in silicon doped primarily with arsenic, assume

$$\mu_n = \{1417 \div [1 + (N_i \div 9.68\text{E}16)^{0.68}]\} + 52.2, \text{ in cm}^2/\text{V-sec.}$$

(In 12 through 15, N_i is the total impurity concentration in n- or p-type material, compensated or not.)

(*13 may be used as an approximation for holes as minority carriers, likewise *14 for minority electrons.)

16. Metal gate work functions should be assumed to be

$$\phi_{\text{M,AI}} = 4.1 \text{ V for aluminum,}$$

$$\phi_{\text{M,Pt}} = 5.3 \text{ V for platinum,}$$

$$\phi_{\text{M,Au}} = 4.75 \text{ V for gold}$$

17. The electron affinity of SiO_2 is

$$\chi_{\text{SiO}_2} = 1.00 \text{ V.}$$

18. Planck constant

$$h = 6.625\text{E}-34 \text{ J-s} = 4.135\text{E}-15 \text{ eV-s, (1 eV} = 1.602\text{E}-19 \text{ Joule).}$$

19. free electron mass

$$m_0 = 9.11\text{E}-28 \text{ g.}$$

20. Boltzmann constant,

$$k = 1.38066\text{E}-23 \text{ J/K}$$

21. Electron charge,

$$q = 1.60218\text{E}-19 \text{ Coulomb}$$

22. Permittivity of free space,

$$\epsilon_0 = 8.854\text{E}-14 \text{ Fd/cm}$$

23. Relative permittivity of silicon,

$$\epsilon_r = 11.7$$

24. Relative permittivity of silicon dioxide, $\epsilon_{\text{rOx}} = 3.9$

25. The breakdown voltage of an abrupt (step) junction (asymmetrical or one-sided) diode with doping on the lightly doped side of N_B is $V_B = 60(E_g/1.1)^{3/2} (10^{16}/N_B)^{3/4} \text{ V}$. The critical field for breakdown is modeled as $E_{\text{crit}} = (120\text{V} \cdot qN_B / (\epsilon_r \epsilon_0))^{1/2} \cdot (E_g/1.1)^{3/4} \cdot (10^{16}/N_B)^{3/8}$

26. Each part is worth [x] points, as given in the problem.

1. The SPICE model for the junction capacitance is $C_j = CJ0 \{1 - V_a/VJ\}^{-M}$, where V_a is the voltage across the depletion region and $CJ0$, VJ and M are SPICE parameters.

a. [5] Give the defining equation of any additional small signal capacitance in the SPICE diode model **in terms of V_a** for the case where IS , N and RS are sufficient to fit the static diode data (*i.e.*, ISR and IKF are not significant). Define any additional SPICE parameters required to model this capacitance.

The diffusion capacitance, C_d , is in parallel with C_j .

$C_d = TT/r_d$, where TT is a SPICE parameter,

$r_d = N \cdot V_t / I_d$, and

$I_d = IS \exp[(V_a - I_d \cdot RS) / (N \cdot V_t)]$.

Note that I_d is nonlinear in V_a but is nonetheless a function of V_a .

b. [5] Is the SPICE junction capacitance model always a good fit to the data – or can you cite a device which does not fit this model?

The pin diode of part A of Project 1 fits the SPICE model only in the range $V_a > V_{PT}$.

V_{PT} is the punch-through voltage at which the depletion region width reaches to the N^+n^- junction. For $V_a > V_{PT}$, the depletion region does not reach to the N^+n^- junction.

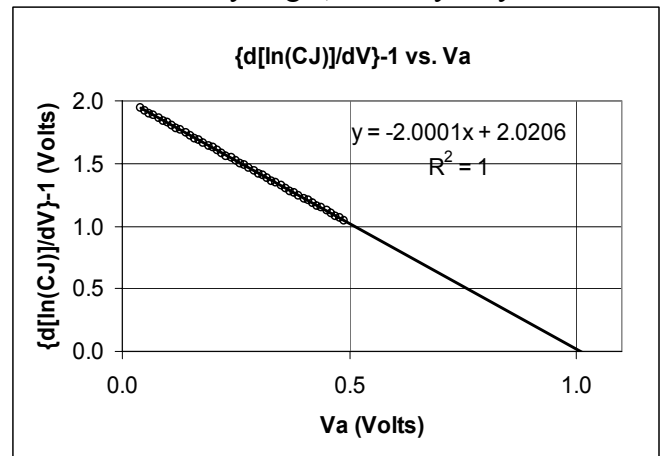
c. [5] The lectures defined a method of plotting the junction capacitance as $[d\{\ln(C_j)\}/dV_a]^{-1}$ vs. V_a in cases where the data can be fit to the SPICE model. Such a plot would then give a straight line plot with a slope of $-M^{-1}$, and intercept of VJ/M (if the data fits the SPICE model). When you applied this process to the data in Part A of the project, you did not get a single straight line plot for the data, but two line segments. Which voltage range gave M and VJ values which are reasonable. What values did you get, and why do you believe such values are reasonable?

The graph shown is a plot of $[d\{\ln(C_j)\}/dV_a]^{-1}$ vs. V_a for $V_a > V_{PT} \sim 0$.

The slope is -2 , so $m = \frac{1}{2}$, also $VJ \sim 1$ from the voltage axis intercept.

The remainder of the data will not fit this function with a slope of $1/3 < m < 1/2$, or for a VJ value less than $E_{g,Si}$.

Consequently, the SPICE model will not realistically fit the observations for $V_a < V_{PT}$.

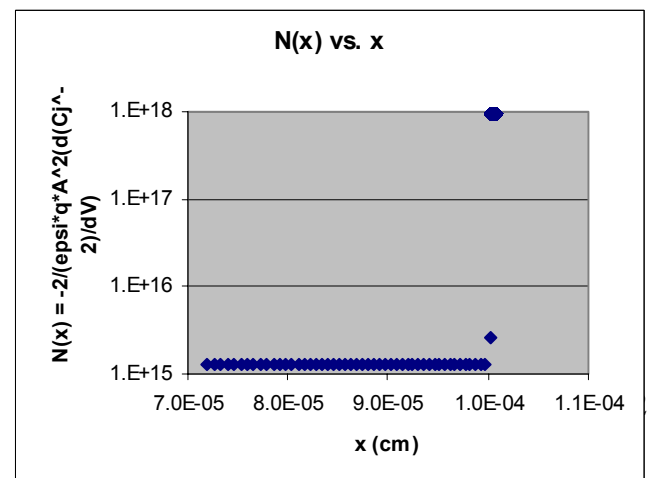


d. [5] Relate the observations you made about the C_j vs. V_a function in part a to the $N(x)$ relationship you obtained for the data given.

$N(x) = -2 / (\epsilon_{si} \cdot q \cdot A^2 \cdot d(C_j^{-2}) / dV)$ is shown in the plot.

The depth $x = \epsilon_{si} \cdot A / C_j$.

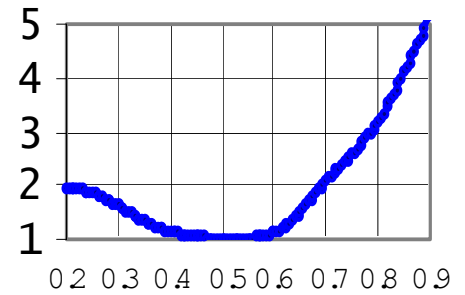
The N value shifts from $1.3E15/cm^3$ to $9.5E17/cm^3$ at $x = 1E-4$ cm (which corresponds to $V_a = 0 = V_{PT}$).



2. In the range in which the low-level injection current and the recombination current are both significant (usually for $V_a < 600$ mV), the function of V_a defined as $N_{\text{eff}}(V_a) \equiv \{dV_a/d[\ln(I_D)]\}/Vt$ ranges from ~ 2 for low voltages to ~ 1 . In the same range, the function of V_a defined as $IS_{\text{eff}}(V_a) \equiv \exp[\ln(I_D) - V_a/(N_{\text{eff}}*V_t)]$ can be used to estimate IS and ISR.

a. [5] Describe how you would select the value of N_{eff} that would be the most reliable estimate for N and whether you expect this estimate will be too large or too small. Be sure to make your discussion in terms of maximum and minimum values of N_{eff} and the precise definition of voltage data used. Relate this answer to the size and sign of error you observed in doing this in Part D1 of Project 1.

The plot shows a typical result of $N_{\text{eff}}(V_a)$. In this case, for the range of $0.2 \text{ V} < V_a < 0.6 \text{ V}$, N_{eff} has a maximum value $N_{\text{eff,max}}$. As discussed in class, the range in which $I_{d,\text{inj}} > I_{d,\text{rec}}$ is the region in which N_{eff} approximates N. The best estimate of N is therefore the minimum value, $N_{\text{eff,min}}$, which occurs at some voltage $V_a = V_{a,N_{\text{eff,min}}}$. If there is noise in the data, some averaging is appropriate. Furthermore, since $I_{d,\text{rec}}$ is small but not negligible in this range, it has the effect of increasing $N_{\text{eff,min}}$, so $N_{\text{eff,min}} >$ the true N.



N_{eff} vs. V_{ext}

b. [5] Describe how you would select the value of N_{eff} that would be the most reliable estimate for NR and whether you expect this estimate will be too large or too small. Be sure to make your discussion in terms of maximum and minimum values of N_{eff} and the precise definition of voltage data used. Relate this answer to the size and sign of error you observed in doing this in Part D1 of Project 1.

Similarly, for the range of $0.2 \text{ V} < V_a < 0.6 \text{ V}$, N_{eff} has a maximum value $N_{\text{eff,max}}$. As discussed in class, the range in which $I_{d,\text{inj}} < I_{d,\text{rec}}$ is the region in which N_{eff} approximates NR. The best estimate of NR is therefore the maximum value, $N_{\text{eff,max}}$, which occurs at $V_a = V_{a,N_{\text{eff,max}}}$.

If there is noise in the data, some averaging is appropriate, and many times the end-point (the lowest voltage) will not be reliable due to errors in calculating the finite difference approximation of the derivative. Furthermore, since $I_{d,\text{inj}}$ is small but not negligible in this range, it has the effect of decreasing $N_{\text{eff,max}}$, so $N_{\text{eff,max}} <$ the true NR.

c. [5] Describe how you would select the value of IS_{eff} that would be the most reliable estimate for IS and whether you expect this estimate will be too large or too small. Be sure to make your discussion in terms of the precise definition of voltage data used. Relate this answer to the size and sign of error you observed in doing this in Part D1 of Project 1.

The range in which $I_{d,\text{inj}} > I_{d,\text{rec}}$ is the region in which N_{eff} approximates N. Likewise in this range, the IS_{eff} function best approximates IS. The best estimate of IS is therefore the IS_{eff} function calculated at $V_a = V_{a,N_{\text{eff,min}}}$. Since N is overestimated, examination of the IS_{eff} function indicates that $IS_{\text{eff}}(V_{a,N_{\text{eff,min}}}) >$ the true IS. As before, some averaging is appropriate.

d. [5] Describe how you would select the value of IS_{eff} that would be the most reliable estimate for ISR and whether you expect this estimate will be too large or too small. Be sure to make your discussion in terms of

the precise definition of voltage data used. Relate this answer to the size and sign of error you observed in doing this in Part D1 of Project 1.

The best estimate of ISR is the IS_{eff} , function calculated at $V_a = V_{a,Neff,max}$. Since NR is underestimated, examination of the IS_{eff} function indicates that $IS_{eff}(V_{a,Neff,max}) < \text{the true ISR}$. As before, some averaging is appropriate. You should verify that your error has the same sign as predicted in parts a, b, c, and d.

3. In the high level injection range, $I_d \sim (IS \cdot IKF)^{1/2} \exp[(V_a - I_d \cdot RS)/(2 \cdot N \cdot V_t)]$, where V_a is the potential across the depletion region. In the high level injection range, one might expect that $N_{eff} \sim 2$, however in the data you generated for part D, the N_{eff} value would have been > 2 . This is due to the effect of the series resistance RS. From the small-signal model, one would expect that the differential conductance dI_d/dV_a should be the conductance represented by the series combination of RS and r_d (i.e., $dI_d/dV_a = [RS + r_d]^{-1}$). **Note in this case, there was an error in the problem statement. V_a above should be replaced by V_x the external voltage, and the relationship between the two is $V_a = V_x - I_d \cdot RS$. You will be graded on treating the problem consistently - as below.**

a. [5] From the definition of the high level injection current above, show that $r_d = 2NV_t / I_d$.

The differential intrinsic diode conductance is defined as,

$$g_d = dI_d/dV_a = d\{(IS \cdot IKF)^{1/2} \exp[V_a/(2 \cdot N \cdot V_t)]\}/dV_a = I_d/(2 \cdot N \cdot V_t),$$

so

$$r_d = 2 \cdot N \cdot V_t / I_d.$$

b. [5] From the definition of the high level injection current above, show that $dI_d/dV_a = 1/(RS + r_d)$.

The total extrinsic diode resistance is defined as,

$$g_{dx} = dI_d/dV_x = (d/dV_x)\{(IS \cdot IKF)^{1/2} \exp[(V_x - I_d \cdot RS)/(2 \cdot N \cdot V_t)]\} \\ = I_d \cdot [1 - (dI_d/dV_x) \cdot RS] / (2 \cdot N \cdot V_t),$$

so, solving for $r_{dx} = 1/g_{dx} = RS + 1/g_d$

c. [5] Use the value you extracted for N in part D1 of Project 1 to determine r_d at the highest three voltage data points of the data you generated in part C.

The value of r_{di} at each of the points specified as the top three voltage points, V_i , where $i = 1, 2, \text{ and } 3$ (where $I_{di} = I_d(V_i)$) is given by, $r_{di} = 2 \cdot N \cdot V_t / I_{di}$

d. [5] Use the information in problem 3c to calculate RS (based on the formula in 3b). Be careful as to how you calculate dI_d/dV_a and the value of r_d used. Discuss any difference between this value and the result you found in doing the project.

Using the central difference, there is only one data point ($i = 2$) where g_{dx} and I_d are known for the same condition.

$$g_{dx(i=2)} = [I_{d(i=3)} - I_{d(i=1)}] / [V_{(i=3)} - V_{(i=1)}] \\ = I_{d(i=2)} \cdot [1 - ([I_{d(i=3)} - I_{d(i=1)}] / [V_{(i=3)} - V_{(i=1)}]) \cdot RS] / (2 \cdot N \cdot V_t),$$

so

$$RS = [V_{(i=3)} - V_{(i=1)}] / [I_{d(i=3)} - I_{d(i=1)}] - 2 \cdot N \cdot V_t / I_{d(i=2)}$$

If the data points for $i = 1, 2,$ and 3 are in the high level injection range, this value of RS should be more reliable than other methods, since this method uses an exact estimate of g_d , whereas the other methods applied to the static data assume it is negligible. Consequently, this method should give a smaller value of RS than approximating RS to the high current limit of V_x/I_d or dV_x/dI_d .

4a. [5] In the project, you are asked to set RS to a value such that at IKF , $IKF \cdot RS / VKF = 10\%$. What is the effect of doing this?

This means that at VKF , $V_a = V_x + I_d \cdot RS$ is about $0.9 \cdot V_a$. As a consequence, the RS and IKF contributions to I_d begin to be significant at about the same value of V_x .

b. [5] Will your extracted parameters (IKF , RS) be closer to your preset values if we increase the 10% function above to a value of 20%. Why?

The effect will be small by only changing a factor of 10%. Values more like 100% are necessary for a significant effect

5. [5] Assume $N_A > N_D$ for a p-n junction. If we N_A is doubled and N_D is decreased by 50% of the original value, will the $CJ0$ SPICE parameter increase or decrease? Prove your conclusion.

Since $CJ0 \sim (N_A \cdot N_D / (N_A + N_D))^{1/2}$, then the change described above will not change the numerator, and will have a net increase in the denominator, so $CJ0$ will decrease.

Criteria Used in Grading Test 2 for Project 1

1.a.

Equation for C_d 2 points

Equation for r_d 2 points

Equation for I_s 1 point

1.b.

No..... 4 points

PIN..... 1 point

1.c.

Give the value you obtained in the project..... 3 points

Explain reason..... 2 points

2 (for a, b, c and d)

Give the method to select best estimate2 points

The extracted values are larger or smaller than the true values1 point

Explain why they are larger or smaller than their true values1 point

Related your report data1 point

3.a and 3.b

You can get full credit **ONLY** if you correctly use the formula for the differential in your answer

3.d.

If you use central difference correctly, you will get full points

Otherwise (forward difference, etc), you will get 4 points if your result is correct.

4.b.

No.....3 points

Reasonable explanation for this answer..... 2 points

5

Decrease.....2 points

Show your answer mathematically.....3 points

Total: 75