Project 2 Test - EE 5342 (print last name) ________________________ (print first name) ____Solution____
9:30 AM, Tuesday April 27, 2004, 105 Nedderman Hall
80 minutes allowed (last four digits of your student #) _____________ (e-mail if new)______________________
Instructions: Seat Number ____________

1. Do your own work. DO NOT REMOVE THE STAPLE ON THIS EXAM.
2. You may use a legal copy of the text by Massobrio and Antognetti. You may write notes in your text. You may NOT pass a book or note sheet to another student, or class notes or previously solved problems. **You may use your Project 1 solution and must submit it with this test in your exam packet**.
3. Calculator allowed. You may NOT share a calculator with another student.
4. Where values or equations are given on this cover sheet, use them in lieu of any other source. If a value is not given, explicitly state definitions and assumptions that you use.
5. Where possible, calculate parameters rather than read them from a graph.
6. Do all work in the spaces provided on this exam paper. If you write on the back of a sheet, make the notation "PTO" in your solution in order to assure that material written on the back of the page is evaluated for a grade. AN EXTRA BLANK SHEET IS ATTACHED AT THE BACK OF THE EXAM.
7. Show all calculations, making numerical substitutions and giving numerical results where possible.
8. **The total for the test is 75. Up to 25 additional points will be given for the Project report**.
9. Unless stated otherwise,
   $T = 300K$, $V_t = 25.852$ mV
10. Unless otherwise stated, the material is silicon (300K) with
    $n_i = 1.45E10\ cm^{-3}$ $N_c = 2.8E19\ cm^{-3}$ $q\chi_{Si} = 4.05$ eV
    $E_{g,Si} = 1.124$ eV. $N_v = 1.04E19\ cm^{-3}$
11. For the work function of poly silicon, use $\phi_{n+} = \chi_{si} = 4.05$ V
    $\phi_{p+} = \chi_{Si} + E_{g,Si}/q = 5.174$ V.
12. For minority carrier (either electrons or holes) lifetime in silicon, use the relationship
    $\tau_{min} = (4.5E\text{-}5\ sec)/(1 + N_i/1E17 + (N_i/5E17)^2)$,
    where N_i = the total impurity concentration in cm^{-3}
13. For holes in silicon doped primarily with boron*, assume
    $\mu_p = \{470.5 \div [1+(N_i \div 2.23E17)^{0.719}]\}+44.9$, in cm^2/V-sec.
14. For electrons in silicon doped primarily with phosphorous*, assume
    $\mu_n = \{1414 \div [1+(N_i \div 9.2E16)^{0.711}]\}+68.5$, in cm^2/V-sec.
15. For electrons in silicon doped primarily with arsenic, assume
    $\mu_n = \{1417 \div [1+(N_i \div 9.68E16)^{0.68}]\}+52.2$, in cm^2/V-sec.
    (In 12 through 15, N_i = the total impurity concentration in n- or p-type material, compensated or not.)
    (*13 may be used as an approximation for holes as minority carriers, likewise *14 for minority electrons.)
16. Metal gate work functions should be assumed to be
    $\phi_{M,Al} = 4.1$ V for aluminum, $\phi_{M,Pt} = 5.3$ V for platinum, $\phi_{M,Au} = 4.75$ V for gold
17. The electron affinity of SiO_2 is $\chi_{SiO2} = 1.00$ V.
18. Planck constant $h = 6.625E\text{-}34$ J-s = 4.135E-15 eV-s, (1 eV = 1.602E-19 Joule).
19. free electron mass $m_o = 9.11E\text{-}28$ g.
20. Boltzmann constant, $k = 1.38066E\text{-}23$ J/K
21. Electron charge, $q = 1.60218E\text{-}19$ Coulomb
22. Permittivity of free space, $\varepsilon_o = 8.854E\text{-}14$ Fd/cm
23. Relative permittivity of silicon, $\varepsilon_r = 11.7$
24. Relative permittivity of silicon dioxide, $\varepsilon_{rOx} = 3.9$
25. The breakdown voltage of an abrupt (step) junction (asymmetrical or one-sided) diode with doping on the lightly doped side of N_B is $V_B = 60(Eg/1.1)^{3/2}\ (10^{16}/N_B)^{3/4}$ V. The critical field for breakdown is modeled as $E_{crit} = (120V \cdot qN_B/(\varepsilon_r\varepsilon_0))^{1/2} \cdot (Eg/1.1)^{3/4} \cdot (10^{16}/N_B)^{3/8}$
26. Each part is worth [x] points, as given in the problem.

1. This question refers to data taken for a forward gummel setup ($v_{CBext} = 0$, 350 mV < v_{BEext} < 950 mV).

a. [5] A particular set of data has the characteristic that $i_C*(dv_{BEext}/di_C)$ has a nearly constant value of 26 mV for 350 mV < vBEext < 450 mV. What is the best value to assume for the extracted value of NF?

Ic*(dvBEext/diC) = dvBE/d(lniC) = NF*Vt = 26 in this region,
since Vt = 25.852 mV, NF ~ NFeff = 26/25.852
NF = 1.005725

b. [5] At v_{BEext} = 400 mV, i_C = 10 nA. What is the best value to assume for the extracted value of IS?

ISeff = exp[ln(i) - vBEext/(Neff*Vt)] = iC*exp(-vBEext/NeffVt) = 10nA*exp(-400/26)
ISeff = 2.0823E-15

c. [5] There are two ways one can obtain an extracted value for the parameter BF. One can extract the value ISeff ~ IS/BF from the ISeff function applied to the i_B data in the range where $i_B*(dv_{BEext}/i_B)$ is minimum. One can also extract BF directly from the maximum value observed for the beta = i_C/i_B ratios. Use the G-P static model equations to determine which is likely to be the best method.

In the first case, (IS/BF)eff1 is calculated from data which theoretically represents iB = [IS/BF]*exp[vBE/(NFVt)] + ISE*exp[vBE/(NEVt)]. The most accurate estimate for NF is NFeffiC from the iC data (which is not equal to NFeffiB), so using this as NFeff, we have

$$\begin{aligned}(IS/BF)eff1 &= iB * \exp[-vBEext/(NeffVt)]\\ &= \left(\frac{IS}{BF}\exp\left(\frac{vBE}{NF \bullet VT}\right) + ISE\exp\left(\frac{vBE}{NE \bullet VT}\right)\right) * \exp\left[\frac{-vBE}{Neff1 \bullet VT}\right]\\ &= \frac{IS}{BF}\left[1 + \exp\left(\frac{vBE}{NF \bullet Vt}\left(1 - \frac{NF}{NFeff1}\right)\right) + \frac{ISE \bullet BF}{IS}\exp\left(\frac{vBE}{Vt}\left(\frac{1}{NE} - \frac{1}{NFeff1}\right)\right)\right]\end{aligned}$$

which, although the leading term is IS/BF from iB1, has a complex dependence on ISE and NE and includes a term in the difference between NF and NFeff. In the second case,

$$beta = \frac{iC}{iB} = \frac{IS\exp\left(\frac{vBE}{NF \bullet VT}\right)}{\frac{IS}{BF}\exp\left(\frac{vBE}{NF \bullet VT}\right) + ISE\exp\left(\frac{vBE}{NE \bullet VT}\right)} = \frac{BF}{1 + BF\frac{ISE}{IS}\exp\left(\frac{vBE}{Vt}\left(\frac{1}{NE} - \frac{1}{NF}\right)\right)}$$

so, beta = BF/(1+ILE/IBF) is only slightly less than BF, since IBF >> ILE in the range where beta is at it's largest value. Essentially the deviation is the same as that expressed in the ISE*BF/IS term in (IS/BF)eff1.

d. [5] Use the values you obtained in your extraction for Project 2 to support your conclusion in part c.

For the data given, $beta_{max}$ is 113.6, and $[IS/((IS/BF)eff1)]_{max}$ = 95.66. Considering the analytical characteristics of the response in part c, the larger value is better. This conclusion is correct as the data were generated using BF = 123.

2. The data provided for h11 was taken for a range of i_B values in the forward active mode (i.e. $v_{BE} > 0$ and $v_{BC} < 0$). The collector to substrate bias was chosen so this junction is reverse biased by 2 or 3 volts.

a. [5] The intent of this data is to generate a network from the small signal model of the bjt so that h_{11} is primarily determined by the base-emitter branch of the G-P model. Why is the substrate to collector junction reverse biased and why is the h_{11} parameter measured?

> Vcs is chosen to reverse bias the collector-substrate junction and thus reduce the value of Cjs as much as possible. H11 is defined as (dvbe/dib)vbc=0. For these conditions, the small-signal equivalent circuit becomes RBB in series with the parallel combination of r_π, C_π and C_{je}, in series with RE. Consequently, H11 samples the desired branch primarily.

b. [5] Consideration of the small-signal model leads one to conclude that $\mathrm{Re}\{h_{11}\} = RBB(i_B) + RE$ for $f \gg f_c$, and $\mathrm{Re}\{h_{11}\} = RBB(i_B) + r_\pi + RE$ for $f \ll f_c$. Assuming the junction capacitance can be neglected, define the characteristic frequency, f_c in terms of IS, BF, NF and TF (the forward base transit time).

> From a, we see that at frequencies much less than $f_c = 1/(2\pi RBB(C_\pi + C_{je}))$, h11 = RB + r_π + RE, since the capacitance are primarily open circuit. At frequencies much higher than f_c, we have h11 = RB + RE, since the capacitances are essentially a short-circuit
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> If the junction capacitance can be neglected, the characteristics frequency, f_c, is $f_c = 1/(2\pi RBB C_\pi)$, where C_π is: C_π=TF*q*IS*exp(vBE/kT)/kT

3. The model for RBB is given by the following equations:

RBB = RBM + (RB-RBM)/QB, $Q_B = \left(1 + \frac{V_{B'C'}}{VAF} + \frac{V_{B'E'}}{VAR}\right) \bullet \left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{BF \bullet I_{BF}}{IKF} + \frac{BR \bullet I_{BR}}{IKR}}\right)$, (when IRB = 0).

a. [5] If RB = 1.2 kOhm and RBM = 120 Ohm, for forward gummel data, at BF*i_B ~ BF*I_{BF} = iC*Q_B = IKF, find the value of RBB. Neglect the VAF, VAR terms in Q_B.

> In the limit described above, $Q_B = \left(\frac{1}{2} + \sqrt{\frac{1}{4} + 1}\right) = \frac{\sqrt{5}+1}{2} = 1.618$, so
>
> RBB = 1.2k+(1.2k-120)/1.618 = 787.5 Ohm at iB ~ IKF/BF

b. [5] When IRB > 0, the model for RBB is given by the following equations:

RBB = RBM + 3(RB-RBM)*(tan(z)-z)/(ztan^2(z)), $z = \frac{\pi^2}{24}\sqrt{\frac{IRB}{i_B}}\left[\sqrt{1 + \frac{144 i_B}{\pi^2 IRB}} - 1\right]$. What is the value of RBB at i_B = IRB?

> In the case of ib = IRB, RBB = (RB+RBM)/2 = 660 Ohms.

3c. [5] How could one decide whether to use the IRB = 0 or IRB > 0 model based on the data given? Based on this, justify the value you chose for IRB in your solution to Project 2.

> The question is whether the apparent halfway transition from RB to RBM is at IKF/BF. If so, the IRB = 0 model is appropriate. Looking at the beta data, beta falls to 50% beta$_{max}$ when the currents are iC = 18 mA and iB = 0.347 mA. Looking at the h11 data, we see h11 ~ (RB+RBM)/2 when iB = 1 μA, which is much less than IKF/BF.

Consequently, the IRB > 0 (~ 1 μA) model should be chosen.

4. Another method to obtain RB and RE data is the reflyback method. Base current is measured for a range of base-emitter voltages with the collector open-circuited (iC = 0). In this case, it is <u>assumed</u> RE ~ dvCE/diB and RBM ~ d(vBC)/diB.

a. [5] Why is the value for RBB assumed to approximate RBM?

The data are taken at high currents, hence RBB ~ RBM for either model

b. [5] Use the model equations to develop reasons this method is inferior to the h11 method?

The problem is that for iC = 0, vC is neither the node voltage at the internal base node B′ or the internal emitter node at E′. Using the GP definition for
iC = (IS/QB)*[exp(vBE/NFVT)-exp(vBC/VRVt)] – (IS/BR)*[exp(vBC/NRVt)-1] - ISC*[exp(vBC/NCVt)-1],
it is clear that for iC = 0, neither vBC nor vBE is zero, so the equations above cannot be correct.

5. Define the circuits and input sweeps used to take the data for extraction of

a. [5] VAF and VAR.

To extract VAF, use the forward early circuit. In the circuit, set vBE: 0.7V<vBE<0.9V, sweep vCE from 0.2V to 5V to obtain the collector current as a function of vCE.
To extract VAR, use the reverse early circuit. In the circuit, set vBC: 0.7V<vBC<0.9V, sweep vEC from 0.2V to 5V to obtained the emitter current as a function of vEC

b. [5] Give the equations to generate the effective VAF and VAR extraction values.

VAFeff=iC/[∂iC/∂vBC]vBE
VAReff=iE/[∂iE/∂vBE]vBC

5c. [5] What are the conditions on vBE, vBC, etc., for extracting VAF ~ VAFeff ***or*** VAR ~ VAReff?

The most accurate VAF value is obtained at vBC=0
The most accurate VAR value is obtained at vBE=0

d). [5] In VAF extraction, we typically measure the data by setting VBE in the range between 0.7V to 0.9V. Why? What is the effect if we significantly increase and/or decrease this range?

Because we want to avoid the high-level injection and recombination current in the measured current data. In the VAF, if VBE is too high, high level inject current will significantly contribute to the collect current. Thus, the equation $i_C = \frac{I_S}{Q_B} e^{\frac{V_{BE}}{NV_T}}$ only gives a poor approximation. The extracted VAF will have large error. Similar situation happens if VBE is too small because of the recombination current.