Nonlinear Design of Three Axes Autopilot for a Tactical Aerospace Vehicle

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Abstract- The nonlinearity and coupling of the aerospace vehicle dynamics can be linearized and decoupled by well known feedback linearization techniques. The fundamental problem of feedback linearization technique is that it is not able to handle the model uncertainty. Significant research works have been carried out in order to overcome the above aspect. In this paper, a new robust nonlinear controller structure using feedback linearization technique is proposed. The design is carried out in two time scale separation. The fast dynamics (inner body rate loop) and slow dynamics (outer lateral loop) are controlled separately by two nonlinear controllers. The performances of the design in terms of robustness, minor coupling between the longitudinal motion and lateral motion, stability and guaranteed tracking are shown in a realistic 6-DOF simulation.

I. INTRODUCTION

The basic requirement for an autopilot is fast response because of the short amount of time involved in the end game. Secondly minimum tracking error is an obvious requirement if the interceptor is to achieve a hit to kill miss. Finally robustness w.r.t model uncertainties and decoupling between longitudinal and lateral motion are important for the interceptor to achieve its objective in the actual real life environments.

The highly nonlinear nature of the aerospace vehicle dynamics [9, 12] due to the severe kinematics and inertial coupling of the vehicle aerodynamics is a challenge to the autopilot design which is required to have satisfactory performance for all flight conditions catering for different engagements.

Classically, interceptor autopilots are designed using linear control approaches [10]. Traditional approach is to first linearize the vehicle short period dynamics for each axis about an operating condition, and then to apply linear control theory to synthesize a feedback controller for each axis separately. This process is repeated at multiple operating conditions and the controllers are then scheduled with respect to the flight conditions. The assumption in the classical approach is not valid at high angle of attack because of dynamic parameter uncertainties and coupling between lateral and longitudinal motion. Severe roll rate coupling can destabilize the lateral autopilot. It may cause control saturation and may cause track loss of the interceptor homing head as target tracker or seeker is having limited gimbal freedom from physical considerations.

A natural idea for handling the nonlinear dynamics is to design the autopilot on the basis of the more accurate nonlinear model, thus leading to nonlinear autopilots. One of such strategies is well known feedback linearization [7,8] approach which uses the feedback and or coordinates transform to linearize the nonlinear system. Then it addresses the design issues on the linearized system thus obtained.

Devaud E., Siguerdidjane H. and etc [1,2] presented a three axe interceptor autopilot design using classical linear time invariant control and static and dynamic approximate input output linearizing feedback control. In a realistic test bed, it performs well for nominal plant but performance degrades significantly with model uncertainty level. Menon P. K. and etc. [3,4] presented the linearizing transformation technique to the autopilot design of air to air interceptor systems. They have shown performances fewer than ±10% perturbations in aerodynamic force and moment models. Huang J., Lin C.F., J.R. Cloutier etc [5] investigates the feasibilities of autopilot design for highly maneuverable bank to turn vehicle using feedback linearization based approach. Here, assumption is that coupling between force and control deflection is neglected. All these paper works show the comparable performances for well known model. But model uncertainty is a challenge. Most of the paper works show after feedback linearized plant, a robust controller or sliding mode controller is used to handle plant uncertainty. But this scheme also degrades when degree of uncertainty is high [6].

In the approach of the research, the nonlinear controller is designed corresponds to a basic application of linearizing techniques combined with input output linearization and time scale separation. The fast dynamics (inner loop) and the slow dynamics (outer loop) are linearized based on the input-output feedback linearization technique. The model uncertainty is handled by designing a new controller structure for both the loops. It gives the property of decoupling, robust in presence of model uncertainty and satisfactory performances.

The main feature of the research is the consideration of a three-axis time variant nonlinear interceptor model. The vehicle aerodynamic parameters are used into a look up table form. Full inertia tensor, sensors and actuators modeling with physical limit are also taken into account.

I. MATHEMATICAL MODEL OF PLANT DYNAMICS

There are six equations of motion with six degree of freedom consist of three translations and three rotations along and about
the interceptor \([x_b, y_b, z_b]\) axes. The nonlinear differential equations for the interceptor are written in fin frame as [11]:

\[
\dot{U} = rv - qw + \frac{1}{m} [T_x - QSD D_0] - g_x \\
\dot{v} = pw - ru + \frac{1}{m} [T_y + QS C_{NB} + C_{l_h} \delta_y + \frac{D}{2V_m} (-C_{\gamma} \beta + C_{\gamma r})] - g_y \\
\dot{w} = qU - pv + \frac{1}{m} [T_z + QS C_{NA} - C_{l_h} \delta + \frac{D}{2V_m} (C_{\eta} q - C_{\eta \dot{a}})] - g_z
\]

\[
\hat{p} = \frac{1}{l_{xx}} [-\hat{L} p + T_{mx} + QSD \frac{D}{2V_m} (C_{l_h} p - C_{l_h} \delta r + C_{\gamma})] \\
\hat{q} = \frac{1}{l_{yy}} [-\hat{L} q + T_{my} - (l_{xx} - l_{yy}) p r + QSD (-C_{mr} q \delta p + C_{MA} + \frac{D}{2V_m} (C_{mq} q + C_{m\dot{z}} \dot{a}))]] \\
\hat{r} = \frac{1}{l_{zz}} [-\hat{L} r + T_{mz} - (l_{xx} - l_{yy}) p q + QSD (-C_{mr} q \delta r + C_{MB} + \frac{D}{2V_m} (C_{mr} r + C_{m\dot{z}} \dot{a})])]
\]

In these expressions, \([U, v, w]\) is the linear velocity, \([p, q, r]\) is the angular rate and \([\delta, \delta_p, \delta_r]\) is control deflection derived from four actuator deflections. The aerodynamic coefficients are used in terms of look up table form and the nomenclatures are given as: 

- \(C_{l_0}\): axial force or drag coefficient, 
- \(C_{NA}\) and \(C_{NB}\) are the force coefficient (component of normal and side force coefficient) along pitch and yaw axis. Similarly \(C_{MA}\) and \(C_{MB}\) are the Moment Coefficient. 
- \(C_\gamma\): Rolling Moment Coefficient. 

All are used as a function of mach no, resultant angle of attack and maneuver plane roll orientation. The control force effectiveness \((C_{\delta}, C_{i_0}, C_{i_e})\) are function of mach no and control deflection \(\delta\) and used as a look up table form.

The pitching moment of the interceptor is coupled to the yawing motion on account of roll rate. Again yawing motion induces forces in the pitch plane if roll motion is present. This is undesirable as moderate roll rate can unstaclble the lateral autopilot due to cross coupling.

II. NONLINEAR CONTROLLER WITH TIME SCALE SEPARATION.

A two time scale static linearizing control law may be designed on an approximate model of the interceptor. In this approach, state \(p, q\) and \(r\) (body rates) are identified as faster dynamic responses, while, \(\alpha, \beta\) and \(\phi\) are characterized as slow state variables. The angular rates \(p, q\) and \(r\) strongly depend upon the fin deflection \(\delta\). Thus to start with a fast state controller for body rate \(p, q\) and \(r\) are designed. Having designed a fast state controller, a separate approximate inversion procedure is carried out to design the slow state controller for \(\alpha, \beta\) and \(\phi\). It may be noted that, such a model reduction method was possible as there is significant difference in the time scale between the fast and slow state in the open loop dynamics of interceptor. As stated earlier the design of this controller depends on two time scale separation. The outer loop is slow dynamics and the inner loop is fast dynamics. The outer loop controller takes the commanded acceleration and current acceleration as input and generates the rate command which works as an input to the inner loop. The basic block diagram of the nonlinear controller is shown in figure-1.

III. INNER LOOP CONTROLLER DESIGN

The purpose of the inner loop design is to decouple the pitch roll and yaw channel and the equivalent transfer function between the control new input \(V_i\) and output \(y_i\) (body rates \(p, q\) and \(r\) ) is that of an integrator. A nonlinear Luenberger observer [13] is also designed to estimate the states for dynamic feedback control law. Here, there are three inputs and three outputs available. Let the nonlinear system is defined as

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= h(x)
\end{align*}
\]

Here, \(x = [U, v, w, p, q, r]^T\) is 6x1 the state vector,
\(u = [\delta_r, \delta_p, \delta_y]^T\) is the 3x1 control input vector and \(y = [p, q, r]^T\) is the 3x1 vector of system output. Referring to equations (1), relative degree is [1,1,1], it may be redefined that \([p, q, r]^T = M_f(x) + E_f(x)[\delta, \delta_p, \delta_y]^T\) where \(M_f(x) = \begin{bmatrix} L_1 h_1 & L_2 h_2 & L_3 h_3 \\ L_1 h_2 & L_2 h_2 & L_3 h_2 \\ L_1 h_3 & L_2 h_3 & L_3 h_3 \end{bmatrix}\) and the decoupling matrix \(E_f(x) = \begin{bmatrix} L_{g_1} h_1 & L_{g_2} h_1 & L_{g_3} h_1 \\ L_{g_1} h_2 & L_{g_2} h_2 & L_{g_3} h_2 \\ L_{g_1} h_3 & L_{g_2} h_3 & L_{g_3} h_3 \end{bmatrix}\)
The 3×3 matrix $E_j(x)$ is invertible over the region. Then the input transformation

$$\begin{bmatrix} \delta_r & \delta_p & \delta_y \end{bmatrix}^T = -E_j^{-1}(x) [V_I - M_I(x)] \quad \text{--- (4)}$$

The suffix $I$ stands for inner loop. The above three equations of the simple form $\begin{bmatrix} \dot{\rho} & \dot{\varphi} & \dot{\theta} \end{bmatrix}^T = [v_{11} \ v_{12} \ v_{13}]^T$. Hence the control law decouples the longitudinal motion and lateral motion. The inner loop control structure is shown in figure-2.

In practice, the system description is rarely known exactly. The model uncertainty is not able to cater by the feedback linearization control law as shown in equation-(4). Now our objective is how to control the plant uncertainty. Basically after feedback linearized plant, a robust controller is designed to handle the plant uncertainty in most of the papers. In this research, a robust feedback linearization scheme based on model reference adaptive control is proposed to handle the model uncertainty. The scheme shows significant performances and robustness of the scheme will be presented through an exhausted 6-DOF simulations.

III ROBUST FEEDBACK LINEARIZATION CONTROLLER

For simplicity, let we denote the nominal plant (model) output is $y_M$ and actual plant output is $y$. The output equation may be expressed as

$$\begin{align*}
\dot{y} &= M_I(x) + E_I(x)u \\
\dot{y}_n &= M_{In}(x_n) + E_{In}(x_n)u
\end{align*} \quad \text{--- (5)}$$

The model uncertainty is included in $M_I(x)$ and $E_I(x)$. The input output relationship of the nominal plant as discussed earlier is $y_n = V_{In}$, where the relationship between the linear control $V_{In}$ and the plant input $u$ is given by $u = E_{In}^{-1}(x_n) [V_{In} - M_{In}(x_n)]$. Our objective is to derive a new linear control $v_I$ and the relationship between this new input $V_I$ to FBLC (feedback linearization controller) and actual plant input $u$ such that it will linearize and decouple the output equation in the form $\dot{y} = V_I$. Now doing some mathematical formulation, we can express $V_I$ in terms of $V_{In}$ and as well as in terms of $u$ as follows

$$V_I - V_{In} = \dot{y} - \dot{y}_n \Rightarrow V_{In} = V_I - (\dot{y} - \dot{y}_n) \quad \text{--- (6)}$$

Hence, the control input is obtained as

$$u = E_{In}^{-1}(x_n) [V_{In} - (\dot{y} - \dot{y}_n) - M_{In}(x_n)] \quad \text{--- (7)}$$

The inner loop block diagram with robust feedback linearization control law is shown in figure-3. The same input signal $u$ goes to model as well as plant. The actual output is compared with the expected output and the error signal $(\dot{y} - \dot{y}_n)$ is feedback to the reference input $V_I$. Hence, the equivalent transfer function between the input signal $V_I$ and
the actual output \( y \) is just like an integrator. It may be noted that under perturbed cases also, the robust scheme holds the linearity and decoupling properties.

![Diagram](Image)

**Figure-3: Modified feedback linearization controller**

### IV. UNCERTAINTY LEVEL WITH NEW CONTROLLER

If no constraint on control input \( u \), then the above modified scheme looks like an integrator between the plant output \( y \) and new linear controller input \( V_I \) for any level of uncertainty. In practice, the actuation servo has deflection limitation and finite rate limitation. Here, we will derive the limit of uncertainty bound for which the actuation system operates in linear zone. Let us rewrite the plant output equation

\[
\dot{y} = (M_{In} + \Delta M_{In}) + (E_{In} + \Delta E_{In})u \quad \text{--- (8)}
\]

Where, \( M_{In}, E_{In} \) represent the nominal part of the plant while \( \Delta M_{In}, \Delta E_{In} \) represent the uncertainty. After a little derivation, it may be shown that

\[
u = u_n + \left[ \Delta \dot{y}_n - \frac{\Delta M_{In}}{E_{In} + \Delta E_{In}} \right] \quad \text{--- (9)}
\]

where, \( u_n \) is the control requirement for nominal plant and \( \Delta \dot{y}_n = \dot{y} - \dot{y}_n \). Let the maximum control deflection is \( u_{\text{max}} \). Therefore, the bound of uncertainty should lie such that

\[
\frac{\Delta \dot{y}_n - \Delta M_{In}}{E_{In} + \Delta E_{In}} \leq u_{\text{max}} - |u_n| \quad \text{--- (10)}
\]

### V. LINEAR CONTROLLER DESIGN OF INNER LOOP

The inner loop of the dynamic inversion control law controls the fast states \( p, q \) and \( r \). This loop calculates the virtual control surface deflection commands \([v_1, v_2, v_3]\) from the rate commands \([\phi, f_z, f_y]\) given by the slow dynamics \([\dot{\phi}, f_z, f_y]\). In any controller, desired dynamics must be imposed to obtain the desired result. There are many ways to design the desired dynamics based upon robustness requirement. Let us assume that the desired dynamics between the demanded body rate and sensed body rate is first order, where the time constant or closed loop BW is the designed parameter. The desired dynamics used are given by

\[
\dot{p} = \omega \, f_p \left( p_d - p \right), \quad \dot{q} = \omega \, f_q \left( q_d - q \right) \quad \text{and} \quad \dot{r} = \omega \, f_r \left( r_d - r \right)
\]

where \( \omega \, f_p, \omega \, f_q \) and \( \omega \, f_r \) are the BW of the inner rate loop.

Hence may be written that

\[
\begin{align*}
\dot{v}_1 &= \omega \, f_p \left( p_d - p \right) \\
\dot{v}_2 &= \omega \, f_q \left( q_d - q \right) \\
\dot{v}_3 &= \omega \, f_r \left( r_d - r \right)
\end{align*}
\]

--- (11)

### VI. OUTER LOOP NONLINEAR CONTROLLER DESIGN

The command to the outer loop is the demanded lateral generated by the guidance law as per interceptor target kinematics. The outer loop basically controls the flight path rate of the interceptor. The outer loop generates the body rate demand to the inner loop. Therefore, the plant considered for outer loop which input is body rate and output is lateral acceleration. Referring the governing equation, it may be noted that, the output equation of lateral acceleration is not able to control directly through \( p, q \) and \( r \) as the main component of body rate term included in force coefficient term \( C_{Nq} \) and \( C_{Np} \). Hence, the acceleration is not the state of interceptor; the acceleration command must be changed in angle of attack \( \alpha \) command and side slip \( \beta \) command. But \( \alpha \) and \( \beta \) are not the measurable quantities, an observer [13] is imposed to find out \( \alpha \) and \( \beta \) from the measurable quantities \( f_x \) and \( f_y \). The system output equations \( f_x \) and \( f_y \) are given as

\[
\begin{align*}
f_z &= \rho \, V_m^2 \, S_{ref} \, \alpha \left( 1 - \frac{h}{l_c} \right) \\
f_y &= \rho \, V_m^2 \, S_{ref} \, \beta \left( 1 - \frac{h}{l_c} \right)
\end{align*}
\]

--- (12)

In the expression, \( \rho \) is the air density, \( S \) is the reference area, \( m \) is the mass, \( h \) is the static margin and \( l_c \) is the control moment arm. Differentiating the above equation, we can write

\[
\begin{align*}
\dot{f}_z &= \frac{\partial f_z}{\partial \alpha} \alpha + \frac{\partial f_z}{\partial \alpha} \alpha \\
\dot{f}_y &= \frac{\partial f_y}{\partial \beta} \beta + \frac{\partial f_y}{\partial \beta} \beta
\end{align*}
\]

--- (13)

Therefore, the demanded lateral may be converted into equivalent angle of attack command. Once the \([\alpha, \beta, f_c] \) are known, the slow state can be inverted to get the commanded rates as the nonlinear controller structure is shown in figure-4.

The slow dynamics can be expressed in the form of

\[
\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix}^T = \hat{M}_{\alpha} + \hat{E}_{\alpha} \left[ \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right]^T \\
\Rightarrow \left[ \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right]^T = \hat{E}_{\alpha}^{-1} \left[ \begin{bmatrix} \alpha \\ \beta \\ \phi \end{bmatrix} \right]^T \quad \text{--- (14)}
\]
Where, $v_o$ is the linear control output of the outer loop. The above equations linearize the slow dynamics. The above input-output relation is decoupled, in addition to being linear. Since new input $v_{oi}$ only affects the corresponding output, but not the others. This control law is also known as decoupling control law or non-interacting control law. But this decoupling law as stated above does not guaranteed the robustness in the presence of parameter uncertainty or unmodelled dynamics. The problem is due to the fact that the exact model of the nonlinear system is not available in performing feedback linearization. The sensitivity to modeling errors may be particularly severe when the linearizing transformation is poorly conditioned. To overcome the problem, the same concept of RFLC technique as described in case of inner loop design is applied here and shown in figure-5.

![Figure-4: Structure of the outer loop of the autopilot](image)

![Figure-5: Outer loop controller structure with robust feedback linearization scheme](image)

**VII SIMULATION RESULTS**

The design is thoroughly validated in a full scale 6-DOF platform considering all the plant parameter variations. For off nominal conditions, the following plant uncertainties are considered. $\Delta v_m = \pm 21\%$, $\Delta C_N$, $\Delta C_S = \pm 10\%$, $\Delta C_l = 100\%$, $\Delta C_{\eta, \zeta} = \pm 10\%$. Plant inertia variation is $\pm 10\%$. All are taken in additive sense. A conventional linear three loop lateral autopilot [10] and a PI type roll [9] autopilot are also designed to establish a base line for control system performance comparison. All the cases, the nominal as well as off nominal cases, the performances of the nonlinear autopilot are carried out and compared with linear controller. The dynamic profiles are scaled and then presented here.

**A. Autopilot Performances by Applying Step Body Rate Demand**

In 6-DOF, body rate is forced and are applied to inner loop as demand for different time intervals for different channels.
Figure -6 shows the comparative body rate responses for off-nominal condition.

Figure-6 : Body rate profiles with forced body rate demand

It may point out from the figure that the ignition of body rate in one channel is not affects the other two channels. It proves the evidence of decoupling. But, linear controller fails to decouple the motions. The autopilot response is almost critically damped in nature.

B. Autopilot Performances by Applying Step Latax Demand

Latax demand is forced as outer loop demand for pitch and yaw channel, whereas, roll channel demand is zero.

Figure-7 : A/P Performances for step latax demand

A step latax demand is applied simultaneously for both pitch and yaw plane for off-nominal condition. The corresponding latax profiles are shown in figure-7. The body rate profiles are shown in figure-8. Here, also significant amount of decoupling among the three axes may be found compared to linear controller. The roll rate is almost silent for the nonlinear controller which is the main objective of the design. The corresponding control deflection profiles are shown in figure-9.

Figure-9: Control deflection profiles for step latex demand

C. Autopilot Performances by Applying latex as per closed loop guidance

Latax demand is applied as generated by the guidance law for a typical flight trajectory with one pitch down maneuver and followed by a high sustained pitch up maneuver. A Gaussian noise of three sigma value $\sigma = 0.5m/s^2$ is
considered for both accelerometer outputs. The latex profiles are shown in figure-10. Body rate profiles are shown in figure-11.

Figure-11: Body rate profiles for a typical flight trajectory

Figure-12: $\alpha, \beta$ profiles for a typical flight trajectory

The corresponding alpha and delta profiles are shown in figure-12 and figure-13.

Figure-11: $\delta$ profiles for a typical flight trajectory

Here also, significant decoupling and smoother responses are found for nonlinear controller even during the high maneuver phases. The summary of performances considering nominal and off nominal conditions are shown in table-1.

<table>
<thead>
<tr>
<th>Performance Index</th>
<th>Performances of Nonlinear A/P w.r.t Linear A/P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>1.2 times faster</td>
</tr>
<tr>
<td>Time Constant</td>
<td>1.2 times faster</td>
</tr>
<tr>
<td>Latax overshoot</td>
<td>1/5 times lesser</td>
</tr>
<tr>
<td>Steady state error</td>
<td>1/10 times lesser</td>
</tr>
<tr>
<td>Max roll rate</td>
<td>1/20 times lesser</td>
</tr>
<tr>
<td>Control deflection</td>
<td>40% lesser*</td>
</tr>
</tbody>
</table>

(* For linear controller, due to roll induced yaw rate, more control efforts are required)

CONCLUSION

This research demonstrates the design of a nonlinear controller using feedback linearization technique for the aerospace vehicle control problem associated with severe coupling of longitudinal and lateral motion. A new controller structure is proposed to tackle significant plant uncertainty. The nonlinear autopilot design methodology is carried out in two time scale separation. The design is validated here through a full scale 6-DOF simulation and a comparison is also made with linear traditional controller. A significant performances improvement may found for nonlinear controller. These promising results may provide a new avenue to meet the ever stringent performance requirements of modern and future flight vehicle.

REFERENCES

