DYNAMIC MODELING OF THERM. ACTUATORS

HISTORY AND PREVIOUS WORK – A lot of activity:

- transient FEA, identification techniques (ARX model) third-order autoregressive-moving average difference equation - experimental, reduced order modeling, Static FEA to get Dynamic model [Senturia]

ISSUES:
1. Repeatedly done Transient FEA is slow.
2. Experimentally Obtained Dynamic Model – require actual device.

WE WANT
1. Practical & fast development of the simple dynamic model (static FEA) – easy to use (ODE - MATLAB).
2. To develop easy-to-use tables connecting geometrical parameters of the actuator to the parameters of the dynamic model
APPROACH

1. Define the Structure of the Lumped Model

2. Establish Finite Elements Model (FE) – for electrothermal (ET) and mechanical (M) simulations - as accurate as possible

3. Do ET Simulation for several voltages – record thermal fluxes through areas of interest, temperature distributions.

4. Define average temperature for each ET step – calculate corresponding thermal capacitances and conductances.

5. Apply temperature distribution as a load to the static mechanical FEA – get deflections.

6. Do modal analysis - to check thermal & mechanical band overlapping.
LUMPED MODEL

\[ q = G_e(\bar{T})v^2 \]

\[ C_{th}(\bar{T})\ddot{\bar{T}} + G_{cond}(\bar{T})\Delta\bar{T} + G_{conv}(\bar{T})\Delta\bar{T} + G_{rad}(\bar{T})\Delta\bar{T} = q \]

\[ \bar{m}(\bar{T})\ddot{x} + d\dot{x} + k(\bar{T})x = k\alpha(\bar{T}) \]
ELECTRO-THERMAL FE MODEL – PARAMETER EXTRACTION

\[
\sum_{i=1}^{N} q_i = 0
\]

\[
q_{\text{conv}} = h \sum_{i=1}^{K} T_i A_i
\]

\[
q_{\text{rad}} = \sigma \varepsilon F \sum_{i=1}^{K} A_i \left( T_i^4 - T_{\text{room}}^4 \right)
\]

\[
q_{\text{cond}} = q - q_{\text{conv}} - q_{\text{rad}}
\]

\[
G_{\text{cond}}(\bar{T}) = \frac{dq_{\text{cond}}(\bar{T})}{d\bar{T}}
\]

\[
G_{\text{conv}}(\bar{T}) = \frac{dq_{\text{conv}}(\bar{T})}{d\bar{T}}
\]

\[
G_{\text{rad}}(\bar{T}) = \frac{dq_{\text{rad}}(\bar{T})}{d\bar{T}}
\]

\[
G_e(\bar{T}_j) = q(\bar{T}_j) v_j^{-2}
\]
MECHANICAL FE MODEL – PARAMETER EXTRACTION

\[ \ddot{m}(\bar{T}) \ddot{x} + d \dot{x} + k(\bar{T}) x = k \alpha(\bar{T}) \quad \ddot{m}(\bar{T}) \ddot{x} + d \dot{x} + k(\bar{T}) x = \alpha_f(\bar{T}) \]

\[ \ddot{x} = \dot{x} = 0 \]

\[ x = \alpha(\bar{T}) \]

\[ x(\bar{T_j}) = \alpha(\bar{T_j}) \]

\[ k(\bar{T_j}) x_j(\bar{T_j}) + k(\bar{T_j}) \Delta x(\bar{T_j}) = \alpha_f(\bar{T_j}) + F_L \]

\[ \bar{m}(\bar{T_j}) = \frac{1}{x_j^2} \sum_{i=1}^{N} x_i^2(\bar{T_j}) m_i \]

\[ k(\bar{T_j}) = \frac{F_L}{\Delta x(\bar{T_j})} \]
$T = T_{room}$

$\Delta x = \Delta y = \Delta z = 0$

$F_{\mu N}$ vs $T_{avg} [K]$

$G_{th \, tot}$, $G_{th \, air}$, $G_{th \, poly}$ vs $T_{avg} [K]$

$x [\mu m]$ vs $T_{avg} [K]$
RESULTS - VERIFICATION

Extracted parameters (previous page) are used to build nonlinear model.

Obtained dynamic model, in the Ordinary Differential Equation (ODE) form, is easily simulated in MATLAB, enabling the subsequent design/investigation of the actuation signal.

Simulation (right) shows good alignment with experimental results.
THERMAL MODELING – CONCLUSIONS

1. Method - combined analytical and FEA approach – requires several simple static FEA to determine the parameters – states: Velocity, Deflection and Temperature.

2. The fictitious, average temperature is introduced as a state variable to preserve the thermal energy balance inside the model.

3. Resulting model – evaluated by conducting numerical integration of ODE.

4. It is flexible and enables a separate introduction of various phenomena and external forces.

5. Resulting parameters are intuitive and have physical meaning that can be easily related to the geometry and material properties of the device.

Detailed description of the method is given in: