Synchronization in Biology, Chemistry, and Physics

Collective Motion in Animals, Fish, and Insects

Add discussion. Refer to refs.

1.1 Collective Motion in Animal Groups

Equation Chapter 1 Section 1

The collective motions of animal social groups are among the most beautiful sights in nature. Each individual has its own inclinations and motions, yet the aggregate motion makes the group appear to be a single entity with its own laws of motion, psychology, and responses to external events. Flocks of birds, herds of animals, and schools of fish are aggregate entities that take on an existence of their own due to the collective motion instincts of their individual members. The aggregate turns and wheels to pursue its objectives such as seeking food or migrating, and to avoid predators and obstacles. Such synchronized and responsive motion makes one think of choreographed motions in a dance, yet they are a product not of planned scripts, but of instantaneous decisions and responses by individual members. In this section we study the mechanisms for such synchronized choreographic behavior of groups in terms of instantaneous decisions made by their members. The material in this section is from Reynolds [].

Figure. A flock of birds wheels in synchrony. Courtesy pbs.org

Figure. A flock of birds appears to be a single composite entity in its motion and behaviors. Courtesy watt-works.com
Figure. A flock of geese on migration. Courtesy Freepohoto.com

Figure. A school of fish in synchronized motion. Courtesy gmagazine.com.au

Figure. A school of fish responding as a single organism. Courtesy aquariumprosmn.com

Figure. A herd of bison on the move in search of food. Courtesy shelledy.mesa.k12.co.us

Figure. Wildebeest herd on migration. Courtesy arkive.org

Figure. Wildebeest stampede to avoid danger. Courtesy allposters.com
Collective motions allow the group to achieve what the individual cannot. Benefits of aggregate motion include defense from predators, social and mating advantages, and group foraging for food. In migrating bird flocks, the energy required by individual birds in flying is reduced by remaining in the wingtip vortex upwash of those ahead. Choreographed motions of groups of lions, wolves, and jackals allow more efficient pursuit of prey. In collective motion situations, the important entity becomes the group, not the individual. Such behavior has been observed in human crowd panic and mob situations, where there is pressure to follow the group’s collective lead, not think on one’s own [ref].

Reynolds’ Rules of Motion for Individuals in Collective Groups

To reproduce the collective motion of an animal group in computer animation has been a challenge. It would be impossible to script the motion of each individual using planned motions or trajectories. Analysis of groups based on social behaviors is complex, yet the individuals in collectives appear to follow simple rules that make their motion efficient, responsive, and practically instantaneous. The cumulative motion of animal groups can be programmed in computer animation by endowing each individual with the same few rules that allow it to respond to the situations it encounters. The responses of the individuals accumulate to produce the combined motion of the group.

In large groups, each individual is aware only of the motions of its immediate neighbors. The field of perceptual awareness of the individual changes for different types of animal groups and in different motion scenarios. In flocking motion, birds are aware only of a few neighbors ahead of them or beside them. In schools of fish, shock waves transmitted by the water allow individuals to be aware instantaneously of motions of neighbors that they may not even see. In animal herds, vibrations of the earth allow individuals to be aware of the general motion tendencies of others that may not be close by.

The collective motion of large groups can be captured by using a few simple rules governing the behavior of the individuals. Individual motions in a group are the result of the balance of two opposing behaviors: a desire to stay close to the group, and a desire to avoid collisions with other individuals [34]. Reynolds has captured the tendencies governing the
motions of individuals through his three rules.

**Reynolds’ Rules:**
1. Collision avoidance: avoid collisions with neighbors
2. Velocity matching, match speed and direction of motion with neighbors
3. Flock centering: stay close to neighbors

**Implementing Reynolds’ Rules in Dynamical Motion Systems**
Reynolds’ rules capture very well the collective motion of animal groups and can also be used as control schemes for human systems such as vehicle formations. Consider a set of $N$ agents $\{i \in N\}$ and let the communication between the agents be modeled by a general directed graph. There are many mechanisms for implementing Reynolds’ rules in dynamical systems control. Of importance is the definition of an individual’s ‘neighborhood’. Agents seek to avoid collisions. Therefore, define a circle of radius $\rho_c$ which each agent seeks to keep clear of other agents.

Define the collision neighborhood for agent $i$ as $N_i^c = \{ j: r_{ij} \leq \rho_c \}$ where the distance between nodes $i$ and $j$ is

$$
r_{ij} = |x_j - x_i| = \sqrt{(p_j - p_i)^2 + (q_j - q_i)^2} \tag{1.1.1}
$$

Define the interaction radius $\rho > \rho_c$ and the interaction neighborhood by $N_i = \{ j: r_{ij} \leq \rho \}$. It is noted that radii $\rho_c, \rho$ are different for different animal groups and different vehicles. Moreover, for some groups, such as flocks of birds in migration, the collision and interaction neighborhoods are not circular.

The dynamics used to simulate the individual group members can be very simple, yet realistic results are obtained. Consider agent motion in 2-D according to the dynamics

$$
\dot{x}_i = u_i \tag{1.1.2}
$$

with states $x_i = [p_i, q_i]^T \in \mathbb{R}^2$ where $(p_i(t), q_i(t))$ is the position of node $i$ in the $(x, y)$-plane. This is a simple point-mass dynamics with velocity control inputs $u_i = [u_{pi}, u_{qi}]^T \in \mathbb{R}^2$.

A suitable law for collision avoidance is given by

$$
u_i = - \sum_{j \in N_i^c} c_{ij} (x_j - x_i) \tag{1.1.3}
$$

which causes agent $i$ to turn away from other agents inside the collision neighborhood $N_i^c$. A suitable law for flock centering is given by

$$
u_i = \sum_{j \in N_i \setminus N_i^c} a_{ij} (x_j - x_i) \tag{1.1.4}
$$

which causes agent $i$ to turn towards other agents inside the interaction neighborhood $N_i$ and outside the collision neighborhood $N_i^c$. These protocols can be written in terms of the components of velocity as

$$
u_{pi} = \sum_{j \in N_i \setminus N_i^c} a_{ij} (p_j - p_i) \tag{1.1.5}$$
The relation between the collision avoidance gains $c_{ij}$ and the flock centering gains $a_{ij}$ determines different response behaviors insofar as agents prefer to flock or prefer to avoid neighbors. The relation between the radii $\rho, \rho_c$ is also instrumental in designing different behaviors.

Alternative protocols for collision avoidance and flock centering are given respectively by

$$u = -\sum_{j \in N_i \setminus N_{ij}} c_{ij} \frac{(x_j - x_i)}{r_{ij}} = -\sum_{j \in N_i \setminus N_{ij}} c_{ij} \frac{(x_j - x_i)}{|x_j - x_i|}$$

$$u = \sum_{j \in N_i \setminus N_{ij}} a_{ij} \frac{(x_j - x_i)}{r_{ij}} = \sum_{j \in N_i \setminus N_{ij}} a_{ij} \frac{(x_j - x_i)}{|x_j - x_i|}$$

which are normalized by dividing by the distance between agents. Note that the sum is over components of a unit vector. Therefore, these laws prescribe a desired direction of motion and result in motion of uniform velocity. By contrast, the laws (1.1.3), (1.1.4) give velocities that are larger if one is further from one’s neighbors.

These protocols guarantee consensus behavior in terms of $(x, y)$ position, yet do not include velocity control. Simple motions of a group of $N$ agents in the $(x, y)$-plane can alternatively be described by the node dynamics

$$\dot{p}_i = V_i \cos \theta_i$$

$$\dot{q}_i = V_i \sin \theta_i$$

where $V_i$ is the speed and $\theta_i$ the heading of agent $i$. Velocity matching can be achieved by using the local voting protocol to reach heading consensus according to

$$\dot{\theta}_i = \sum_{j \in N_i} a_{ij} (\theta_j - \theta_i)$$

and a second local voting protocol to match speeds according to

$$\dot{V}_i = \sum_{j \in N_i} a_{ij} (V_j - V_i)$$

As a third alternative, one can use the Newton’s law agent dynamics

$$\dot{x}_i = v_i$$

$$\dot{v}_i = u_i$$

with vector position $x_i \in \mathbb{R}^n$, speed $v_i \in \mathbb{R}^n$, and acceleration input $u_i \in \mathbb{R}^n$. This is more realistic than (1.1.2) for motion of physical bodies in $n$ dimensions. For motion in the 2-D plane, one would take $x_i = [p_i, q_i]^T$ where $(p_i(t), q_i(t))$ is the position of node $i$ in the $(x, y)$-plane. Consider the distributed position/velocity feedback at each node given by the second-order local neighborhood protocols

$$u_i = c \sum_{j \in N_i} a_{ij} (x_j - x_i) + c\gamma \sum_{j \in N_i} a_{ij} (v_j - v_i) = \sum_{j \in N_i} ca_{ij} ((x_j - x_i) + \gamma (v_j - v_i))$$

(1.13)
where $c > 0$ is a stiffness gain and $c\gamma > 0$ is a damping gain. This is based on local voting protocols in both position and velocity so that each node seeks to match all its neighbors’ positions as well as their velocities. Thus, this protocol realizes Reynolds’ rules for flock centering and velocity matching. A protocol such as (1.1.3) could be added for collision avoidance. Alternatively, offsets could be added for desired relative separation as in

$$u_i = c \sum_{j \in N_i} a_{ij} (x_j - x_i - \Delta_{ji}) + c\gamma \sum_{j \in N_i} a_{ij} (v_j - v_i)$$

(1.1.14)

where $\Delta_{ji} \in R^n$ are the desired offsets of node $i$ from node $j$. This law contains both flock centering and collision avoidance aspects. See the Section on Higher-Order Consensus **.

Alternative methods to local cooperative protocols for implementing Reynolds’ rules include potential field approaches. The methods covered in the Chapter on Potential Field Motion Control **, including approaches for obstacle avoidance and goal seeking, are intimately related to the topics in this section.

Connectivity of Animal Groups and Vehicle Formations

The connectivity of the directed graphs just discussed depends on $r_{ij}$ the distances between nodes $i$ and $j$. As agents move, these distances change and so the neighbors in the collision avoidance region $N_i^c$ and the flocking region $N_i$ change. This is called a distance graph topology, and the graph has time-varying edges. Nevertheless, the discussion in the Section on Dynamically Changing Interaction Topologies ** reveals that the group will reach consensus if the edges vary in such a way that the union of graphs over each of an infinite set of time intervals has a spanning tree.

However, there is no guarantee that the edges will not split so that the graph is no longer strongly connected or has a spanning tree. For instance, on reaching an obstacle a flock of birds often splits into two groups, and reforms into a single flock on the other side. Many papers have been written about defining potential fields or motion protocols that guarantee the group does not lose connectivity [Lei Guo, etc.].

1.2 Leadership in Animal Groups on the Move

We have seen that information can be transferred locally between individual neighboring members of animal groups, yet result in collective synchronized motions of the whole group. Local motion control protocols are based on a few simple rules that are followed by all individuals. However, in many situations, the whole group must move towards some goal, such as along migratory routes or towards food sources. In these cases, only a few informed individuals may have pertinent information about the required directions of motion. In this section we will study how a small percentage of informed individuals can have a global impact that results in control of the whole group towards desired goals. The material in this section is from [Couzin 2005].

Some species have evolved specialized mechanisms for conveying information about location and direction of goals. One example is the waggle dance of the honeybee that recruits
hive members to visit food sources. However, in many groups such as schools of fish, bird flocks, and mammal herds on the move evidence supports the idea that the decisions made by all individuals follow simple local protocols based on nearest neighbors similar to Reynolds’ Rules. Mechanisms of information transfer in groups involve questions such as how information about required motion directions, originally held by only a few informed individuals, can propagate through an entire group by simple mechanisms that are the same for every individual. It is not clear how individuals recognize leaders who are informed, or even if this is required, or how conflict is resolved when several informed individuals differ in their motion preferences.

**Protocols for Leadership by a Small Percentage of Informed Individuals**

The setup used in [Couzin 2005] is as follows. Consider a set of \( N \) agents \( \{ i \in N \} \) and let the communication between the agents be modeled by a general directed graph. The edges of the graph correspond to agents that are close together in some sense depending on the specific animal group considered, as discussed in the previous section. Each agent has a position vector, a direction vector, and a speed of motion. One dynamics that captures this for motion in the \((x, y)\)-plane is the discrete-time dynamics

\[
\begin{align*}
p_i(k+1) &= p_i(k) + V_i(k)T \cos \theta_i(k) \\
q_i(k+1) &= q_i(k) + V_i(k)T \sin \theta_i(k)
\end{align*}
\]

where \( V_i \) is the speed of agent \( i \), \( \theta_i \) its heading, and \( x_i = [p_i, q_i]^T \) its position. The time index is \( k \) and the sampling period is \( T \). The direction vector of agent \( i \) is

\[
v_i(k) = [V_i(k) \cos \theta_i(k) \quad V_i(k) \sin \theta_i(k)]^T
\]

Each agent turns away from others in a collision avoidance region \( N_i^c \) according to

\[
\theta_i(k+1) = -\sum_{j \in N_i^c} c_{ij} \frac{(x_j(k) - x_i(k))}{|x_j(k) - x_i(k)|}
\]

which causes agent \( i \) to turn away from other agents inside the collision neighborhood. If agents are not detected within the collision avoidance region, then each agent will turn towards and align with its neighbors in an interaction region \( N_i \) according to

\[
\theta_i(k+1) = \sum_{j \in N_i \setminus N_i^c} c_{ij} \frac{(x_j(k) - x_i(k))}{|x_j(k) - x_i(k)|} + \sum_{j \in N_i \setminus N_i^c} c_{ij} \frac{v_j(k)}{|v_j(k)|}
\]

This is converted to a unit vector by

\[
\hat{\theta}_i(k+1) = \frac{\theta_i(k+1)}{|\theta_i(k+1)|}
\]

A percentage \( p \) of the group consists of informed individuals \( i \), each of whom have a preferred desired direction of motion given by a unit reference direction vector \( r_i \). The heading control protocols are therefore taken as

\[
\bar{\theta}(k+1) = \frac{\hat{\theta}_i(k+1) + g_i r_i}{|\hat{\theta}_i(k+1) + g_i r_i|}
\]
with $g_i \geq 0$ a leader weighting gain. If $g_i = 0$, agent $i$ is not informed, exhibits no preferred direction of travel, and merely aligns with his neighbors. If $g_i > 0$, agent $i$ is an informed individual and, as $g_i$ increases, agent $i$ exhibits a stronger desire to move in its reference direction $r_i$.

1.3 Human Crowd behavior During Panic Situations

Equation Section (Next)
Vicsek paper

1.4 Motion in Physical Particle Systems

Equation Section (Next)
Vicsek

1.5 Kuramoto Oscillators

Equation Section (Next)

$N$ Kuramoto oscillators have dynamics

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_j \sin(\theta_j - \theta_i) \quad (1.5.1)$$

with oscillation frequency $\omega_i$ and coupling gain $K$ [Strogatz 2000]. The oscillators are said to synchronize if $\dot{\theta}_i(t) - \dot{\theta}_j(t) \to 0$ as $t \to \infty \forall i, j$.

Kuramoto took the oscillation frequencies as distributed according to a probability density function. He showed that there is a coupling gain $K_c$ below which the oscillators remain incoherent, i.e. do not synchronize. Above this gain the incoherent state becomes unstable, and the oscillators split into two groups—those with oscillation frequencies close to the mean synchronize to a mean frequency $\bar{\theta}$, while the others drift relative to this group.

In [Chopra and Spong CDC 2005], synchronization is studied using the Lyapunov function $V = \frac{1}{2} \theta^T \dot{\theta}$, with $\theta = [\theta_1 \cdots \theta_N]^T$. It is shown that $\dot{V} = -\frac{K}{N} \sum_{i,j} \cos(\theta_i - \theta_j)(\dot{\theta}_i - \dot{\theta}_j)^2$. It is shown that above some coupling gain, and if the initial phase differences are in a certain compact set, the oscillators converge to the mean frequency $\bar{\theta} = \frac{1}{N} \sum_i \omega_i$. 

Equation Section (Next)

1.6 Synchronization of Electric Power Systems

In this section we apply the concepts of synchronization of coupled systems to distributed electric power systems control. Consider a power system with $N$ generators. The dynamics of each node $i$ are given by the flux decay mechanical and generator electrical equations, which have the basic form [Ortega 2005]

$$\dot{\delta}_i = \omega_i$$
$$\dot{\omega}_i = -D_i \omega_i + \frac{\omega_m}{M_i} P_{mi} - E_i \sum_{j \in N_i} E_j Y_{ij} \sin(\delta_i - \delta_j) \quad (1.6.1)$$
$$\dot{E}_i = -a_i E_i + u_i + b_i \sum_{j \in N_i} E_j \cos(\delta_i - \delta_j)$$

and the turbine and valve dynamics

$$\dot{P}_{mi} = -c_i P_{mi} + d_i X_{Ei} \quad (1.6.2)$$
$$\dot{X}_{Ei} = h_i X_{Ei} + g_i (P_c - \frac{1}{\omega} \omega)$$

The power flow terms are summations over the neighborhood of node $i$, which consists of all generators connected via transmission lines to node $i$. We have assumed the lossless case for simplicity. These equations have both a field excitation control $u_i$ and a power control input $P_c$.

The power system transient stability problem occurs at a fast time scale, and is concerned with selecting $u_i$ for fault recovery. For this problem the mechanical power $P_{mi}$ (which varies more slowly) is generally assumed constant, so only the three flux equations are considered. Classical techniques design AVR and PSS to damp interarea oscillations. Linear design methods are often used. Soos [2002] used optimal control techniques. Jiang and Dorsey [1994] used Lyapunov methods to design local adaptive controllers. In Wang [1997] a nonlinear feedback linearization method was used. Neural network learning control was used in Cheng [1997]. Pourboghrat [2004] uses sliding mode methods. Ortega [2005] used a nonlinear passivity based approach for general lossy systems. Partial differential equations were solved for the controls, which have the form

$$u_i = k_1 E_i - k_2 \omega_i - b \sum_{j \in N_i} E_j \cos(\delta_i - \delta_j) + \sum_{j \in N_i} k_{ij} \omega_j \quad (1.6.3)$$

where $k_{ij}$ are nonlinear functions.

On the other hand, the load frequency control (LFC) problem generally ignores the fast exciter dynamics (third equation) and uses linear methods to design the power control input $P_c$. ACE is often used. The literature is rich. Wang [1994] used a robust adaptive control method based on solution of a local decoupled Riccati equation. Geromel [1985] uses output feedback design to get decentralized local LFC controllers.

Venayagamoorthy [2003, 2004] has used nonlinear neurocontrollers, based specifically on DHP, for power system stability control in a multiarea system. The controllers only required local measurements. He has achieved superior results and implemented his controller on microalternators at Univ. of Natal in Durban.

Recently proposed controllers based on FACTS allow the control of real and/or reactive power flow in AC transmission lines by modifying the reactance. Li [2006] designed a FACTS controller using feedback linearization. Mishra [2006] designed a NN FACTS controller. Ray
and Venayagamoorthy [preprint 2006] has designed a GCSC FACTS optimal controller based on HDP. Recurrent NN are used to identify the power system dynamics.

Synchronous, renewable, and CHP generators have different dynamics, yet they share the basic swing equation for rotating generators, given for the $i$-th generator as (assuming the lossless case)

$$\dot{\omega}_i = -D_i \omega_i + \frac{\alpha_a}{M_i} P_m - E_i^2 G_u - P_{ei} = -D_i \omega_i + \frac{\alpha_a}{M_i} P_m - E_i^2 G_u - E_i \sum_{j \in N_i} E_j Y_{ij} \sin(\delta_i - \delta_j)$$  \hspace{1cm} (1.6.4)

$$\dot{E}_i = -a_i E_i + u_i + b_i \sum_{j \in N_i} E_j \cos(\delta_i - \delta_j)$$  \hspace{1cm} (1.6.5)

with frequency $\omega_i = \dot{\delta}_i$, mechanical power $P_{mi}$, electrical power $P_{ei}$, and admittance $Y_{ij}$ capturing the networked interconnection structure between generators. A frequency equilibrium is characterized by frequency synchronization $\omega_i = \omega_j$, $\forall i, j$ and balanced power flow $Q_i(\delta) = P_m - P_{ei} - E_i^2 G_u = 0$, $\forall i$. In [Dorfler 2010] it was shown that these interconnected systems are generalized Kuramoto oscillators [Kuramoto 1984], and conditions for synchronization to a common frequency are given.
References

