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Cooperative Control of Multi-Agent Systems on Communication Graphs

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http://www.UTA.edu/UTARI/acs
He who exerts his mind to the utmost knows nature's pattern. The way of learning is none other than finding the lost mind. Man's task is to understand patterns in nature and society.
It is man’s obligation to explore the most difficult questions in the clearest possible way and use reason and intellect to arrive at the best answer.

Man’s task is to understand patterns in nature and society.

The first task is to understand the individual problem, then to analyze symptoms and causes, and only then to design treatment and controls.

Ibn Sina 1002-1042
(Avicenna)
Patterns in Nature and Society
1. Natural and biological structures

Many of the beautiful pictures are from a lecture by Ron Chen, City U. Hong Kong
Pinning Control of Graphs
Distribution of galaxies in the universe
Front and back covers: Diagrams of the Abstract Syntax Tree (AST) internal representation of a simple C++ program. The firework burst patterns are namespaces and different parts of the internal classes, functions and code within functions. The front cover shows the AST for source code, while the back cover shows a similar AST used to represent the structure of binary executables for analysis.

Source: Dan Quinlan, Center for Applied Scientific Computing, Lawrence Livermore National Laboratory
The Egyptian Twitter Network
Arab users in Red, English users in purple
J.J. Finnigan, Complex science for a complex world

The internet

ecosystem
Professional Collaboration network
Barcelona rail network
Airline Route Systems
2. Motions of biological groups

Fish school

Birds flock

Locusts swarm

Fireflies synchronize
Figure 1: Simulation of pedestrians moving with identical desired velocity $v_d = v_o$ towards the 1-m-wide exit of a room of size $15 \times 15$ m. 

- **a:** Snapshot of the simulation. Dynamic simulations are available at http://angel.elte.hu/~panic/
- **b:** Leaving times of pedestrians for various desired velocities $v_o$. Irregular outflow due to clogging is observed for high desired velocities ($v_o \geq 1.5$ m s$^{-1}$, red plus signs).
- **c:** Under conditions of normal walking, the time for 200 pedestrians to leave the room decreases with growing $v_o$. Desired velocities higher than 1.5 m s$^{-1}$ reduce the efficiency of leaving, which becomes particularly clear when the outflow $J$ is divided by the desired velocity $v_d$. This is due to pushing, which causes additional friction effects. Moreover, above a desired velocity of about $v_o = 5$ m s$^{-1}$ (corresponding to dashed lines in c and d) people are injured and become non-moving obstacles for others, if the sum of the magnitudes of the radial forces acting on them divided by their circumference exceeds a pressure of 1,600 N m$^{-1}$ (ref. 5).

Owing to the above ‘faster-is-slower effect’, panics can be triggered by pedestrian counterflows, which cause delays to the crowd intending to leave. This makes the stopped pedestrians impatient and pushy which may be described by increasing the desired velocity according to $\dot{v}_d(t) = [1 - \rho(t)]\dot{v}_d(0) + \rho(t)v_{d}^{\text{max}}$, where $\dot{v}_d(0)$ is the initial, and $v_{d}^{\text{max}}$ the maximum desired velocity. The time-dependent parameter $\rho(t) = 1 - \bar{v}_d(t)/v_{d}^{\text{max}}$, where $\bar{v}_d(t)$ denotes the average speed in the desired direction of motion, is a measure of impatience. Altogether, long waiting times increase the desired velocity, which can produce inefficient outflow. This further increases the waiting times, and so on, so that this tragic feedback can eventually trigger panics. It is therefore imperative to have sufficiently wide exits and to prevent counterflows when big crowds want to leave.

Helbring, Farkas, Vicsek, Nature 2000
Communication Graph

N nodes (agents) interconnected by communication links. Each agent can only get information from its neighbors.

\[ N_i \quad \text{In-neighbors of node } i \]

Each agent has dynamics

\[ \dot{x}_i = Ax_i + Bu_i \]

Study the interaction of control and communication
1. Random Graphs – Erdos and Renyi

N nodes
Two nodes are connected with probability $p$ independent of other edges

**Phase Transition**

$m = \text{number of edges}$

There is a critical threshold $m_0(n) = N/2$ above which a large connected component appears – giant clusters
J.J. Finnigan, Complex science for a complex world

Connectivity - degree distribution is Poisson
Homogeneity - all nodes have about the same degree

Poisson degree distribution
most nodes have about the same degree
ave(k) depends on number of nodes
2. Small World Networks - Watts and Strogatz

Start with a regular lattice
With probability $p$, rewire an edge to a random node.

Connectivity - degree distribution is Poisson
Homogeneity – all nodes have about the same degree

Small diameter (longest path length)
Large clustering coeff. - i.e. neighbors are connected

Regular

Small-world

Random

$p = 0$ Increasing randomness $p = 1$

Watts & Strogatz, Nature 1998
Phase Transition
Diameter and Clustering Coefficient

Clustering coefficient

Nr of neighbors of i = 4
Max nr of nbr interconnections = 4x3/2 = 6
Actual nr of nbr interconnects = 2
Clustering coeff = 2/6 = 1/3

Figure 2 Characteristic path length \( L(p) \) and clustering coefficient \( C(p) \) for the family of randomly rewired graphs described in Fig. 1. Here \( L \) is defined as the number of edges in the shortest path between two vertices, averaged over all pairs of vertices. The clustering coefficient \( C(p) \) is defined as follows. Suppose that a vertex \( v \) has \( k_v \) neighbours; then at most \( k_v(k_v - 1)/2 \) edges can exist between them (this occurs when every neighbour of \( v \) is connected to every other neighbour of \( v \)). Let \( C_v \) denote the fraction of these allowable edges that actually exist. Define \( C \) as the average of \( C_v \) over all \( v \). For friendship networks, these
3. Scale-Free Networks—Barabasi and Albert

Start with $m_0$ nodes
Add one node at a time:
    connect to $m$ other nodes
with probability

$$P(i) = \frac{d_i + 1}{\sum_j (d_j + 1)}$$

i.e. with highest probability to biggest nodes
(rich get richer)

Nonhomogeneous- some nodes have large degree, most have small degree
Scale-Free- degree has power law degree distribution

$$P(k) = \frac{2m^2}{k^3}$$
4. Proximity Graphs

Randomly select $N$ points in the plane
Draw an edge $(i,j)$ if distance between nodes $i$ and $j$ is within $d$

When is the graph connected?
for what values of $(N,d)$
What is the degree distribution?

Mobile Sensor Networks Project with Singapore A-Star
N nodes (agents) interconnected by communication links. Each agent can only get information from its neighbors.

Each agent has dynamics \( \dot{x}_i = Ax_i + Bu_i \)

Study the interaction of control and communication
The Interaction Between Communication and Control

The way we communicate decides the way we evolve dynamically
Flocking

Reynolds, Computer Graphics 1987

Reynolds’ Rules:
Alignment: align headings
\[ \dot{\theta}_i = \sum_{j \in N_i} a_{ij} (\theta_j - \theta_i) \]

Cohesion: steer towards average position of neighbors- towards c.g.
Separation: steer to maintain separation from neighbors
The Power of Synchronization

Coupled Oscillators

Diurnal Rhythm
Synchronization on Good Graphs

Chris Elliott fast video

Regular mesh
Synchronization on Gossip Rings

Chris Elliott weird video
Distributed Adaptive Control for Multi-Agent Systems
Consensus Control for Swarm Motions

\[ \dot{\theta}_i = \sum_{j \in N_i^c} a_{ij}(\theta_j - \theta_i) \]  
heading angle

\[ \dot{x}_i = V \cos \theta_i \]
\[ \dot{y}_i = V \sin \theta_i \]

Convergence of headings

Nodes converge to consensus heading
Consensus Control for Formations

Formation - a Tree network

Heading Consensus using Equations (21) and (22)

\[ \dot{\theta}_i = \sum_{j \in N_i^c} a_{ij} (\theta_j - \theta_i) \quad \text{heading angle} \]

\[ \dot{x}_i = V \cos \theta_i \]

\[ \dot{y}_i = V \sin \theta_i \]

Nodes converge to heading of leader
Define $\xi_{ij}$ as the trust that node i has for node j

$$\xi_{ij} \in [-1, 1]$$

-1 .................. 0 .................. 1
Distrust no opinion complete trust

Define trust vector of node i as

$$\xi_i = \begin{bmatrix} \xi_{i1} \\ \xi_{i2} \\ \vdots \\ \xi_{iN} \end{bmatrix} \in R^{N^2}$$

Trust node i has for node 3

N vector

Standard local voting protocol

$$\dot{\xi}_i = u_i = \sum_{j \in N_i} a_{ij} (\xi_j - \xi_i)$$

Difference of opinion with neighbors

Closed-loop trust dynamics

$$\dot{\xi} = -(L \otimes I_N)\xi$$
Trust Propagation & Consensus

Nodes 1, 2, 4 initially distrust node 5
initial trusts are negative

Other nodes agree that node 5 has negative trust

Convergence of trust
Trust-Based Control: Swarms/Formations

Trust dynamics
\[ \dot{\xi}_i = \sum_{j \in N_i} a_{ij} (\xi_j - \xi_i) \]

Motion dynamics
\[ \dot{\theta}_i = \sum_{j \in N_i^c} \xi_{ij} a_{ij} (\theta_j - \theta_i) \]
\[ \dot{x}_i = V \cos \theta_i \]
\[ \dot{y}_i = V \sin \theta_i \]

Convergence of trust
Convergence of headings
Nodes converge to consensus heading
Causes Unstable Formation

Trust-Based Control: Swarms/Formations

Malicious Node

\[ \dot{\theta}_i = \sum_{j \in N_i^c} \xi_{ij} a_{ij} (\theta_j - \theta_i) \]

Node 5 injects negative trust values

Internal attack
Malicious node puts out bad trust values
i.e. false information
c.f. virus propagation

Divergence of trust
Divergence of headings

Causes Unstable Formation
Trust-Based Control: Swarms/Formations

**CUT OUT Malicious Node**

\[ \dot{\theta}_i = \sum_{j \in N_i^c} \xi_{ij} a_{ij} (\theta_j - \theta_i) \]

-hearing angle

Node 5 injects negative trust values

If node 3 distrusts node 5, Cut out node 5

Other nodes agree that node 5 has negative trust

Convergence of trust

Convergence of headings

Restabilizes Formation

Work by Sajal Das
Balancing HVAC Ventilation Systems

Work with SIMTech – Singapore Inst. Manufacturing Technology

SIMTech 5th floor temperature distribution
Automated VAV control system

LEGENDS
- VAV box
- Room thermostat
- Air diffuser
- Extra WSN temp. sensors
Adjust Dampers for desired Temperature distribution

Temperature dynamics

\[ x_i(k + 1) = x_i(k) + f_i(x) + u_i(k) \quad \text{Unknown } f_i(x) \]

Control damper position based on local voting protocol

\[ u_i(k) = \frac{1}{n_i + 1} \gamma_i(k) \sum_{j \in N_i} a_{ij} (x_j(k) - x_i(k)) \]

\[ \gamma_i(k) = 1, \frac{1}{2}, \frac{1}{4}, \ldots \]

Under certain conditions this converges to steady-state desired temp. distribution

Open Research Topic - HVAC Flow and Pressure control
Herd and Panic Behavior During Emergency Building Egress
Work with Bari Polytechnic University, Italy (David Naso) – comes April 24

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Helbring, Farkas, Vicsek, Nature 2000
Crowd Panic Behavior
Modeling Crowd Behavior in Stress Situations

\[ m_i \frac{dv_i}{dt} = m_i \frac{v_i^0(t)e_i^0(t) - v_i(t)}{\tau_i} + \sum_{j \neq i} f_{ij} + \sum_{W} f_{iW} \]

desired speed \( v_i^0 \) in a certain direction \( e_i^0 \),

\[ f_{ij} = \left\{ \begin{array}{l} A_i \exp\left[\frac{(r_{ij} - d_{ij})}{B_i}\right] + kg(r_{ij} - d_{ij}) \right\} n_{ij} + kg(r_{ij} - d_{ij})\Delta v_{ij} \]

where the function \( g(x) \) is zero if the pedestrians do not touch each other \( (d_{ij} > r_{ij}) \), and is otherwise equal to the argument \( x \).
Synchronization

\[ \dot{x}_i = f_i(x_i) + g(x_i)u_i \]

\[ y_i = h(x_i) \quad \text{passive} \]

\[ V_i(x(t)) - V_i(x(0)) \leq \int_0^T u_i^T(s)y_i(s) \, ds \]

Storage function

Synchronize if \( y_i(t) \to y_j(t), \ \text{all } i, j \)

Local voting protocol with OUTPUT FEEDBACK

\[ u_i = \sum_{j \in N_i} K(y_j - y_i) \]

Result -
Let the communication graph be balanced. Then the agents synchronize.

Circadian rhythm
Synchronization: Ron Chen – Pinning Control

Connected undirected graphs \( \dot{x}_i = f(x_i) + u_i, \quad y_i = Cx_i \)

Leader node dynamics \( \dot{x}_0 = f(x_0) \)

\[ d_i = \sum_{j \neq i} a_{ij} = \sum_{j \neq i} a_{ji} \quad \text{In-degree = out-degree} \]

\[ a_{ii} = -d_i \quad \text{Diffusivity condition} \]

Pinning Control – inputs to some nodes

\[ \dot{x}_i = f(x_i) + c \sum_j a_{ij} C(x_j - x_i) + c b_i C(x_0 - x_i) \]

\[ \dot{x} = \left( f(x) - c(L + B) \otimes Cx \right) + cB \otimes C \mathbb{1} x_0 \]

\[ L + B = D + B - A \quad \text{has e-vals } \lambda_i \]

Results –

Node motions synchronize if \( \left( \frac{\partial f(x)}{\partial x} - c \lambda_i C \right) \) is stable

Pin to the biggest node = highest degree node= highest social standing- c.f. Baras

Must have control gain \( c \) big enough
Synchronization of Chaotic node dynamics – Ron Chen

Pinning control of largest node (for increasing coupling strengths)

Chen’s attractor node dynamics

Fig. 8. Specifically pinning the biggest node of 19 degrees in a 50-node network generated by the B–A scale-free model: (a)–(d) are stabilizing phases with different coupling strengths. (a) $c = 0$, $\delta = 0$, (b) $c = 10$, $\delta = 1000$, (c) $c = 15$, $\delta = 1000$, (d) $c = 20$, $\delta = 1000$. 
Our revels now are ended. These our actors,
As I foretold you, were all spirits, and
Are melted into air, into thin air.

The cloud-capped towers, the gorgeous palaces,
The solemn temples, the great globe itself,
Yea, all which it inherit, shall dissolve,
And, like this insubstantial pageant faded,
Leave not a rack behind.

We are such stuff as dreams are made on,
and our little life is rounded with a sleep.

Prospero, in The Tempest, act 4, sc. 1, l. 152-6, Shakespeare