UTA EE5306 PhD Diagnosis Exam (Fall 2011)  
Electromagnetics

Instructions:  
• Verify that your exam contains 7 pages (including the cover sheet).  
• Some space is provided for you to show your work. Only if more space is needed, you may show your work on the back of the exam sheet.  
• The point values listed on this exam serve only as a guideline. The Department reserves the right to make modifications to the weighting of the problems.  
• You may use a calculator.

I Choose to work on Problems _____ and _______ (Choose only 2 from the 3 problems).

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible Points</th>
<th>Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Total Score (Choose 2 Problems)</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. (50 points) There is an infinitely large plate made of perfect conductor and located at \( z = 0 \). To the left of \( z = 0 \), there is only air. An incident plane wave traveling in the region \( z < 0 \) has electric field

\[
E^i(\mathbf{r}) = (\sqrt{2} \hat{x} + 2j \hat{y} + E_{z_0} \hat{z})e^{-j\omega \sqrt{\epsilon_0 \mu_0} (z + x)/\sqrt{2}} \text{ (V/m)}.
\]

(a) Find \( E_{z_0} \).
(b) What is the polarization of the incident plane wave?
(c) When this incident wave hits the conducting plate, what is the electric field associated with the reflected wave?
(d) What is the polarization of the reflected wave?

---

Figure for the problem
2. (50 points) In an air-filled rectangular waveguide with dimensions \( a = 3\sqrt{2} \) cm and \( b = a / 2 \), the guided wave is given by

\[
\vec{E}(x,y,z,t) = \hat{y} E_0 \sin\left(\frac{\pi}{a} x\right) \sin\left(\omega t - \frac{\pi}{a} z\right)
\]

\[
\vec{H}(x,y,z,t) = \hat{x} H_0 \sin\left(\frac{\pi}{a} x\right) \sin\left(\omega t - \frac{\pi}{a} z\right) + \hat{z} H_0 \cos\left(\frac{\pi}{a} x\right) \cos\left(\omega t - \frac{\pi}{a} z\right)
\]

where \( E_0 \) and \( H_0 \) are real constants.

(a) What is the mode of this wave? Indicate \( \text{TE}_{mn} \) or \( \text{TM}_{mn} \) and the mode numbers \( m \) and \( n \).
(b) Determine the frequency of this wave.
(c) Determine the phase velocity in \( \hat{z} \) direction in terms of the light speed \( c \).
(d) Determine the cut-off frequency of this mode.
3. (50 points) A Hertzian dipole is made of two opposite charges ±q separated by an
infinitesimally small distance l. The electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ of a Hertzian at a
very large distances ($kr \ll 1$) in free-space are

$$\mathbf{E}(r, t) = -\hat{\theta} \frac{k^2 q l}{4 \pi \epsilon_0 r} \sin \theta \cos(\omega t - kr)$$

$$\mathbf{H}(r, t) = -\hat{\phi} \frac{\omega k q l}{4 \pi r} \sin \theta \cos(\omega t - kr)$$

(a) Find the Poynting vector $\mathbf{S}$ in time domain, and determine the time-averaged power density
vector $\langle \mathbf{S} \rangle$.

(b) Determine the average power $P$ radiated by the Hertzian dipole.

(c) Consider a radio station 15 km away from the city. The transmitting antenna tower may be
modeled as a Hertzian dipole antenna of dipole moment $ql$. Determine the radiation power $P_r$
needed to maintain a 25 mV/m field strength in the city.