UTA PhD Diagnosis Exam (Fall 2012)

Linear Systems / Controls

Instructions:
• Verify that your exam contains 7 pages (including the cover sheet).
• Please be sure to use blank paper to write your answers. If more space is needed, please ask the
instructor for extra paper. DO NOT WRITE ON THE BACK OF A SHEET!
• The point values listed on this exam serve only as a guideline. The Dept reserves the right to
make modifications to the weighting of the problems.
• Calculator is okay.

I Choose to work on Problems _____ and _______ (Choose only 2 from the 3
problems).

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<tr>
<th>Problem</th>
<th>Possible Points</th>
<th>Scores</th>
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<td>1</td>
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<td>2</td>
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<td>3</td>
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<td>Total Score (Choose 2 Problems)</td>
<td>100</td>
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1. Show all work, do by hand.

\[
\dot{x} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u = Ax + Bu, \quad y = \begin{bmatrix} 3 & 1 \end{bmatrix} x = Cx
\]

a. Find poles.
b. Find resolvent matrix \( \Phi(s) = (sI - A)^{-1} \) and state transition matrix \( \phi(t) = e^{At} \).
c. Find transfer function \( H(s) = C(sI - A)^{-1} B \).
d. Find impulse response.
e. Find output \( y(t) \) if \( u(t) = 3e^{-3t} u_{s+3}(t), \ x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \).
2. A system has the transfer function given by:

\[ H(s) = \frac{1}{s^3 + 3s^2 - 16s + 12}. \]

a. Is the system stable? Why?

b. Draw the canonical controllable form realization diagram for this system.

c. The system is connected in closed-loop using a PD controller. Draw a block diagram of the closed-loop system and compute the resulting closed loop transfer function.

d. Find the PD control gains necessary to place all closed loop poles at -1.
3. Consider a system with state-space system matrices
\[ A = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \end{bmatrix}, D = 0. \]

The system is controlled using full state-feedback of the form \( u(t) = -Kx + r \), where \( r \) is a reference input signal.

a. Show that the system is observable.

b. Show that there exist control gain matrices \( K \) making the closed-loop system unobservable.

c. Find one such gain matrix of the form \( K = [1 \ k2] \), for which the closed-loop system is unobservable and compute the closed-loop transfer function with this gain.