UTA PhD Diagnosis Exam (Fall 2012)

Digital Signal Processing

Instructions:
- Verify that your exam contains 7 pages (including the cover sheet).
- Please be sure to use blank paper to write your answers. If more space is needed, please ask the instructor for extra paper. DO NOT WRITE ON THE BACK OF A SHEET!
- The point values listed on this exam serve only as a guideline. The Dept reserves the right to make modifications to the weighting of the problems.
- Calculator is okay.

I Choose to work on Problems _____ and _______ (Choose only 2 from the 3 problems).

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible Points</th>
<th>Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Total Score (Choose 2 Problems)</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Problem 1: [50 pts] Find Discrete Time Fourier Transforms (DTFTs) of the following sequences. Use closed forms if possible.

(a) [10 pts] (.5)^n u(n) where u(n) = 1 for n ≥ 0 and u(n) = 0 for n < 0.
(b) [10 pts] n ⋅ (.5)^n u(n)
(c) [10 pts] cos(3n) [u(n) – u(n-N)].
(d) [10 pts] δ(sin(3n))
(e) [10 pts] ((.5)^n/n!)-u(n)
Problem 2: [50 pts] Let an FIR (finite impulse response) filter have the impulse response \( h(n) = a^n [u(n) - u(n-N)] \), which is a\(^n\) for \( 0 \leq n \leq N-1 \) where \( |a| < \infty \). Two possible closed forms for \( H(z) \) are

\[
H_1(z) = \frac{1-a^N z^{-N}}{1-az^{-1}} , \quad H_2(z) = \frac{a^N z^{-N} - 1}{a z^{-1} - 1}
\]

(a) [12 points] Give the region of convergence (R.O.C.) for \( H(z) \) (not for \( H_1(z) \) or \( H_2(z) \)).

(b) [13 points] If long division is used to find \( h_1(n) \) from \( H_1(z) \), give the first \( h_1(n) \) term, the second \( h_1(n) \) term, and the last \( h_1(n) \) term found, in the order in which they are found. Is \( h_1(n) = h(n) \) ?

(c) [12 points] If long division is used to find \( h_2(n) \) from \( H_2(z) \), give the first \( h_2(n) \) term, the second \( h_2(n) \) term, and the last \( h_2(n) \) term found, in the order in which they are found. Is \( h_2(n) = h(n) \) ?

(d) [13 points] The stable difference equation from \( H_1(z) \) (assuming \( |a| < 1 \)) is

\[
y_1(n) = x(n) - a^N x(n-N) + a \cdot y_1(n-1)
\]

Solving this difference equation for \( a \cdot y_1(n-1) \), multiplying both sides by \( 1/a \), replacing \( n \) by \( (n+1) \) everywhere, and replacing \( y_1() \) by \( y_2() \) everywhere, give the difference equation for \( H_2(z) \). Is this difference equation stable?
Problem 3: [50 pts] Passing a discrete signal $x(n)$ through a discrete system $T$ yields an output signal $y(n)$. The system $T$ performs these operations:

1. It up-samples the signal $x(n)$ by 2, creating $x_1(n)$ as $x_1(n) = x(n/2)$ for $n$ even, $x_1(n) = 0$ for $n$ odd.
2. It filters $x_1(n)$ with a linear time-invariant (LTI) system whose impulse response is $h_1(n)$, creating $y_1(n)$,
3. Then the system down-samples $y_1(n)$ by 2, as $y(n) = y_1(2n)$.

(a) [10 points] Prove that up-sampling is not an LTI operation.

(b) [15 points] Convolve $h_1(n)$ with $x_1(n)$ to get $y_1(n)$. Express $y_1(n)$ in terms of $h_1(n)$ and $x(n)$.

(c) [15 points] Find $h(n)$, the impulse response of $T$, in terms of $h_1(n)$.

(d) [10 points] Is $T$ an LTI system? Why?