Problem 1: [50 pts] Find Discrete Time Fourier Transforms (DTFTs) of the following sequences. Use closed forms if possible.

(a) [10 pts] \((0.5)^nu(n)\) where \(u(n) = 1\) for \(n \geq 0\) and \(u(n) = 0\) for \(n < 0\).
(b) [10 pts] \(n(0.5)^nu(n)\)
(c) [10 pts] \(\cos(3n) [u(n) - u(n-N)]\).
(d) [10 pts] \(\delta(\sin(3n))\)
(e) [10 pts] \(((0.5)^n/n!)u(n)\)
Problem 2: [50 pts] Let an FIR (finite impulse response) filter have the impulse response \( h(n) = a^n [u(n) - u(n-N)] \), which is a^n for \( 0 \leq n \leq N-1 \) where \( |a| < \infty \). Two possible closed forms for \( H(z) \) are

\[
H_1(z) = \frac{1 - a^N z^{-N}}{1 - az^{-1}}, \quad H_2(z) = \frac{a^N z^{-N} - 1}{az^{-1} - 1} = \frac{a^{-N} - (N-1)}{1 - a^{-N} z^{-1}}
\]

(a) [12 points] Give the region of convergence (R.O.C.) for \( H(z) \) (not for \( H_1(z) \) or \( H_2(z) \)).

(b) [13 points] If long division is used to find \( h_1(n) \) from \( H_1(z) \), give the first \( h_1(n) \) term, the second \( h_1(n) \) term, and the last \( h_1(n) \) term found, in the order in which they are found. Is \( h_1(n) = h(n) \)?

(c) [12 points] If long division is used to find \( h_2(n) \) from \( H_2(z) \), give the first \( h_2(n) \) term, the second \( h_2(n) \) term, and the last \( h_2(n) \) term found, in the order in which they are found. Is \( h_2(n) = h(n) \)?

(d) [13 points] The stable difference equation from \( H_1(z) \) (assuming \( |a| < 1 \)) is

\[
y_1(n) = x(n) - a^N x(n - (N-1)) + a \cdot y_1(n-1)
\]

Solving this difference equation for \( a \cdot y_1(n-1) \), multiplying both sides by \( 1/a \), replacing \( n \) by \( (n+1) \) everywhere, and replacing \( y_1() \) by \( y_2() \) everywhere, give the difference equation for \( H_2(z) \). Is this difference equation stable?
Problem 3: [50 pts] Passing a discrete signal \( x[n] \) through a discrete system \( T \) yields an output signal \( y[n] \). The system \( T \) performs these operations:

1. It up-samples the signal \( x(n) \) by 2, creating \( x_1(n) \) as \( x_1(n) = x(n/2) \) for \( n \) even, \( x_1(n) = 0 \) for \( n \) odd.
2. It filters \( x_1(n) \) with a linear time-invariant (LTI) system whose impulse response is \( h_1(n) \), creating \( y_1(n) \),
3. Then the system down-samples \( y_1(n) \) by 2, as \( y(n) = y_1(2n) \).

(a) [10 points] Prove that up-sampling is not an LTI operation.
(b) [15 points] Convolve \( h_1(n) \) with \( x_1(n) \) to get \( y_1(n) \). Express \( y_1(n) \) in terms of \( h_1(n) \) and \( x(n) \), using no divisions by 2.
(c) [15 points] Find \( h(n) \), the impulse response of \( T \), in terms of \( h_1(n) \).
(d) [10 points] Is \( T \) an LTI system? Why?

10 (a) **Counter Example to LTI.**
   Given \( x(n) \), \( T[x(n)] = x_1(n) = x(n/2) \), \( x_1(2n-10) = x(\frac{n}{2} - 5) \)
   \( T[x(2n-10)] = x(\frac{n}{2} - 10) \neq x(\frac{n}{2} - 5) \)

15 (b) \( x_1(n) = \sum_{k=0}^{5} h_1(k)x_1(n-k) = \sum_{k=0}^{5} h_1(n-k)x(k) \)
   \( = \sum_{k=0}^{5} h_1(n-2k)x(k) \)

15 (c) \( y_1(2n) = \sum_{k=0}^{5} h_1(2(n-k))x(k) \)
   So \( h(n) = h_1(2n) \)

10 (d) **Yes LTI.** Because it's expressable as a convolution.