UTA PhD Diagnosis Exam (Spring 2012)

Random Signals and Noise

Instructions:
- Verify that your exam contains 10 pages (including the cover sheet).
- Some space is provided for you to show your work. If more space is needed, please ask the instructor for extra paper. DO NOT WRITE ON THE BACK OF A SHEET!
- The point values listed on this exam serve only as a guideline. The Dept reserves the right to make modifications to the weighting of the problems.
- Calculator is okay.

I Choose to work on Problems _____ and _______ (Choose only 2 from the 3 problems).

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1. (50 points) A stochastic counting process with independent-increments is said to be a Poisson random process, \( N(t) = N_t \), where \( t \geq 0 \), if

(i) \( P(N_0 = 0) = 1 \), and

(ii) for all \( 0 \leq t_0 < t \), and \( \lambda > 0 \),

\[
P(N_t - N_{t_0} = k) = \frac{\left( \lambda (t_1 - t_0) \right)^k}{k!} e^{-\lambda (t_1 - t_0)}
\]

Consider a Poisson random process \( N_t \), \( t \geq 0 \), with a mean arrival rate of \( \lambda = 5 \).

Provide an explicit expression for the trivariate joint probability

\[
P(N_{t_1} = n_1, N_{t_2} = n_2, N_{t_3} = n_3) \quad \text{with} \quad t_3 > t_2 > t_1.
\]
2. \( X(t) \) be a continuous time Gaussian Random process with mean function and covariance function given by:

\[
m_X(t) = 3t \quad \text{and} \quad C_X(t_1, t_2) = 9 \, e^{-2|t_1 - t_2|}
\]

a) Find \( P[X(3) < 6] \) \[Use attached Q table as needed\] 20 pts

b) \( P[X(1) + X(2) > 2] \) 25 pts

c) Is this a stationary, wide sense stationary or a non-stationary process? Justify your answer. 5 pts
3.

[50 pts] Let $X_n$ be a random sequence defined for $n \geq 1$, with initial pdf

$$f_X(x_0) := f_X(x_0; 0) = N(0, \sigma_0^2)$$

for a given $\sigma_0 > 0$ and transition pdf

$$f_X(x_n|x_{n-1}) := f_X(x_n|x_{n-1}; n, n - 1) \sim N(\rho x_{n-1}, \sigma_w^2)$$

with $|\rho| < 1$ and $\sigma_w > 0$.

(a) Determine the unconditional density of $X_n$ at an arbitrary time $n \geq 1$.
(b) Determine the limit of the variance of $X_n$ as $n \to \infty$. 