UTA PhD Diagnosis Exam (Spring 2012)

Digital Signal Processing

Instructions:
• Verify that your exam contains 10 pages (including the cover sheet).
• Some space is provided for you to show your work. If more space is needed, please ask the instructor for extra paper. DO NOT WRITE ON THE BACK OF A SHEET!
• The point values listed on this exam serve only as a guideline. The Dept reserves the right to make modifications to the weighting of the problems.
• Calculator is okay.

I Choose to work on Problems _____ and _______ (Choose only 2 from the 3 problems).

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Problem 1: [50 pts] Find $z$-transforms of the following sequences in closed form, and their regions of convergence (R.O.C.). For sequences containing $x()$, find the transforms in terms of $X()$, using real coefficients only. Assume the R.O.C. of $X(z)$ is $|z| > 0.5$

(a) [10 pts] $u(n-5)$ where $u(n) = 1$ for $n \geq 0$ and $u(n) = 0$ for $n < 0$.
(b) [10 pts] $n \cdot u(n-5)$
(c) [10 pts] $c^n x(n)$, where $c$ can be complex.
(d) [10 pts] $n^2 x(n)$
(e) [10 pts] $\frac{0.5}{n!} \cdot [(j)^n + (-j)^n] \cdot u(n)$
Problem 2: [50 pts] Let $x(n)$, $h(n)$ and $y(n)$ denote complex sequences with DTFTs $X(e^{j\omega})$, $H(e^{j\omega})$ and $Y(e^{j\omega})$. In parts (a) and (b), find frequency domain expressions for $C$.

(a) [13 points] \[ C = \sum_{n=-\infty}^{\infty} x(n) \cdot y^*(n) \]

(b) [12 points] \[ C = \sum_{n=-\infty}^{\infty} x(n) \cdot y(-n) \]

Find the numerical values of

(c) [13 points] \[ \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{1 - \frac{1}{2} e^{-j\omega}} \frac{1}{1 - \frac{1}{3} e^{j\omega}} d\omega \]

(d) [12 points] \[ \sum_{n=-\infty}^{\infty} \frac{\sin(.5n)}{\pi n} \cdot \frac{\sin(2n) - \sin(.25n)}{\pi n} \]
**Problem 3: [50 pts]** A linear system processes a discrete-time input signal as follows:

Step 1. upsampling the signal by 2,

Step 2. Convolving the result from Step 1 with a filter $h[n]$,

Step 3. Downsampling the result from Step 2 by 2 yielding the output signal.

Prove that the above system is LTI and determine the corresponding impulse response of the system in terms of $h[n]$. 
