UTA PhD Diagnosis Exam (Spring 2013)

Random Signals and Noise

Instructions:
- Verify that your exam contains 7 pages (including the cover sheet).
- Please be sure to use blank paper to write your answers. If more space is needed, please ask the instructor for extra paper. DO NOT WRITE ON THE BACK OF A SHEET!
- The point values listed on this exam serve only as a guideline. The Dept reserves the right to make modifications to the weighting of the problems.
- Calculator is okay.

I Choose to work on Problems ____ and _____ (Choose only 2 from the 3 problems).

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible Points</th>
<th>Scores</th>
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<td>1</td>
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(1) Consider a random process \( X(t) = A \sin(\omega t + \Psi) \) where \( A \) and \( \Psi \) are independent random variables. \( A \) is uniformly distributed on \([0, 1]\).

(a) Determine a probability density function of \( \Psi \) so that \( X(t) \) is Wide-Sense Stationary.

(b) Using the random variable \( \Psi \) from part (a), prove or disprove that \( X(t) \) is second-order ergodic.
(2) Let $X$ and $Y$ be Gaussian random variables with probability density functions $\mathcal{N}(a, b)$ and $\mathcal{N}(c, d)$, respectively. Suppose that $Z = X + Y$,

(a) Prove that $Z$ is also Gaussian.
(b) Find the conditional probability density function of $X$ given that $Z = 1$. 
(3) Consider a linear time invariant system shown in the figure below. Let $X(t)$ and $N(t)$ be WSS and mutually uncorrelated random processes with zero means. The power spectral densities (PSD) for $X(t)$ and $N(t)$ are $S_{XX}(f)$ and $S_{NN}(f)$, respectively. The frequency response of the system is $H(f) = \mathcal{F}\{h(t)\}$.

(a) Derive the PSD of the output $Y(t)$.

(b) Derive the cross-power spectral densities $S_{XY}(f)$ and $S_{YX}(f)$. 