In the table below, check which two problems you wish to be graded. Problems not selected will not be graded. If all three boxes happen to be marked, only the first two problems will be graded. If you need extra space to complete a problem do not write on the back. Place the ID number and the problem number on all additional sheets used. There should be 9 pages following this sheet.

Designation TABLE

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### Table 3.3 Carrier Action Equation Summary.

#### Equations of State

\[
\begin{align*}
\frac{\partial n}{\partial t} &= \frac{1}{q} \mathbf{v} \cdot \mathbf{J}_N + \frac{\partial n}{\partial t} \bigg|_{\text{thermal}} + \frac{\partial n}{\partial t} \bigg|_{\text{other processes}} \\
\frac{\partial p}{\partial t} &= -\frac{1}{q} \mathbf{v} \cdot \mathbf{J}_P + \frac{\partial p}{\partial t} \bigg|_{\text{thermal}} + \frac{\partial p}{\partial t} \bigg|_{\text{other processes}}
\end{align*}
\]

\[
\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L
\]

\[
\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L
\]

#### Current and R–G Relationships

\[
\mathbf{J}_N = J_{N_{\text{diff}}} + J_{N_{\text{drift}}} = q \mu_n n \mathbf{E} + qD_N \nabla n
\]

\[
\mathbf{J}_P = J_{P_{\text{drift}}} + J_{P_{\text{diff}}} = q \mu_p p \mathbf{E} - qD_P \nabla p
\]

\[
\mathbf{J} = \mathbf{J}_N + \mathbf{J}_P
\]

#### Key Parametric Relationships

\[
L_N = \sqrt{D_N \tau_n}
\]

\[
L_P = \sqrt{D_P \tau_p}
\]

\[
\frac{D_N}{\mu_n} = \frac{kT}{q}
\]

\[
\frac{D_P}{\mu_p} = \frac{kT}{q}
\]

\[
\tau_p = \frac{1}{c_p N_T}
\]

\[
\tau_n = \frac{1}{c_n N_T}
\]

#### Resistivity and Electrostatic Relationships

\[
\rho = \frac{1}{q(\mu_n n + \mu_p p)}
\]

\[
\rho = \frac{1}{q \mu_p N_A} \quad \text{... p-type semiconductor}
\]

\[
\rho = \frac{1}{q \mu_n N_D} \quad \text{... n-type semiconductor}
\]

\[
\mathcal{E} = \frac{1}{q} \frac{dE_q}{dx} = \frac{1}{q} \frac{dE_p}{dx} = \frac{1}{q} \frac{dE_i}{dx}
\]

\[
V = -\frac{1}{q} (E_c - E_{\text{ref}})
\]

#### Quasi-Fermi Level Relationships

\[
F_N = E_i + kT \ln \left( \frac{n}{n_i} \right)
\]

\[
J_N = \mu_n n \nabla F_N
\]

\[
F_p = E_i - kT \ln \left( \frac{p}{n_i} \right)
\]

\[
J_P = \mu_p p \nabla F_P
\]

\[
\frac{n(x)}{n(x')} = e^{\frac{q(\phi(x) - \phi(x'))}{kT}}
\]
\[ \mathbf{E} = -\nabla V \]
\[ \nabla \cdot \mathbf{E} = \sigma \varepsilon \]

\[ \gamma = \frac{1}{1 + \left( \frac{D_E}{D_B} \frac{L_B}{L_E} \frac{N_B}{N_E} \right) \frac{\sinh(W/L_B)}{\cosh(W/L_B)}} \quad (11.31) \]

\[ \alpha_T = \frac{1}{\cosh(W/L_B)} \quad (11.32) \]

\[ \alpha_{dc} = \gamma \alpha_T = \frac{1}{\cosh(W/L_B) + \left( \frac{D_E}{D_B} \frac{L_B}{L_E} \frac{N_B}{N_E} \right) \frac{\sinh(W/L_B)}{\cosh(W/L_B)}} \quad (11.33) \]

and

\[ \beta_{dc} = \frac{1}{\alpha_{dc}} - 1 = \frac{1}{\cosh(W/L_B) + \left( \frac{D_E}{D_B} \frac{L_B}{L_E} \frac{N_B}{N_E} \right) \frac{\sinh(W/L_B)}{\cosh(W/L_B)} - 1} \quad (11.34) \]

Expressions for the total emitter and collector currents are next obtained by simply adding their respective \( n \) and \( p \) components.

\[ I_B = qA \left[ \left( \frac{D_E}{L_B} n_{B0} + \frac{D_B}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right) (e^{qV_{EB}/kT} - 1) \right. \]
\[ - \left. \left( \frac{D_E}{L_B} p_{B0} \frac{1}{\sinh(W/L_B)} \right) (e^{qV_{EB}/kT} - 1) \right] \quad (11.35) \]

\[ I_C = qA \left[ \left( \frac{D_E}{L_B} p_{B0} \frac{1}{\sinh(W/L_B)} \right) (e^{qV_{EB}/kT} - 1) \right. \]
\[ - \left. \left( \frac{D_B}{L_C} n_{C0} + \frac{D_C}{L_B} p_{B0} \frac{\cosh(W/L_B)}{\sinh(W/L_B)} \right) (e^{qV_{EB}/kT} - 1) \right] \quad (11.36) \]

Finally, if desired, an explicit expression for the base current could be established employing \( I_B = I_E - I_C \).
Chapter 4 Formulas

\[ \phi = \frac{1}{q} (E_i - E_t) \]  
\[ E_c = \frac{d\phi}{dx} = -\frac{kT}{q} \frac{1}{n} \frac{dn}{dx} = \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \]  
\[ d\phi = \frac{kT}{q} \frac{dn}{n} \]  
\[ n = n_i \exp \left( \frac{q\phi}{kT} \right) \]  
\[ \frac{d^2 \phi}{dx^2} = -\frac{q}{\varepsilon_s} (p - n + N_d - N_a) = \frac{q}{\varepsilon_s} \left( 2n_i \sinh \frac{q\phi}{kT} + N_a - N_d \right) \]

Quasi-neutrality for gradual doping

\[ E_x = -\frac{kT}{q} \frac{1}{N_d} \frac{dN_d}{dx} \] or \[ \frac{kT}{q} \frac{1}{N_a} \frac{dN_a}{dx} \]

Since

\[ n = n_i e^{\frac{q\phi}{kT}} \]  
\[ \phi_n = \frac{kT}{q} \ln \frac{N_d}{n_i} \]  
\[ \phi_p = -\frac{kT}{q} \ln \frac{N_a}{n_i} \]  
\[ \phi_i = \phi_n - \phi_p = \frac{kT}{q} \ln \left( \frac{N_d N_a}{n_i^2} \right) \]

For an abrupt junction:

\[ x_d = x_p + x_n = \left[ \frac{2\varepsilon_s}{q} \left( \frac{1}{N_d} + \frac{1}{N_a} \right) (\phi_i - V_a) \right]^{1/2} \]

When the doping is symmetrical around the junction where:

\[ N_d - N_a = ax^b \]

\[ x_d = \left[ \frac{2^{b+1} (b+2) \varepsilon_s (\phi_i - V_a)}{qa} \right]^{1/(b+2)} \]

\[ E_{\text{max}} = \frac{2(\phi_i - V_a)}{x_d} \]

For np heterojunctions:

\[ \phi_i = \chi_2 - \chi_1 + \frac{\varepsilon_{\text{g2}}}{q} - \frac{kT}{q} \ln \frac{N_{c2} N_{v2}}{N_{d1} N_{a2}} \]

Even when \( \chi_1 \neq \chi_2 \):

\[ E_{c2} - E_{c1} = \varepsilon_{\text{g2}} - kT \ln \frac{N_{c1} N_{v2}}{N_{d1} N_{a2}} \]

General pm homojunction:

1
\[ N_d - N_n = ax^b \]
\[ C = A \left[ \frac{qae^{b+1}}{2^{b+1}(b + 2)\phi_1} \right]^{1/(b+2)} \times \frac{1}{(1 - V_a/\phi_1)^{1/(b+2)}} \]  \hspace{1cm} (4.3.8)
\[ C = \frac{C_0}{(1 - V_a/\phi_1)^\gamma} \]

where
\[ \gamma \equiv \frac{1}{b + 2} \]

Avalanche Breakdown:
\[ \alpha \approx KE\exp\left(-\frac{B}{E}\right) \]  \hspace{1cm} (4.4.13)
\[ M = \frac{1}{1 - (|V_R|/BV)^n} \]  \hspace{1cm} \(2 < n < 6\) \hspace{1cm} (4.4.14)
\[ E_{\text{max}} = \left( \frac{2qN_a|V_R|}{\epsilon_s} \right)^{1/2} \]  \hspace{1cm} (4.4.15)

From Sze, p. 104, an approximate breakdown voltage is:
\[ BV = 60 \left( \frac{E_g}{1.1} \right)^{3/2} \left( \frac{N_B}{10^{16}} \right)^{-3/4} \]  \hspace{1cm} abrupt junction \( N_B \) lightly doped side
\[ BV = 60 \left( \frac{E_g}{1.1} \right)^{6/5} \left( \frac{a}{3 \times 10^{20}} \right)^{-2/5} \]  \hspace{1cm} graded junction
Chapter 6 Formulas

For an npn transistor:

\[ J_p = 0 = q\mu_n p E_x - qD_p \frac{dp}{dx} \]  \hfill (6.1.1)

\[ J_n = qD_n \frac{d(np)}{dx} \frac{d}{p} \]  \hfill (6.1.4)

For short base and \( n(x) \) varies linearly in the base:

\[ J_n = J_s \left[ \exp \left( \frac{qV_{BC}}{kT} \right) - \exp \left( \frac{qV_{BE}}{kT} \right) \right] \]  \hfill (6.1.7)

where

\[ J_s = \frac{qD_n n_i^2}{x_B N_a B} \]  \hfill (6.1.8)

or where

\[ Q_B = q \int_0^{x_B} p \, dx \]  \hfill (6.1.13)

\[ J_s = \frac{q^2 n_i^2 D_n}{Q_B} \]  \hfill (6.1.15)

\[ G.N. = \int_0^{x_B} N_n(x) \, dx = \frac{Q_B}{q} = \frac{q n_i^2 D_n}{J_s} \]  \hfill (6.2.3)

\[ n'_n = n_{p0} \left( e^{qV_{BE}/kT} - 1 \right) \left( 1 - \frac{x}{x_B} \right) \quad 0 \leq x \leq x_B \]  \hfill (6.1.16)

Base recombination current:

\[ I_{RB} = \frac{qA_E}{\tau_n} \int_0^{x_B} n' \, dx \]  \hfill (6.2.5)

\[ = \frac{qA_E n_i^2 x_B}{2N_a \tau_n} \left[ \exp \left( \frac{qV_{BE}}{kT} \right) - 1 \right] \]  \hfill (6.2.6)

\[ \alpha_T = 1 - \left| \frac{I_{RB}}{I_{EB}} \right| = 1 - \frac{x_B^2}{2\tau_n D_n} = 1 - \frac{x_B^2}{2T_n^2} \]  \hfill (6.2.8)

Recombination of holes injected into the emitter results from the built-in field \( E_0 \) plus and added field, \( E_n \), that comes from \( V_{BE} \approx 0 \).

\[ J_n = q\mu_n (n_{p0} + p'_n)(E_0 + E_n) + qD_n \left( \frac{dn_{n0}}{dx} + \frac{dp'_n}{dx} \right) \]

\[ J_p = q\mu_p (p_{n0} + p'_p)(E_0 + E_n) - qD_p \left( \frac{dp_{n0}}{dx} + \frac{dp'_p}{dx} \right) \]  \hfill (6.2.12)

where \( p'_n = n'_p \ll n_0 \).

\[ J_n \approx q\mu_{mn} n_{n0} E_n + qD_n \frac{dp'_n}{dx} \]  \hfill (6.2.13)
\[ J_P \approx q\mu_P p' E_0 - qD_p \frac{dp'}{dx} \]

(6.2.15)

The hole current in the emitter is:

\[ I''_{PE} = - \frac{qD_p n_i^2 A_E \left( e^{qV_{BE}/kT} - 1 \right)}{\int_{x_n}^{x_E} N_{DE}(x) \, dx} \]

(6.2.18)'

or for uniform doping

\[ I''_{PE} = -\frac{qA_E n_i^2 D_{PE}}{N_{DE} x_E} \left( e^{qV_{BE}/kT} - 1 \right) \]

(6.2.10)

\[ \gamma = \frac{1}{1 + |I_{PE}|/|I_{nE}|} \]

(6.2.19)

\[ = \frac{1}{1 + (x_B N_{ab} D_{PE})/(x_E N_{de} D_{NB})} \]

(6.2.20)

\[ \alpha = \gamma \alpha_T \]

(6.2.21)

\[ = \frac{1}{1 + (GN_B D_{PE})/(GN_E D_{NB})} \]

Ebers-Moll Equations:

\[ I_E = -I_{ES} \left( e^{qV_{BE}/kT} - 1 \right) + \alpha_R I_{CS} \left( e^{qV_{BC}/kT} - 1 \right) \]

(6.4.7a)

\[ I_C = -I_{CS} \left( e^{qV_{AC}/kT} - 1 \right) + \alpha_F I_{ES} \left( e^{qV_{BE}/kT} - 1 \right) \]

(6.4.7b)

\[ I_S = \alpha_F I_{ES} = \alpha_R I_{CS} \]

(6.4.8)

\[ I_F = I_{ES} \left( e^{qV_{BE}/kT} - 1 \right) \]

(6.4.9a)

\[ I_R = I_{CS} \left( e^{qV_{BC}/kT} - 1 \right) \]

(6.4.9b)

\[ I_{CBD} = I_{CS}(1 - \alpha_R \alpha_F) \]

(6.4.15)

\[ I_{CEO} = \frac{I_{CBD}}{1 - \alpha_F} \]

(6.4.16)

\[ V_{CEsat} = \frac{kT}{q} \ln \frac{\text{Num}}{\text{Den}} \]

(6.4.17)

\[ \text{Num} = 1 - \frac{I_C}{I_B}(1 - \alpha_R) \]

\[ \text{Den} = \alpha_R \left[ 1 - \frac{I_C}{I_B} \left( \frac{1 - \alpha_F}{\alpha_F} \right) \right] \]

\[ BV_{CEO} = \frac{BV_{CBD}}{\beta^{1/m}} \quad m \approx 4 \]

(6.5.1)

For the HBT using (5.3.35) and (5.3.36):

\[ \gamma = \frac{1}{1 + J_P/J_n} \]
\[
\frac{J_0}{J_p} = \frac{D_p}{D_b} \frac{L_p N_d e^{\Delta \varepsilon_b / kT}}{L_n N_a}
\]

\[
J_n = \frac{q e^{\Delta \varepsilon_x / kT}}{\int_0^{W_B} \frac{N_{ab}}{D_n n_i^2} \, dx}
\]

(6.6.2)
Chapter 8 Formulas

The flat-band voltage for MOS structure is the difference of the work functions between the metal and the semiconductor.

\[V_{FB} = \Phi_M - \Phi_S = \Phi_{MS}\]  \hspace{1cm} (8.1.1)

In silicon, \(\phi_p\) and \(\phi_s\) are the bulk and surface potentials.

Drop across depletion region = \(\phi_s - \phi_p\)

\[x_d = \sqrt{\frac{2 \epsilon_S |\phi_p - \phi_s|}{qN_a}}\]  \hspace{1cm} (8.3.5)

Inversion begins with \(\phi_s = 2|\phi_p|\)

\[x_{d,max} = \sqrt{\frac{4 \epsilon_S |\phi_p|}{qN_a}}\]  \hspace{1cm} (8.3.6)

\[Q_{max} = -qN_n x_{d,max}\]  \hspace{1cm} (8.3.7)

Non-equilibrium with Gate, Channel, and Body voltages

\[x_{d,max} = \sqrt{\frac{2 \epsilon_S (2|\phi_p| + V_C - V_B)}{qN_a}}\]  \hspace{1cm} (8.3.8)

\[Q_d = -\sqrt{2 \epsilon_S qN_n (|\phi_p| V_C - V_B)}\]  \hspace{1cm} (8.3.9)

\[Q_s = Q_n + Q_d\]  \hspace{1cm} (8.3.14)

\[= -C_{ox}(V_C - V_{FB} - V_{bs}) - (\phi_s - \phi_p) - Q_d\]  \hspace{1cm} (8.3.15)

MOS capacitance under certain conditions:

\[C = \frac{1}{1/C_{ox} + 1/C_s}\]  \hspace{1cm} (8.4.4)

\[C_s = \frac{\epsilon_s}{x_d}, \quad C_{ox} = \frac{\epsilon_{ox}}{x_{ox}}\]

\[\Delta V_{FB} = -\frac{Q_{ox} x_1}{\epsilon_{ox}} = -\frac{1}{C_{ox}} \int_0^{x_1} \frac{x}{x_{ox}} \rho(x) \, dx\]  \hspace{1cm} (8.5.4)

\[V_{FB} = \Phi_{MS} - \frac{Q_l}{C_{ox}} - \frac{1}{C_{ox}} \int_0^{x_1} \frac{x}{x_{ox}} \rho(x) \, dx\]  \hspace{1cm} (8.5.6)
PROBLEM 1

A $pn$ junction has the doping profile sketched below. Throughout this problem, assume the carrier concentrations may be neglected ($n = 0$, $p = 0$) in the $0 \leq x \leq x_i$ region of the diode.

(a) What is the built-in voltage across the junction? Justify your answer.

(b) Invoking the depletion approximation, make a sketch of the charge density inside the diode. Label significant $\rho$ and $x$ values.

(c) Obtain an analytical solution for the electric field, $\mathcal{E}(x)$, at all points inside the depletion region ($-x_p \leq x \leq x_n$). Show all work and make a sketch of the deduced $\mathcal{E}(x)$ versus $x$.

(d) In a standard $pn$ step junction $N_A x_p = N_D x_n$. How are $x_n$ and $x_p$ related here?
PROBLEM 2

The energy band diagram given below characterizes a Si step junction diode maintained at room temperature. Note that $E_v(-\infty) = E_c(+\infty)$. Also, $x_n + x_p = 2 \times 10^{-4}$ cm, $A = 10^{-3}$ cm$^2$, $\tau_n = \tau_p = \tau_0 = 10^{-6}$ sec, $\mu_n(p$-side) = 1352 cm$^2$/V-sec, $\mu_p(n$-side) = 459 cm$^2$/V-sec, $K_S = 11.8$, and $\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm.

(a) What is the magnitude of the reverse-bias voltage ($V_A$) being applied to the diode? Explain how you arrived at your answer.

(b) Determine $V_{bi}$, the built-in voltage.

(c) Compute the recombination-generation current flowing through the diode at the pictured bias point.

(d) Compute the diffusion current flowing through the diode at the pictured bias point.
PROBLEM 3

The energy band diagram for a $p$-Si/SiO$_2$/n-Si (SOS) capacitor under flat-band conditions is given below. To achieve the pictured state, there must of course be a non-zero voltage applied to the SOS-C gate. The SOS-C is ideal except for a non-zero workfunction difference. $T = 300$ K, $N_A(p$-side) = $10^{15}$/cm$^3$, $N_D(n$-side) = $10^{15}$/cm$^3$, $n_i = 10^{10}$/cm$^3$, $x_0 = 5 \times 10^{-6}$ cm, and $A_G = 10^{-3}$ cm$^2$.

(a) What is the voltage being applied to the $p$-Si/SiO$_2$/n-Si SOS-C to achieve the pictured flat-band condition? Give both the polarity and magnitude of $V_G$.

(b) Sketch the energy band diagram and the associated block charge diagram for the SOS-C when a large positive gate voltage (say $V_G > 5$ V) is applied to the device. Add descriptive words to your sketches as necessary to forestall a misinterpretation of your answer.

(c) Sketch the energy band diagram and the associated block charge diagram for the SOS-C when a large negative gate voltage is applied to the device.

(d) Make a sketch of the high-frequency $C-V_G$ characteristic to be expected from the SOS-C described in this problem. Explain how you arrived at your $C-V_G$ sketch.